Motion of a Cylinder Partially Filled With a Nano-Ferrofluid Suspension in a Magnetic Field

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ABSTRACT

This study describes the motion of a partially full cylinder containing a nano-ferrofluid suspension when subjected to magnetic and nonmagnetic forces. The cylinder is allowed to roll from a state of rest without slipping on a horizontal and an inclined plane while the contained ferrofluid is simultaneously subjected to a magnetic field parallel to the plane and a nonmagnetic force such as gravity. Neglecting viscous dissipation and free-surface distortion, Lagrange's equations of motion are derived and numerically solved for different magnetic strengths. The results point out certain oscillating components and other peculiarities in the motion that may have practical implications. The primary motivation for this study has been the integration of nanoscale materials with mesoscale and macroscale devices.

1. INTRODUCTION

Aferrofluid is a stable colloidal suspension of magnetic particles of approximately 10 nm in size which can experience a force of attraction when subjected to a magnetic field. Ferrofluids have the fluid properties of a liquid and the magnetic properties of a solid. In a magnetic field, the magnetic moments of the nano-particles in the ferrofluid instantaneously align themselves along the field lines, and their magnetization responds immediately with a substantial force as a homogeneous liquid which moves to the region of highest flux. The reason for partial filling is to create a rotational motion of the cylinder as opposed to sliding motion that would occur if the cylinder were completely filled. The rotational motion is a result of unbalanced moment in the magnetic liquid when the cylinder is partially filled.

The development of liquids that exhibit magnetic properties had been described by Neuringer and Rosensweig [1]. Several magneto-mechanical devices have been developed taking advantage of the ferrofluid motion in applied magnetic fields [2], These devices include novel pumps, viscous dampers, accelerometers, gyroscope supports, specific gravity meters, seals, and many others. Ferro-fluid seals have also been used to stop hemorrhage during certain surgical operations. Moskowitz and Ezekiel [3] presented numerous practical applications for lubrication, bearings, and magneto-graphic printing. Jenkins [4] published a continuum theory of magnetic fluids along with some static results. Ferrofluids as magnetically controllable fluids along with several applications had been described by Odenbach [5] and several of his associates in Germany where highly visible and active scientific groups have evolved. Kim et al. [6] have extensively discussed the emerging applications of ferrofluids to biomedical problems. The present problem was first analyzed by Tichenor and Avula [7], and now revisited to emphasize the integration of nanoparticles with the development of mesoscale and macroscale engineering applications.

2. PROBLEM FORMULATION

2.1 System Geometry and Kinematics

The system under study is shown in Fig, 1. The cylindrical container partially filled with a ferro-fluid rolls on an inclined plane in the presence of a magnetic field. Coordinate system (x_1,y_1) is fixed to the inclined plane while (x_2,y_2) is a non-rotating system attached to the center of the cylinder. The rotating coordinate system (x_3,y_3) is used to specify the angular location of the mass center of the liquid. As an analytical procedure to determine the dynamic free surface configuration of a ferrofluid in a magnetic field is complicated, on the basis of experimental evidence, a plane free-surface oscillating in the "first mode" condition during motion is assumed. When the magnetic field is applied to the liquid in the direction away from the rigid boundary the free surface distortions will be minimum because there will be no flow through the boundary.

The center of gravity G for the liquid is specified in terms of the radial distance O_2G, which is designated as d_G (see Fig. 1). This distance is found to be

$$d_G = \frac{2R\sin^3\alpha}{3(\alpha - \sin\alpha\cos\alpha)}, \tag{1}$$

where R is the radius of the cylinder and _ is an angle (in radians) that specifies the liquid level. The mass moment of inertia I_G for the liquid about its center of gravity G is

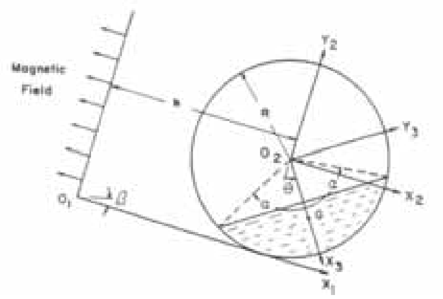

Figure 1: Schematic of the system

$$I_G = \frac{m_L R^2}{2(\alpha - \sin\alpha\cos\alpha)} \times \left[\alpha - (\cos\alpha)\left(\sin\alpha - \frac{2}{3}\sin^3\alpha\right) - \frac{8\sin^6\alpha}{9(\alpha - \sin\alpha\cos\alpha)}\right] \quad (2)$$

where m_L is the liquid mass per unit length of cylinder. From Fig. 1, the radius vector O_1G can be written as

$$r_G = h\mathbf{i}_1 + R\mathbf{j}_1 + d_G(\sin\theta)\mathbf{i}_1 - d_G(\cos\theta)\mathbf{j}_1 \quad (3)$$

where h is the linear displacement of the mass center of the cylinder, measured parallel to the inclined plane, _ is the angle between line O_2G and the negative y_2-axis, and $\mathbf{i}_1$ and $\mathbf{j}_1$ are unit vectors along the x_1 and y_2 -axes, respectively. Then the velocity of point G is

$$\mathbf{v}_G = [\dot{h} + d_G(\cos\theta)\dot{\theta}]\mathbf{i}_1 + d_G(\sin\theta)\dot{\theta}\mathbf{j}_1 \quad (4)$$

where the dot indicates ordinary differentiation with respect to time.

2.2 Lagrange's Equations of Motion

The fluid used in this study is assumed to be nearly inviscid, so the system is conservative and Lagrange's equations are applicable. With the use of equation (4), the kinetic energy T_L of the liquid is given by

$$\mathbf{v}_G = [\dot{h} + d_G(\cos\theta)\dot{\theta}]\mathbf{i}_1 + d_G(\sin\theta)\dot{\theta}\mathbf{j}_1 \quad (5)$$

The kinetic energy T_c of the cylinder may be written as

$$T_c = \frac{1}{2}m_c\dot{h}^2 + \frac{1}{2}m_cR^2\dot{\psi}^2 \quad (6)$$

where m_c is the cylinder mass per unit length, m_cR^2 is the mass moment of inertia with respect to the cylinder axis, z_2 and _ is the angular velocity of the cylinder. Since the cylinder is assumed to roll without slipping on the inclined plane, $h = R$_, and

$$T_c = m_c\dot{h}^2 \quad (7)$$

Using a datum line through point O_1 as a reference, the gravitational potential energy V_g for the system can be written as

$$V_g = (m_L + m_c)g(R\cos\beta - h\sin\beta) - m_L d_G g\cos(\theta - \beta) \quad (8)$$

where _ is the angle of inclination of the inclined plane and g is the gravitational constant.

In the case of the magnetic force, it has been assumed that dF, the magnitude of the magnetic force on an element of fluid, is directly proportional to the mass of the element and inversely proportional to the square of the perpendicular distance from the element to the magnet. For a unit length of cylinder, dF becomes

$$dF = -\epsilon m_L \frac{dA}{R^2(\alpha - \sin\alpha\cos\alpha)}\cdot\frac{1}{\rho^2} \quad (9)$$

where _ is an experimentally determined constant which involves the strength of the magnet and the magnetic properties of the fluid, dA is the cross-sectional area of the fluid element (perpendicular to the axis of the cylinder), $R^2(a$ — sin _ cos _) is the cross-sectional area of the entire body of fluid (also perpendicular to the axis of the cylinder) and _ is the distance from the element to the magnet. Associated with dF is a "magnetic potential" $d\,V_M$ which is defined by the equation

$$dV_M = -\epsilon m_L \frac{dA}{R^2(\alpha - \sin\alpha\cos\alpha)}\cdot\frac{1}{\rho} \quad (10)$$

such that $dF = -d(dV_M)/d$_. Using polar coordinates it can easily be shown that the equation of the liquid surface is

$$r = \frac{R\cos\alpha}{\cos(\phi - \theta)} \quad (11)$$

Using equations (5), (7), and (8), the Lagrangian $L = T - V$ becomes

$$\begin{aligned} L = {} & [(1/2)m_L + m_c]\dot{h}^2 + m_L d_G(\cos\theta)\dot{h}\dot{\theta} \\ & + (1/2)(m_L d_G^2 + I_G)\dot{\theta}^2 \\ & - (m_L + m_c)g(R\cos\beta - h\sin\beta) \\ & + m_L d_G g\cos(\theta - \beta) - V_M \end{aligned} \quad (12)$$

where V_M , the magnetic potential, may be obtained by integration of dV_M over the entire fluid cross section.

For a general system, the Lagrange equations of motion are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{h}}\right) - \frac{\partial L}{\partial h} = 0 \quad (13)$$

and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \quad (14)$$

Using equation (12), equations (13) and (14) yield the equations of motion governing the system in the form:

$$(m_L + 2m_c)\ddot{h} + m_L d_G(\cos\theta)\ddot{\theta} = m_L d_G(\sin\theta)\dot{\theta}^2 + (m_L + m_c)g\sin\beta - \frac{\partial V_M}{\partial h} \quad (15)$$

and

$$m_L d_G(\cos\theta)\ddot{h} + (m_L d_G^2 + I_G)\ddot{\theta} = -d_G m_L g\sin(\theta - \beta) - \frac{\partial V_M}{\partial \theta} \quad (16)$$

Equations (15) and (16) can be expressed in the non-dimensional form:

$$(m + 2)\ddot{h} + mp(\alpha)(\cos\theta)\ddot{\theta} = mp(\alpha)(\sin\theta)\dot{\theta}^2 + (m + 1)\sin\beta - \frac{\partial V_M}{\partial h} \quad (17)$$

and

$$(\cos\theta)\ddot{h} + f(\alpha)\ddot{\theta} = \sin(\beta - \theta) - \frac{\partial V_M}{\partial \theta} \quad (18)$$

In which *p(_)* and *f*(_) are defined for brevity as

$$P(\alpha) = \frac{2\sin^3\alpha}{3(\alpha - \sin\alpha\cos\alpha)} \quad (19)$$

and

$$f(\alpha) = 0.5\cos\alpha + \frac{3(\alpha - \sin\alpha\cos\alpha)}{4\sin^3\alpha} \quad (20)$$

The constant _ that was introduced in equation (9) will appear in the non-dimensional form as $\_{_1}$ = _ / (R^2g), a parameter in the equations of motion.

Equations (17) and (18) constitute a system of two second order nonlinear differential equations that warrant a numerical solution for selected values of _, _, and $\_{_1}$ with initial conditions on _, _, *h* and *h*.

3. RESULTS AND DISCUSSION

Because of the nonlinearity ofthe equations of motion, a numerical solution employing Runge-Kutta integration procedure was sought on a digital computer.

The initial conditions selected in the problem are:

$$\text{At} \quad t = 0 \begin{cases} \theta = \dot{\theta} = 0 \\ \dot{h} = 0 \\ h = 3 \end{cases} \quad (21)$$

The solution consists of determining the linear displacement, velocity and acceleration of the cylinder and the angular displacement, velocity and acceleration of the magnetic liquid mass with the cylinder rolling on horizontal and inclined plane surfaces.

3.1 Motion of the Cylinder and Ferrofluid on a Horizontal Surface

The solution of equations (17) and (18) for the special case of _ = 0 describes the motion of the ferrofluid cylinder on a horizontal surface. Figures 2 and 3 show the linear velocity, and displacement, respectively, for this special case in which _ = 60° and $\_{_1}$ = 0.25, 0.5, and 1.0 were selected in the order of increasing field strength.

Computations reveal that the cylinder is accelerated faster with increasing strength of the magnetic field. Also, the acceleration is slightly oscillatory, but these oscillations which are asymmetric, are damped out faster with increasing magnetic field strength. In a similar fashion, the linear velocity also increases with increasing field strength (Fig. 2) and the cylinder is drawn faster toward the magnet (Fig. 3).

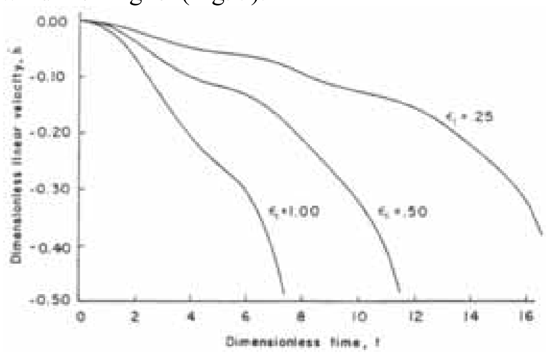

Figure2: Linear velocity of cylinder as a function of time for _ = 0

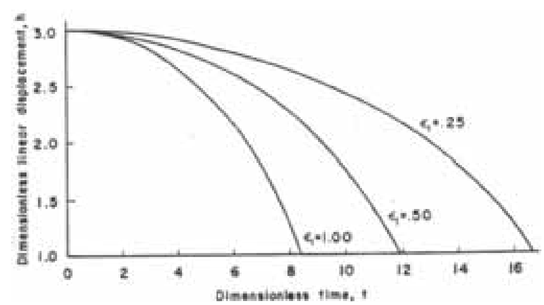

Figure 3: Linear displacement of cylinder as a function of time for _ = 0

The angular acceleration and velocity of the ferrofluid mass also increase with the magnetic field strength, and contain both negative and positive components until the cylinder is finally drawn toward the magnet (not shown). The angular displacement also increases with the magnetic field strength as shown in Fig. 4. At $\_{_1}$ = 0.25, for example, the liquid volume is in a state of oscillatory motion and slowly reaches the magnet. When $\_{_1}$ = 1.0 the oscillations of the liquid mass are damped out fast and the liquid reaches the magnet rather quickly.

3.2 Motion of the Cylinder and Ferrofluid on an Inclined Plane

Keeping the cylinder properties the same, the system is subjected to magnetic fields of varying strengths on an inclined plane at 5° inclination. As shown in Figs. 5 and 6, it requires relatively greater field strength to cause

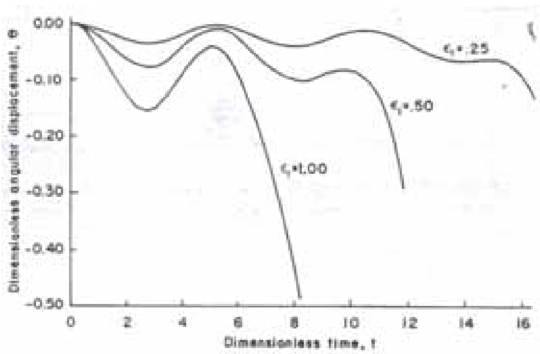

Figure 4: Angular displacement of liquid as A function of time for _ = 0

motion up the plane. Fig. 6 is of special interest. It shows that, if the magnetic field strength is not sufficient, the cylinder will "escape" and roll down the plane as indicated by the linear displacements increasing with respect to time for _$_1$ = 1.0 and 1.25. As the field strength is gradually increased, a value will be reached where, upon release, the cylinder will roll up the plane until it contacts the magnet. This is evident from the decreasing distances from the initial position of the cylinder for _$_1$ = 1.5 and 1.75.

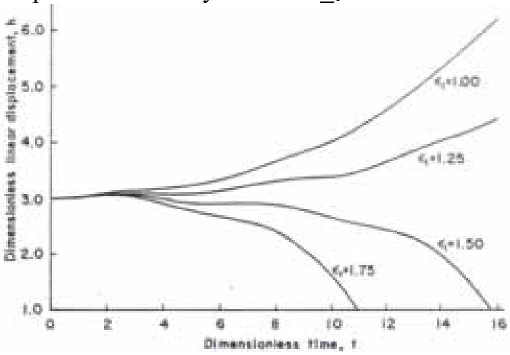

Figure 5: Linear displacement of cylinder as a Function of time for _ = 5˚

With respect to the motion of the ferrofluid mass, when the field strength is not sufficient to pull the cylinder up the plane, in the border line case, the angular acceleration, velocity and displacement are oscillatory. The angular displacement of the liquid mass is shown in Fig. 6. The angular velocity and acceleration are not shown because they are simply the derivatives of the displacement _. In Fig. 6, for _$_1$ = 1.0 and 1.25 as the cylinder "escapes" from the grip of the magnetic field by rolling clockwise, angle _ increases showing a net counterclockwise displacement of the liquid. At higher strengths of the magnetic field, say for _$_1$ = 1.5 and 1.75, the cylinder rolls up the plane in the counterclockwise direction, producing the relative displacement of the liquid in the clockwise direction toward the magnetic field. For example, at _$_1$ = 1.75 the oscillations in the liquid mass subside and the

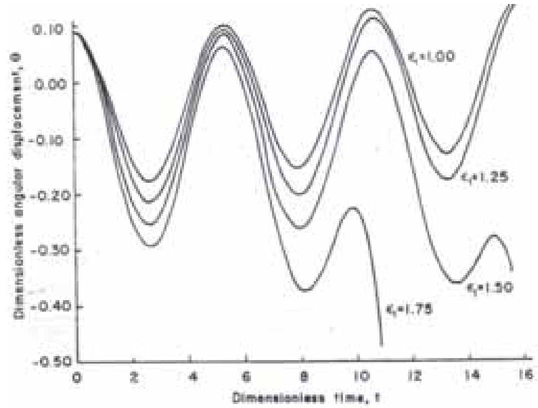

Figure 6: Angular displacement of liquid as a function of time for _ = 5˚

liquid is attracted to the magnet as indicated by _ increasing monotonically in the negative (clockwise) direction.

The present study thus points out certain oscillating components and other peculiarities in the motion characteristics of a cylinder partially filled with a ferrofluid in magnetic fields of varying strengths. Recognizing these oscillations may be useful in magneto-graphic printing in which rotary cylinders filled with magnetic inks are operated, and in other similar devices. The results are however valid for moderate speeds of rotation where free surface distortions can be neglected.

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