

Long-Wavelength Approximation Theory of Light Reflection from Nanoscale Anisotropic Layered Media

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ABSTRACT

The reflection of linearly polarized light from an N-layer system of anisotropic nanometer-size insulating films is investigated in the long-wavelength limit. The approximate expressions for the reflection characteristics of s- and p-polarized light for multilayer systems of anisotropic films are derived. All analytical results are supported by computer-aided analysis made on the basis of general wave propagation theory for anisotropic layered media. It is shown that the accuracy of the long-wavelength approximation for nanoscopic anisotropic layered systems is quite satisfactory: if the thickness of a multilayer divided by the wavelength comprises only a few hundredths, then the error of approximate expressions will be of the order of several percent. The most useful feature of obtained formulas is that they are simply invertible, allowing a direct calculation of the parameters of nanoscale layers.

Keywords: nanoscale anisotropic multilayers, insulating nanofilms, reflection theory

1 INTRODUCTION

Anisotropic ultrathin films and multilayers are of most interest in surface science and nanotechnology. A widespread way to probe nanometric layers is to employ optical reflection methods. But the use of exact equations to resolve the inverse problem; i.e., to determine the parameters of layered structures from reflection characteristics, is rather complicated owing to the complexity of the corresponding exact reflection equations. Generally, the solution of this problem for anisotropic systems can only be found by the use of numerical methods. However, when solving inverse problems with such techniques, one always comes across great difficulties, the main two being the instability and the non-uniqueness of the solution (e.g., strong parameter correlation appears between the thickness and the optical constants of the film). Moreover, since the inverse problem is essentially ill-posed, often this problem may not have a solution at all (or it is impossible to obtain a solution with the desired accuracy) through the presence of both systematic and random measurements errors.

It is also important to note that an analytical algorithm for calculating the parameters of interest directly from the measured data is very fast in comparison, e.g., with regressive type of algorithm. Although the latter is very general by nature, it tends to be computationally slow if more than a few variable parameters are involved. In view of this fact approximate analytical techniques are used to

provide initial guesses at the values of the variable parameters, and regression techniques are then used to fine-tune the desired parameters.

A purpose of this paper, first, is to investigate analytically the reflection characteristics in the long-wavelength limit for an N-layer system of nanoscale anisotropic insulating films on isotropic homogeneous substrates. The second aim is to show by numerical calculations on the basis of exact electromagnetic theory what is the level of accuracy of this approximation for anisotropic films.

So far, the ultrathin multilayers discussed in the literature (see, e.g., Refs. [1-3], and references therein) contain solely isotropic (in)homogeneous films. However, for anisotropic systems, the literature only considers several special one-layer cases with simple geometry (as a surface layer with uniaxial anisotropy [4], a biaxial surface layer with diagonal dielectric tensor [5,6], and a biaxial surface layer where only the Eulerian angle between the x axes of the layer and laboratory has a nonzero value [7,8]) in the long-wavelength approximation (the thickness d of an ultrathin layer is much smaller than an optical vacuum wavelength λ), and it only obtains the corresponding first-order algebraic expressions describing the contribution of the anisotropic insulating surface layer to the reflection characteristics of linearly polarized light from absorbing materials.

2 BASIC FORMULAS

Assuming that all the media are nonmagnetic, we consider the reflection of s- and p-polarized time-harmonic (the complex representation is taken in the form $\exp(-i\omega t)$, where $\omega = 2\pi c/\lambda$, c is vacuum speed of light) electromagnetic plane waves in an ambient medium with real isotropic and homogeneous dielectric constant $\epsilon_a \equiv n_a^2$ from a layered system with plane parallel interfaces consisting of N arbitrarily anisotropic homogeneous dielectric films of thickness $d_i \ll \lambda$ and with real principal dielectric-tensor components in the crystal-coordinate system $\epsilon_{xx}^{(i)}$, $\epsilon_{yy}^{(i)}$, and $\epsilon_{zz}^{(i)}$. The anisotropic films are numbered from ambient to substrate, i.e., index $i = 1, \dots, N$ so that the last film with index $i = N$ is located upon a semi-infinite absorbing, isotropic, and homogeneous substrate with complex dielectric constant $\epsilon_s = \epsilon_{sr} + i\epsilon_{si} \equiv n_s^2$. The orientations of the crystal axes are described by the Euler angles θ_i , φ_i , and ψ_i with respect to a fixed $x y z$ coordinate system (the Cartesian laboratory coordinate system). The laboratory x , y , and z axes are defined as follows. The reflecting surface is the

x y plane, and the plane of incidence is the z x plane, with the z axis normal to the surface of the layered medium and directed into it. The incident light beam in the ambient medium makes an angle ϕ_a with the z axis. The dielectric tensor for i th anisotropic layer in the xyz coordinate system is given by

$$\begin{bmatrix} \epsilon_{11}^{(i)} & \epsilon_{12}^{(i)} & \epsilon_{13}^{(i)} \\ \epsilon_{21}^{(i)} & \epsilon_{22}^{(i)} & \epsilon_{23}^{(i)} \\ \epsilon_{31}^{(i)} & \epsilon_{32}^{(i)} & \epsilon_{33}^{(i)} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \epsilon_{xx}^{(i)} & 0 & 0 \\ 0 & \epsilon_{yy}^{(i)} & 0 \\ 0 & 0 & \epsilon_{zz}^{(i)} \end{bmatrix} \mathbf{A}^{-1} \quad (1)$$

where \mathbf{A} is the coordinate rotation matrix [9].

We use the matrix method for calculating the contributions of ultrathin layers to the reflection characteristics. Since s and p modes are no longer spatially independent of each other (a so-called mode coupling appears) in the anisotropic medium, then, consequently, 4×4 matrices are needed. Dealing directly with first-order Maxwell equations, Berreman showed [10] a general way to calculate the reflection characteristics of an anisotropic layered system from a wave transfer matrix of rank 4 (note that the other way is to work with corresponding second-order wave equations [11]). By Berreman, the 4×4 characteristic matrix, $\mathbf{B}^{(i)}$, such that

$$\begin{bmatrix} E_x^{(i)} \\ H_y^{(i)} \\ E_y^{(i)} \\ -H_x^{(i)} \end{bmatrix} = \mathbf{B}^{(i)} \begin{bmatrix} E_x^{(i)} \\ H_y^{(i)} \\ E_y^{(i)} \\ -H_x^{(i)} \end{bmatrix}_{z=z_{i-1}}$$

or

$$\Psi^{(i)}(z_2) = \mathbf{B}^{(i)} \Psi^{(i)}(z_1), \quad (2)$$

where $z_0 = 0$, $z_i - z_{i-1} = d_i$ is the thickness of the i th layer and $E_{x,y}^{(i)}$ and $H_{x,y}^{(i)}$ are electric and magnetic field components parallel to the interfaces.

A differential form of the equation (2) may be written

$$\frac{\partial}{\partial z} \Psi^{(i)} = \frac{i2\pi}{\lambda} \mathbf{C}^{(i)}(z) \Psi^{(i)}, \quad (3)$$

$$\mathbf{C}^{(i)} = \begin{bmatrix} c_{11}^{(i)} & c_{12}^{(i)} & c_{13}^{(i)} & 0 \\ c_{21}^{(i)} & c_{22}^{(i)} & c_{23}^{(i)} & 0 \\ 0 & 0 & 0 & 1 \\ c_{31}^{(i)} & c_{32}^{(i)} & c_{33}^{(i)} & 0 \end{bmatrix}, \quad (4)$$

$$c_{11}^{(i)} = -n_a \sin \phi_a \epsilon_{13}^{(i)} / \epsilon_{33}^{(i)}, \quad c_{12}^{(i)} = 1 - \epsilon_a \sin^2 \phi_a / \epsilon_{33}^{(i)},$$

$$c_{13}^{(i)} = -n_a \sin \phi_a \epsilon_{23}^{(i)} / \epsilon_{33}^{(i)},$$

$$c_{23}^{(i)} = \epsilon_{12}^{(i)} - \epsilon_{13}^{(i)} \epsilon_{23}^{(i)} / \epsilon_{33}^{(i)},$$

$$c_{43}^{(i)} = \epsilon_{22}^{(i)} - (\epsilon_{23}^{(i)})^2 / \epsilon_{33}^{(i)} - \epsilon_a \sin^2 \phi_a.$$

A fitting procedure for finding $\mathbf{B}^{(i)}$ in the long-wavelength approximation is simply to integrate Eq. (10) and then to use the Taylor-series-like expansion, which in the first order with respect to the small parameter d_i / λ yields:

$$\Psi^{(i)}(z + d_i) = \mathbf{B}^{(i)}(d_i) \Psi^{(i)}(z) = \exp[i2\pi$$

$$\times \mathbf{C}^{(i)}(d_i / \lambda)] \Psi^{(i)}(z) = [\mathbf{I} + i2\pi \mathbf{C}^{(i)}(d_i / \lambda)] \Psi^{(i)}(z).$$

Hence the first-order expression for 4×4 characteristic matrix $\mathbf{B}^{(i)}$ takes the form:

$$\mathbf{B}^{(i)} = \begin{bmatrix} b_{11}^{(i)} & b_{12}^{(i)} & b_{13}^{(i)} & 0 \\ b_{21}^{(i)} & b_{22}^{(i)} & b_{23}^{(i)} & 0 \\ 0 & 0 & 1 & b_{34}^{(i)} \\ b_{41}^{(i)} & b_{42}^{(i)} & b_{43}^{(i)} & 1 \end{bmatrix}, \quad (5)$$

in which

$$b_{11}^{(i)} = 1 + i2\pi c_{11}^{(i)}(d_i / \lambda), \quad b_{12}^{(i)} = i2\pi c_{12}^{(i)}(d_i / \lambda),$$

$$b_{13}^{(i)} = i2\pi c_{13}^{(i)}(d_i / \lambda), \quad b_{21}^{(i)} = i2\pi c_{21}^{(i)}(d_i / \lambda),$$

$$b_{23}^{(i)} = i2\pi c_{23}^{(i)}(d_i / \lambda), \quad b_{34}^{(i)} = i2\pi c_{34}^{(i)}(d_i / \lambda),$$

$$b_{43}^{(i)} = i2\pi c_{43}^{(i)}(d_i / \lambda).$$

Note that $[\mathbf{B}^{(i)}(d_i)]^{-1} = \mathbf{B}^{(i)}(-d_i)$. It is clear that the characteristic transfer matrix \mathbf{B} for an N -layer system is equal to the matrix product of $\mathbf{B}^{(i)}$'s, i.e., $\mathbf{B} = \prod \mathbf{B}^{(i)}$. Thus, for the 4×4 transfer matrix \mathbf{B}^{-1} , which is of present interest, in the first order with respect to the small parameter d_i / λ one can obtain:

$$\mathbf{B}^{-1} =$$

$$\begin{bmatrix} 1 - i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{11}^{(i)} d_i & -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{12}^{(i)} d_i & -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{13}^{(i)} d_i & 0 \\ -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{21}^{(i)} d_i & 1 - i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{22}^{(i)} d_i & -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{23}^{(i)} d_i & 0 \\ 0 & 0 & 1 & -i \frac{2\pi}{\lambda} \sum_{i=1}^N d_i \\ -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{31}^{(i)} d_i & -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{32}^{(i)} d_i & -i \frac{2\pi}{\lambda} \sum_{i=1}^N c_{33}^{(i)} d_i & 1 \end{bmatrix}$$

2 REFLECTION COEFFICIENTS

The electromagnetic field in the isotropic ambient medium is made up of two parts, the incident- and the reflected-wave contributions, $\Psi^{(a)} = \Psi_I^{(a)} + \Psi_R^{(a)}$, and the field in the isotropic substrate matches solely a transmitted wave field, $\Psi^{(s)}$. From the matrix equation

$$\Psi_I^{(a)} + \Psi_R^{(a)} = \mathbf{B}^{-1} \Psi^{(s)}, \quad (6)$$

one can obtain four linear equations:

$$\begin{aligned} E_{Ix}^{(a)} + E_{Rx}^{(a)} &= b_{11} E_x^{(s)} + \frac{n_s}{\cos \phi_s} b_{12} E_x^{(s)} + b_{13} E_y^{(s)}, \\ \frac{n_a}{\cos \phi_a} (E_{Ix}^{(a)} - E_{Rx}^{(a)}) &= b_{21} E_x^{(s)} + \frac{n_s}{\cos \phi_s} b_{11} E_x^{(s)} + b_{23} E_y^{(s)}, \\ E_{Iy}^{(a)} + E_{Ry}^{(a)} &= E_y^{(s)} + n_s \cos \phi_s b_{34} E_y^{(s)}, \\ (n_a \cos \phi_a) (E_{Iy}^{(a)} - E_{Ry}^{(a)}) &= b_{23} E_x^{(s)} + \frac{n_s}{\cos \phi_s} b_{13} E_x^{(s)} + b_{43} E_y^{(s)} \\ &+ n_s \cos \phi_s E_y^{(s)}. \end{aligned}$$

We solve this system for the two cases, first, if $E_{Iy}^{(a)} \equiv 0$ (p-polarized incident wave) and, second, if $E_{Ix}^{(a)} \equiv 0$ (s-polarized incident wave). In the case of p-polarization we obtain for the complex amplitude reflection coefficients $r_{pp} = r_{xx} \equiv E_{Rx}^{(a)} / E_{Ix}^{(a)}$; $r_{ps} = r_{xy} \cos \phi_a$, $r_{xy} \equiv E_{Ry}^{(a)} / E_{Ix}^{(a)}$ the following results (in this paper the first subscript always indicates the incident light):

$$\begin{aligned} r_{pp} &\approx r_p^{(0)} + i 4\pi n_a \cos \phi_a \left\{ \sum_{i=1}^N [(1 - \varepsilon_a \varepsilon_s^{-1} \sin^2 \phi_a) (\varepsilon_{11}^{(i)} - [\varepsilon_{13}^{(i)}]^2 / \varepsilon_{33}^{(i)}) - \varepsilon_s (1 - \varepsilon_a [\varepsilon_{33}^{(i)}]^{-1} \sin^2 \phi_a)] (d_i / \lambda) \right\} \\ &\times (n_s \cos \phi_a + n_a \cos \phi_s)^{-2}, \quad (7) \end{aligned}$$

$$\begin{aligned} r_{ps} &\approx i 4\pi n_a \cos \phi_a \left\{ \sum_{i=1}^N [\cos \phi_s (\varepsilon_{12}^{(i)} - \varepsilon_{13}^{(i)} \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)}) - n_a n_s \sin \phi_a \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)}] (d_i / \lambda) \right\} \\ &\times (n_s \cos \phi_a + n_a \cos \phi_s)^{-1} (n_a \cos \phi_a + n_s \cos \phi_s)^{-1}, \quad (8) \end{aligned}$$

where $r_p^{(0)}$ is the amplitude reflection coefficient from bare ($d_i \equiv 0$) isotropic substrate and is expressed by the standard Fresnel's formula

$$r_p^{(0)} = (n_a \cos \phi_s - n_s \cos \phi_a) / (n_a \cos \phi_s + n_s \cos \phi_a). \quad (9)$$

Analogously, for s-polarized incident wave ($E_{Ix}^{(a)} \equiv 0$) one can obtain that

$$\begin{aligned} r_{ss} &= r_s^{(0)} + i 4\pi n_a \cos \phi_a \left[\sum_{i=1}^N (\varepsilon_{22}^{(i)} - [\varepsilon_{23}^{(i)}]^2 / \varepsilon_{33}^{(i)} - \varepsilon_s) (d_i / \lambda) \right] (n_s \cos \phi_s + n_a \cos \phi_a)^{-2}, \quad (10) \end{aligned}$$

$$\begin{aligned} r_{sp} &= i 4\pi n_a \cos \phi_a \left\{ \sum_{i=1}^N [\cos \phi_s (\varepsilon_{12}^{(i)} - \varepsilon_{13}^{(i)} \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)}) + n_a n_s \sin \phi_a \times \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)}] (d_i / \lambda) \right\} \\ &\times (n_s \cos \phi_a + n_a \cos \phi_s)^{-1} (n_a \cos \phi_a + n_s \cos \phi_s)^{-1}, \quad (11) \end{aligned}$$

$$r_s^{(0)} = (n_a \cos \phi_a - n_s \cos \phi_s) / (n_a \cos \phi_a + n_s \cos \phi_s). \quad (12)$$

For the reflectances $R_{pp} = |r_{pp}|^2$ and $R_{ss} = |r_{ss}|^2$ we obtain from Eqs. (7) and (10) the following expressions accurate to the first order in small parameters:

$$\begin{aligned} R_{pp} &\approx R_p^{(0)} \{1 + 8\pi n_a \cos \phi_a K \varepsilon_{sl} \sum_{i=1}^N [\varepsilon_a (1 - [\varepsilon_a / \varepsilon_{33}^{(i)}] \sin^2 \phi_a) - (\varepsilon_{11}^{(i)} - [\varepsilon_{13}^{(i)}]^2 / \varepsilon_{33}^{(i)}) \cos^2 \phi_a] (d_i / \lambda)\}, \quad (13) \end{aligned}$$

$$\begin{aligned} R_{ss} &\approx R_s^{(0)} \{1 + 8\pi n_a \cos \phi_a [(\varepsilon_a - \varepsilon_s)^2 + \varepsilon_{sl}^2]^{-1} \varepsilon_{sl} \\ &\times \sum_{i=1}^N (\varepsilon_a + [\varepsilon_{23}^{(i)}]^2 / \varepsilon_{33}^{(i)} - \varepsilon_{22}^{(i)}) (d_i / \lambda)\}, \quad (14) \end{aligned}$$

where $R_s^{(0)} = |r_s^{(0)}|^2$, $R_p^{(0)} = |r_p^{(0)}|^2$, $\alpha \equiv \varepsilon_{sR} / |\varepsilon_s|^2$, $|\varepsilon_s|^2 \equiv \varepsilon_{sR}^2 + \varepsilon_{sI}^2$, and

$$\begin{aligned} K &\equiv [1 - 2\varepsilon_a \alpha \sin^2 \phi_a] / [\{\varepsilon_a (1 - \varepsilon_a \alpha \sin^2 \phi_a) - \varepsilon_{sR} \cos^2 \phi_a\}^2 \\ &+ \varepsilon_{sI}^2 \{\cos^2 \phi_a - \varepsilon_a^2 |\varepsilon_s|^{-2} \sin^2 \phi_a\}^2]. \end{aligned}$$

The reflectances $R_{sp} = |r_{sp}|^2$ and $R_{ps} = |r_{ps}|^2$ are equal to zero in the first order in d_i / λ . The second-order formulas take the form:

$$\begin{aligned} R_{\sigma} &= 16\pi^2 \varepsilon_a \cos^2 \phi_a |n_s \cos \phi_a + n_a \cos \phi_s|^{-2} |n_s \cos \phi_s + n_a \cos \phi_a|^{-2} \\ &\times \sum_{i=1}^N \sum_{j=1}^N L_{\sigma}^{(i)} (L_{\sigma}^{(j)})^* (d_i d_j / \lambda^2), \quad (15) \end{aligned}$$

$$L_{\sigma}^{(i)} = \cos \phi_s (\varepsilon_{12}^{(i)} - \varepsilon_{13}^{(i)} \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)}) + P_{\sigma} n_a n_s \sin \phi_a \varepsilon_{23}^{(i)} / \varepsilon_{33}^{(i)},$$

where the asterisk (*) denotes the complex conjugate, $\sigma = sp, ps$ and $P_{sp} = +1$, $P_{ps} = -1$.

As shown by exact numerical calculations (Fig. 1), the error of Eqs. (13) and (14) does not exceed a few percent if the approximate maximum values of $\sum_{i=1}^N d_i / \lambda$ comprises a few hundredths. This is in good agreement with condition

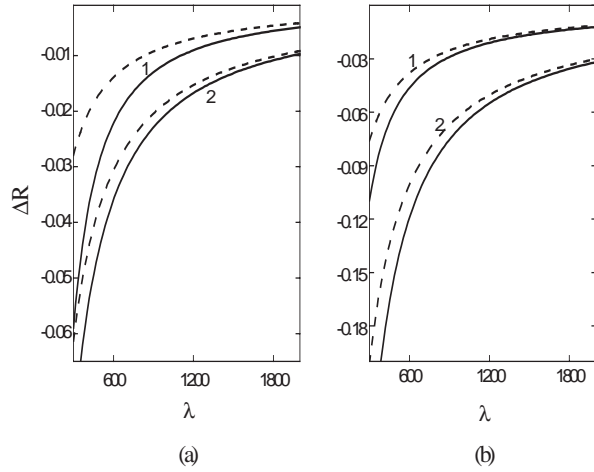


Figure 1: Comparison of long-wavelength limit (dashed curves) and exact computer calculation (solid curves) for the differential reflectances (a) $\Delta R_{pp} = R_{pp} - R_p^{(0)}$ and (b) $\Delta R_{ss} = R_{ss} - R_s^{(0)}$ as functions of wavelength λ for a three-layer system of anisotropic dielectric films with $d_1 = 2$, $d_2 = 4$, and $d_3 = 3$ (the quantities λ and d_i are measured in arbitrary common units) on isotropic absorbing substrate with complex refractive index (1) $n_s = 4 + 2i$ and (2) $2 + 2i$ at incident angle $\phi_a = 50^\circ$. $\epsilon_{xx}^{(1)} = 3$, $\epsilon_{xx}^{(2)} = 2$, $\epsilon_{xx}^{(3)} = 16$, $\epsilon_{yy}^{(1)} = 9$, $\epsilon_{yy}^{(2)} = 3$, $\epsilon_{yy}^{(3)} = 2$, $\epsilon_{zz}^{(1)} = 8$, $\epsilon_{zz}^{(2)} = 16$, $\epsilon_{zz}^{(3)} = 10$, $\phi_1 = 80^\circ$, $\phi_2 = 0^\circ$, $\phi_3 = 40^\circ$, $\psi_1 = 20^\circ$, $\psi_2 = 90^\circ$, $\psi_3 = 60^\circ$, $\theta_1 = 20^\circ$, $\theta_2 = 45^\circ$, $\theta_3 = 20^\circ$. $n_a = 1$.

$\sum_{i=1}^N d_i / \lambda \ll 1 / 2\pi$ used in the derivation of the basic approximate formulas for anisotropic systems. The relative error

$$v = [(\Delta R_\sigma / R_\sigma^{(0)})_{ex} - \Delta R_\sigma / R_\sigma^{(0)}] / (\Delta R_\sigma / R_\sigma^{(0)})_{ex},$$

where $\sigma = pp, ss, sp, ps$, $(\Delta R_\sigma / R_\sigma^{(0)})_{ex}$ was obtained by using the numerical techniques for exact solution of the reflection problem, and $\Delta R_\sigma / R_\sigma^{(0)}$ was calculated by approximate equations are presented in Fig. 2.

In summary, for absorbing substrate mathematical relationships for R_{pp} and R_{ss} take a relatively simple form, i.e., the contributions of dielectric nanofilms to the reflection characteristics can be expressed in terms of up to first order of characteristic small parameter d_i / λ . In comparison with absorbing substrate the fully transparent system is more complicated whereas in the first order the contribution of dielectric nanofilms to the reflectances R_σ from a transparent substrate is equal to zero, and, therefore, this contribution must be expressed in terms of up to second order of parameter d_i / λ . In this case an ultrathin film in multilayer system is sensitive to the presence of other ultrathin films (an interaction between them

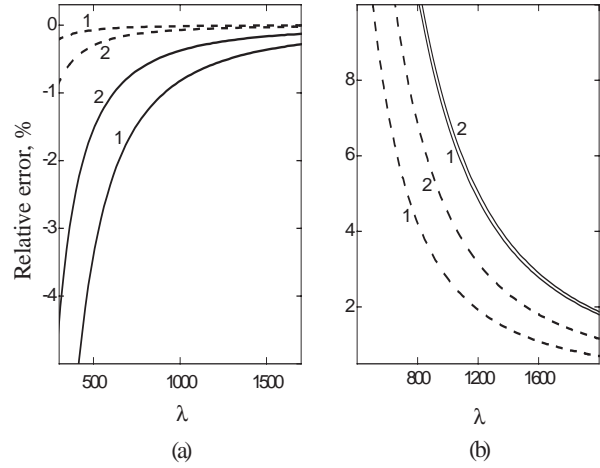


Figure 2: Relative error of long-wavelength limit (a) for $\Delta R_{pp} / R_p^{(0)}$ (solid curves) and $\Delta R_{ss} / R_s^{(0)}$ (dashed curves), and (b) for R_{ps} (solid curves) and R_{sp} (dashed curves) as functions of λ for a three-layer system of anisotropic dielectric films with $d_1 = 2$, $d_2 = 4$, and $d_3 = 3$ on isotropic transparent substrate with real refractive index $n_s = 2$ at incident angle (1) $\phi_a = 60^\circ$ and (2) 30° . $\epsilon_{xx}^{(1)} = 3$, $\epsilon_{xx}^{(2)} = 2$, $\epsilon_{xx}^{(3)} = 4$, $\epsilon_{yy}^{(1)} = 5$, $\epsilon_{yy}^{(2)} = 3$, $\epsilon_{yy}^{(3)} = 2$, $\epsilon_{zz}^{(1)} = 3$, $\epsilon_{zz}^{(2)} = 2$, $\epsilon_{zz}^{(3)} = 4$, $\phi_1 = 80^\circ$, $\phi_2 = 0^\circ$, $\phi_3 = 40^\circ$, $\psi_1 = 20^\circ$, $\psi_2 = 90^\circ$, $\psi_3 = 60^\circ$, $\theta_1 = 20^\circ$, $\theta_2 = 45^\circ$, $\theta_3 = 20^\circ$. $n_a = 1$.

emerges) and the contribution of each layer to the reflection coefficient is not expressed in a purely additive form. Physically this is the most dramatic difference between transparent and absorbing substrates.

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