Active Damping Control in MEMS Using Parametric Pumping

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ABSTRACT

This paper investigates the degree of damping control provided by parametrically pumping a harmonically forced microresonator. It is shown that the parametric excitation terms result in reduced damping levels and thus increases the Q-factor of the mode of vibration to which it is applied. The increased Q-factor of the parametrically enhanced resonator permits reduced forcing levels and thus may provide a suitable method for minimising electrical feedthrough and improving sensor performance.

Keywords: parametric, gyroscope, feedthrough, Hills

1 INTRODUCTION

Parametrically excited systems have been well known for many years and an extensive analysis of such systems has been available in the literature since the 19th century [1]. Parametric excitation has been utilized since the 1960's in microwave, electronic and optoelectronics devices for amplification or harmonic generation [2].

In the MEMS area it is only in the last decade that parametric excitation has been investigated [3-5]. In this paper the application of parametric excitation to a harmonically forced resonator is investigated. A micro ring gyroscope is used as a vehicle to demonstrate the application.

2 EQUATIONS OF MOTION

Figure (1) illustrates the basic form of the ring gyroscope. The resonator takes the form of a planar ring of radius *a*, width *b* and thickness *d* and is supported by a suspension. Actuation is provided electrostatically through a cyclically arrangement of electrodes which are the same thickness as the ring and displaced radially from the outer surface of the ring by a distance h_o.

The electrical energy stored in the capacitance formed between the ring (assume held at Earth potential) and the plurality of electrodes, each biased with a voltage U_k may be expressed in the form

$$E_E = \frac{\varepsilon_o ad}{2h_o}\sum_{k=1}^{p} U_k{}^2 \int_{-\gamma+\theta_k}^{+\gamma+\theta_k}\left[1+\frac{u}{h_o}+\left(\frac{u}{h_o}\right)^2+\left(\frac{u}{h_o}\right)^3+\cdots\right]d\phi \qquad (1)$$

The radial displacement *u* of a point of the centre line of the ring when it is vibrating in the primary mode of the gyroscope may be expressed in terms of the undamped, unforced flexural mode of vibration of an unsupported ring by:

$$u = q_1 \cos n\phi \qquad (2)$$

where q_1 is the generalized coordinates associated with the mode. The radial displacement of the ring is assumed to be small compared to the nominal gap separation h_o and thus terms of order greater than $(u/h_o)^3$ may be ignored.

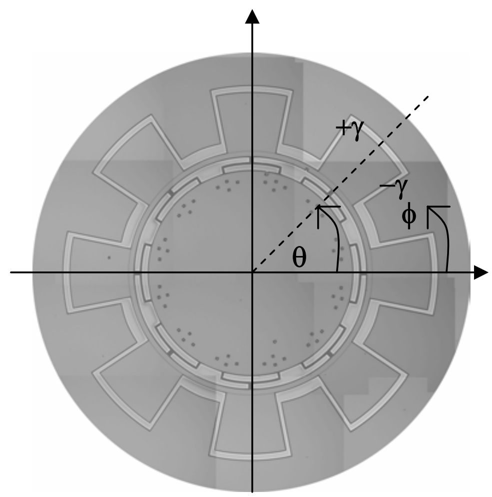

Figure (1) Photograph of Ring Gyroscope

The generalized stiffness and forcing components may be found by calculating dE_E/dq_1, $d^2E_E/dq_1{}^2$. When actuation is provided by the pair of voltages $U(t)$ and $\hat{U}(t)$ applied to two sets of *P* and *Q* electrodes the equation of motion may be expressed in the form

$$m\ddot{q}_1 + 2\xi_1\omega_1 m\dot{q}_1 + kq_1 = \frac{\varepsilon_o ad}{h_o{}^3}\left(U(t)^2 K + \hat{U}(t)^2 \hat{K}\right)q + \frac{\varepsilon_o ad}{2h_o{}^2}\left(U(t)^2 G + \hat{U}(t)^2 \hat{G}\right) \qquad (3)$$

where

$$K=\sum_{k=1}^{P}\overline{k}_k,\ G=\sum_{k=1}^{P}g_k,\ \hat{K}=\sum_{k=1}^{Q}\overline{k}_k,\ \hat{G}=\sum_{k=1}^{Q}g_k$$

with

$$\overline{k}_k=\int_{-\gamma+\theta_k}^{\gamma+\theta_k}\cos^2(n\phi)d\phi \text{ and } g_k=\int_{-\gamma+\theta_k}^{\gamma+\theta_k}\cos(n\phi)d\phi$$

The terms m and k are the generalised mass and stiffness values corresponding to the primary mode of order n and are given by

$$m=m_r\left(1+\frac{1}{n^2}\right),\ m_r=\left(\pi\rho abd\right),\ k=\pi\left(1-n^2\right)\frac{EI}{a^3}$$

By introducing the non-dimensional parameters $\mu=\frac{U^2}{U_p{}^2}$ and $\hat{\mu}=\frac{\hat{U}^2}{U_p{}^2}$ where U_p is the voltage at which electrostatic pull-in occurs, the equation of motion may be written in the form

$$\ddot{q}_1+2\xi_1\omega_1\dot{q}_1+\left\{\omega_1{}^2-\left(\mu+\hat{\mu}\right)D\right\}q_1=\left(\mu+\hat{\mu}\right)F$$

where

$$D=\frac{\varepsilon_o ad}{h_o{}^3}U_p{}^2\frac{K}{m} \text{ and } F=\frac{\varepsilon_o ad}{2h_o{}^2}U_p{}^2\frac{G}{m} \tag{4}$$

and only the case where $P=Q$ has been considered.

3 PERTURBATION ANALYSIS

In normal operation $U,\hat{U}<<U_p$ thus $\mu<<1$. When the excitation voltages U and $\hat{U}$ are periodic μ and $\hat{\mu}$ may be represented by Fourier series

$$\mu=\varepsilon\eta\left(u_o+\sum_{r=1}^{\infty}u_r e^{ir\omega t}+\sum_{r=1}^{\infty}u^*{}_r e^{-ir\omega t}\right) \text{ and}$$

$$\hat{\mu}=\varepsilon\hat{\eta}\left(\hat{u}_o+\sum_{s=1}^{\infty}\hat{u}_s e^{is\hat{\omega}t}+\sum_{s=1}^{\infty}\hat{u}_s^* e^{-is\hat{\omega}t}\right).$$

The equation of motion becomes

$$\begin{aligned}&\ddot{q}_1+2\varepsilon\upsilon_1\omega_1\dot{q}_1+\left\{\omega_1{}^2-\varepsilon D\left(\eta u_o+\hat{\eta}\hat{u}_o\right)\right\}q_1-\\&\varepsilon D\left\{\eta\left(\sum_{r=1}^{\infty}u_r e^{ir\omega t}+\sum_{r=1}^{\infty}u_r{}^* e^{-ir\omega t}\right)+\hat{\eta}\left(\sum_{s=1}^{\infty}\hat{u}_s e^{is\hat{\omega}t}+\sum_{s=1}^{\infty}\hat{u}_s{}^* e^{-is\hat{\omega}t}\right)\right\}q_1=\\&\varepsilon F\left\{\begin{array}{l}\eta\left(u_o+\sum_{r=1}^{\infty}u_r e^{ir\omega t}+\sum_{r=1}^{\infty}u_r{}^* e^{-ir\omega t}\right)+\\\hat{\eta}\left(\hat{u}_o+\sum_{s=1}^{\infty}\hat{u}_s e^{is\hat{\omega}t}+\sum_{r=1}^{\infty}\hat{u}_s^* e^{-is\hat{\omega}t}\right)\end{array}\right\}\end{aligned} \tag{5}$$

Equation (5) represents a Hills equation. As normal operation occurs under vacuum, the damping ratio is small and has been conveniently expressed in terms of the perturbation parameter ε as $\xi_1=\varepsilon\upsilon_1$. With the equation of motion in this form the solution may be investigated using a multiple scales perturbation method. The solution to equation (5) can be written as the asymptotic expansion

$$q_1(\overline{t},\hat{t})=q_1{}^{(0)}(\overline{t},\hat{t})+\varepsilon q_1{}^{(1)}(\overline{t},\hat{t})+\cdots+O\left(\varepsilon^2\right)$$

where $\overline{t}$ and $\hat{t}$ are the slow and fast time scales described by $\overline{t}=\varepsilon t$ and $\hat{t}=t+\cdots+O\left(\varepsilon^2\right)$.

The time derivatives of the generalised coordinates are thus

$$\begin{aligned}\dot{q}_1&=\frac{\partial q_1{}^{(0)}}{\partial\hat{t}}+\varepsilon\frac{\partial q_1{}^{(1)}}{\partial\hat{t}}+\varepsilon\frac{\partial q_1{}^{(0)}}{\partial\hat{t}}+\cdots+O\left(\varepsilon^2\right)\\\ddot{q}_1&=\frac{\partial^2 q_1{}^{(0)}}{\partial\hat{t}^2}+\varepsilon\frac{\partial^2 q_1{}^{(1)}}{\partial\hat{t}^2}+2\varepsilon\frac{\partial^2 q_1{}^{(0)}}{\partial\overline{t}\,\partial\hat{t}}+\cdots+O\left(\varepsilon^2\right)\end{aligned} \tag{6}$$

Substituting equation (6) into equation (5) and equating terms of like powers in ε gives the recursive relations

$$\frac{\partial^2 q_1{}^{(0)}}{\partial\hat{t}^2}+\omega_1{}^2 q_1{}^{(0)}=0 \tag{7}$$

$$\begin{aligned}&\frac{\partial^2 q_1{}^{(1)}}{\partial\hat{t}^2}+\omega_1{}^2 q_1{}^{(1)}=-2\frac{\partial^2 q_1{}^{(0)}}{\partial\overline{t}\,\partial\hat{t}}-2\upsilon_1\omega_1\frac{\partial q_1{}^{(0)}}{\partial\hat{t}}+\\&D\left\{\begin{array}{l}\eta\left(u_o+\sum_{r=1}^{\infty}u_r e^{ir\omega t}+\sum_{r=1}^{\infty}u_r{}^* e^{-ir\omega t}\right)+\\\hat{\eta}\left(\hat{u}_o+\sum_{s=1}^{\infty}\hat{u}_s e^{is\hat{\omega}t}+\sum_{s=1}^{\infty}\hat{u}_s{}^* e^{-is\hat{\omega}t}\right)\end{array}\right\}q_1{}^{(0)}+\\&F\left\{\begin{array}{l}\eta\left(u_o+\sum_{r=1}^{\infty}u_r e^{ir\omega t}+\sum_{r=1}^{\infty}u_r{}^* e^{-ir\omega t}\right)+\\\hat{\eta}\left(\hat{u}_o+\sum_{s=1}^{\infty}\hat{u}_s e^{is\hat{\omega}t}+\sum_{s=1}^{\infty}\hat{u}_s{}^* e^{-is\hat{\omega}t}\right)\end{array}\right\}\end{aligned} \tag{8}$$

The general solution to equation (7) is

$$q_1{}^{(0)}=A_o\left(\overline{t}\right)e^{i\omega_1\hat{t}}+B_o\left(\overline{t}\right)e^{-i\omega_1\hat{t}} \tag{9}$$

The terms resulting in resonance in equation (8) may be explored by expressing the forcing and parametric excitation frequencies in the form

$$r\omega=2\omega_1+\varepsilon r\lambda \text{ and } s\hat{\omega}=\omega_1+\varepsilon r\hat{\lambda} \tag{10}$$

where λ and $\hat{\lambda}$ denote mistuning from the resonant frequencies. To ensure the solution defined by equation (9) is uniform in $\hat{t}$ the terms resulting in an unbounded solution must be removed. This yields the pair of simultaneous equations relating amplitudes A_o and B_o

$$\frac{\partial A_o}{\partial \bar{t}} + \upsilon_1\omega_1 A_o + i\left(\eta u_o + \hat{\eta}\hat{u}_o\right)\frac{D}{2\omega_1}A_o + i\eta u_r \frac{D}{2\omega_1}B_o e^{i\lambda\bar{t}} +$$

$$i\hat{\eta}\hat{u}_s \frac{F}{2\omega_1}e^{i\hat{\lambda}\bar{t}} = 0 \tag{11}$$

$$\frac{\partial B_o}{\partial \bar{t}} + \upsilon_1\omega_1 B_o - i\left(\eta u_o + \hat{\eta}\hat{u}_o\right)\frac{D}{2\omega_1}B_o - i\eta u^*{}_r \frac{D}{2\omega_1}A_o e^{i\lambda\bar{t}} -$$

$$i\hat{\eta}\hat{u}_s^* \frac{F}{2\omega_1}e^{-i\hat{\lambda}\bar{t}} = 0 \tag{12}$$

Assuming a solution to equations (11) and (12) of the form $A_o\left(\bar{t}\right) = A^{(o)}e^{i\frac{\lambda}{2}\bar{t}}$ and $B_o\left(\bar{t}\right) = B^{(o)}e^{i\frac{\lambda}{2}\bar{t}}$ yields

$$\begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix}\begin{bmatrix} A^{(o)} \\ B^{(o)} \end{bmatrix} = \begin{bmatrix} H \\ H^* \end{bmatrix} \tag{13}$$

where

$$a=\upsilon_1\omega_1 + i\left(\frac{\lambda}{2} + \left(\eta u_o + \hat{\eta}\hat{u}_o\right)\frac{D}{2\omega_1}\right),\ b = i\eta u_r \frac{D}{2\omega_1} \text{ and}$$

$$H = -i\hat{\eta}\hat{u}_s \frac{F}{2\omega_1}e^{i\left(\hat{\lambda}-\frac{\lambda}{2}\right)\bar{t}}$$

For the most practical applications the mistuning parameters are related by $\hat{\lambda} = \frac{\lambda}{2}$. The response amplitude obtained from equation (13) for the case where η and $\hat{\eta}$ are both non-zero is be given by

$$\left|A^{(o)}\right|_{on}^{2} = \frac{\begin{vmatrix} H & b \\ H^* & a^* \end{vmatrix}\begin{vmatrix} H & b \\ H^* & a^* \end{vmatrix}^*}{|C|^2} \text{ and } \left|B^{(o)}\right|_{on}^{2} = \frac{\begin{vmatrix} a & H \\ b^* & H^* \end{vmatrix}\begin{vmatrix} a & H \\ b^* & H^* \end{vmatrix}^*}{|C|^2}$$

$$\text{where } C = \begin{vmatrix} a & b \\ b^* & a^* \end{vmatrix}. \tag{14}$$

When the parametric excitation is not applied η=0 and the response amplitudes are given by

$$\left|A^{(o)}\right|_{off}^{2} = \frac{\begin{vmatrix} \bar{H} & 0 \\ \bar{H}^* & \bar{a}^* \end{vmatrix}\begin{vmatrix} \bar{H} & 0 \\ \bar{H}^* & \bar{a}^* \end{vmatrix}^*}{|\bar{C}|^2},\ \left|B^{(o)}\right|_{off}^{2} = \frac{\begin{vmatrix} \bar{a} & \bar{H} \\ 0 & \bar{H}^* \end{vmatrix}\begin{vmatrix} \bar{a} & \bar{H} \\ 0 & \bar{H}^* \end{vmatrix}^*}{|\bar{C}|^2} \tag{15}$$

where $\bar{C} = \begin{vmatrix} \bar{a} & 0 \\ 0 & \bar{a}^* \end{vmatrix}$ with $\bar{a}=\upsilon_1\omega_1 + i\hat{\eta}\hat{u}_o \frac{D}{2\omega_1}$ and

$$\bar{H} = -i\hat{\eta}\hat{u}_s \frac{F}{2\omega_1}e^{i\hat{\lambda}\bar{t}}.$$

QUALITY FACTOR CONTROL

The response described by equation (15) is that of a conventionally forced resonator and thus damping limits the response amplitude. The quality-factor for this case is given by $Q_{off} \approx 1/2\xi$. Applying parametric excitation to a harmonically forced resonator results in a modified response described by equation (14). The parametric excitation is manifested as a term reducing the degree of damping. Thus for a given harmonic forcing amplitude described by $\hat{\eta}$, the response amplitude will be amplified. The degree of amplification when both parametric and external forcing is applied compared to forcing only is described by

$$Gain = \frac{\left|A^{(0)}\right|_{on}}{\left|A^{(0)}\right|_{off}} = \frac{\left|B^{(0)}\right|_{on}}{\left|B^{(0)}\right|_{off}} = N \text{ where } N>1. \tag{16}$$

The quality-factor of the resonator subject to harmonic forcing and parametric excitation is given by

$$Q_{on} = NQ_{off}. \tag{17}$$

From equation (16) it may be shown that infinite gain occurs when $|C| = 0$, which is met when the mis-tuning parameter λ is described by

$$\lambda^2 + 2\frac{D}{\omega_1}\left(\eta u_o + \hat{\eta}\hat{u}_o\right)\lambda + \left(\frac{D}{\omega_1}\right)^2\left(\eta u_o + \hat{\eta}\hat{u}_o\right)^2 -$$

$$\eta^2 |u_r|^2 \left(\frac{D}{2\omega_1}\right)^2 + 4\left(\upsilon_1\omega_1\right)^2 = 0 \tag{18}$$

The roots of equation (18) are

$$\bar{\lambda} = \pm\sqrt{\eta^2 |u_r|^2 \left(\frac{D}{\omega_1}\right)^2 - 4\left(\upsilon_1\omega_1\right)^2} \tag{19}$$

where

$$\bar{\lambda}=\lambda + \left(\eta u_o + \hat{\eta}\hat{u}_o\right)\frac{D}{\omega_1}.$$

Equation (19) defines the boundary of stability for the system subject to parametric excitation. The effect of the d.c. voltage components u_o and $\hat{u}_o$ is to reduce the natural frequency of the system from ω_1 to $\bar{\omega}_1$. Therefore the parameter $\bar{\lambda}$ corresponds to mis-tuning from the natural frequency described by $\bar{\omega}_1$. The minimum value of the parametric scaling factor at which the gyroscope may be parametrically excited is calculated from

$$\eta_{\min} = \frac{2\upsilon_i\omega_i}{|u_r|^2 \frac{D}{\omega_i}}. \tag{20}$$

NUMERICAL EXAMPLE

Table (1) shows the dimensions of the ring gyroscope considered. The natural frequency of the gyroscope has been assumed to equal that of thin planar ring and has the value $\omega_1 = 71250$ rads^{-1} [6]. Furthermore, a value of $Q_{off} = 2000$ has been assumed. Forcing and parametric excitation is provided using two square waves with Fourier components $u_o = \hat{u}_o = \frac{1}{2}$, $|u_r| = \frac{1}{\pi r}$ and $|u_s| = \frac{1}{\pi s}$. The response to only the first harmonics of the excitation will be determined. Thus r=s=1. The boundary of stability defined by equation (19) is shown in figure (3).

Resonator Parameters	
a (mm)	4
b (μm)	175
d (μm)	100
h_o (μm)	5
γ (rads)	π/16

Table (1) Dimensions of Resonator

By choosing the values of $\bar{\lambda}$ and η such that point C of figure (2) is maintained at a fixed distance from the stability boundary, the quality factor will be increased. The stability boundary must be approached from the stable region as shown by the curve BC. Point D resides on the line $\bar{\lambda} = 0$ and is in the immediate vicinity of the point on the stability boundary at which the minimum value of η occurs. Thus at point D both the harmonic forcing and parametric excitation occur at resonance. As point D approaches the stability boundary the amount of gain defined by equation (16) increases quadratically. The minimum value of η on the stability boundary may be determined from equation (20) and yields the value $\eta_{\min} = 495$. The increase in the response amplitude when the quality-factor has been amplified parametrically is shown in figure (3). Excitation was provided at the resonant frequencies such that $\bar{\lambda} = 0$. In each response the harmonic forcing amplitude was maintained constant with $\hat{\eta} = 50$ while the parametric excitation level governed by η is increased. For η=300 and 330 the quality-factor for the parametrically enhanced cases has the values $Q_{on}=5Q_{off}$ and $Q_{on}=10Q_{off}$, respectively.

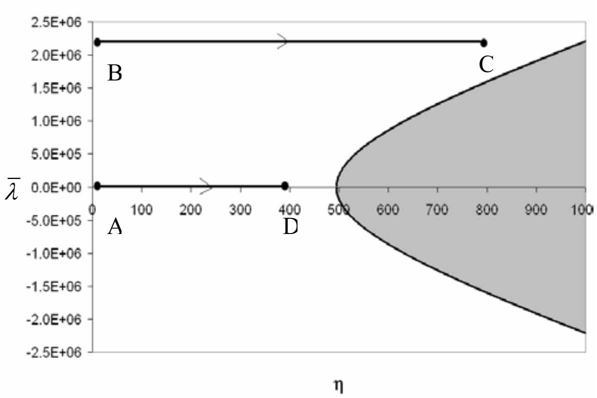

Figure (2) Stability Boundary for Parametric Excitation

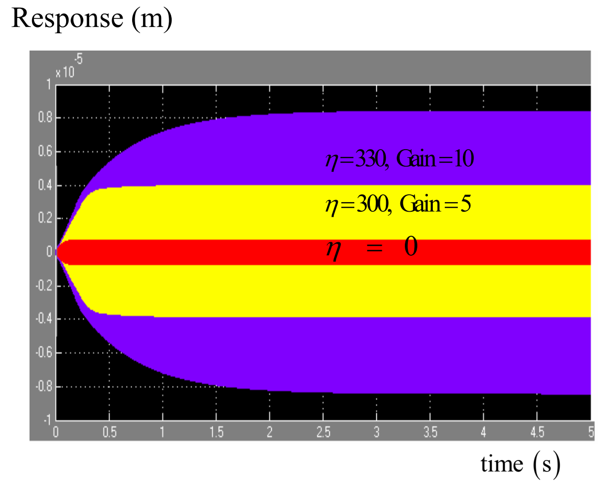

Figure (3) Parametrically Amplified Response

CONCLUSIONS

The quality-factor and thus the response amplitude of a harmonically forced microresonator may be increased by parametric excitation. This may be utilized in an excitation scheme to minimise electrical feedthrough of the harmonic forcing signal and thus improve the signal to noise ratio.

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