Electro-Static Membrane Model in CAD

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ABSTRACT

In this paper the analysis of mistakes often emerging in the micromembrane sensors modeling in mechanical and electrical domain will be presented. Additionally the application of Bibnickow-Galerkin and simple iteration method and its convergence has been discussed.

Keywords: multidomain modelling, MEMS membrane, pressure sensor, von Kármán equation, electric charge density distribution

1 INTRODUCTION

The model simplification is one of the most important problems in the design process. Very often this approach tends to the physical phenomena violation, exceeding assumed domains or using the models (and commercial software) without taking into consideration their fundamental limitations. Unfortunately, there are a lot of publications confirming this problem.

The paper will present the frequently appearing mistakes introduced in the modeling of micromachined pressure sensor based on silicon membrane and taking into account its electrostatic phenomena. Both mechanical and electrostatic phenomena can be wrongly described but the electrical one had the most important modeling consequences since the introduced errors lead to the improper description of the membrane behaviour.

2 MECHANICAL DOMAIN

The mechanical model of micromachined sensors can be described using small and large deflection theory of the clamped thin plate under uniform pressure and with (or without) additional build-in stress [10][13]. Unfortunately, several published solutions of large deflection include some minors' mistakes. In most of cases it can be visible in the simulation result when comparing with FEA. The membrane behaviour can be described using von Kármán set of equations [1], with basic assumptions: quasistationary state, small plate thickness – considered element is a plate which surface deflection is similar to deflection of its central plane:

$$\Delta^2 w = \frac{P}{D} + \frac{h}{D \cdot r} \cdot \left(\partial_r \phi \cdot \partial_{rr} w + \partial_r w \cdot \partial_{rr} \phi \right) \tag{1}$$

$$\Delta^2 \phi = -\frac{E}{r} \frac{\partial w}{\partial r} \frac{\partial^2 w}{\partial r^2} \tag{2}$$

where $w = w(r)$ – membrane deflection $w(r) = w(-r)$; r – radial coordinate, $0 \le r \le R$; R – membrane radius; $\phi = \phi(r)$ – Airy stress function[1]; P – pressure differences between membrane surfaces; D – flexural rigidity $D = E \cdot h^3 / \left(12 \cdot \left(1 - \nu^2\right)\right)$; E, h, ν – Young's modulus, plate thickness and Poisson's ratio, respectively; Δ^2 – biharmonic operator; $\partial_r f \equiv \partial f / \partial r$, $\partial_{rr} f \equiv \partial^2 f / \partial r^2$, ... – differential operators.

This nonlinear equation has been extensively investigated by Berger, Fife [5], Kinghtly [4], Ciarlet [7][8], Fox, Raoult, Simo [11], Block [12], Brilla [6], Christensen [9] and others. Unfortunately the micromembrane behaviour and introduced assumptions presented in the MEMSs literature is often not free from mistakes. The knowledge about the source of errors can improve the future simulation results and released CAD software quality. The main mistake introduced in the publications is taking into account model without taking into consideration the basic assumptions e.g.: apply equation (1) for the membrane with considerable plate thickness, taking into account not enough number of approximation series and base order, ignoring bucking or large deflection effects for operating area where it is required. The another group of awkward is associated with too high simplification level e.g. assuming in equation (1) that term $\partial_r w \cdot \partial_{rr} \phi$ can be neglected – unfortunately this simplification is improper because of $\partial_r w \cdot \partial_{rr} \phi$ value is comparable with $\partial_r \phi \cdot \partial_{rr} w$ depending on membrane place (see Figure 1), therefore cannot be take into consideration for whole membrane surface and lead to additional error in electrical domain.

[1] In some publication $F(r)$ is used instead of $\phi(r)$ Airy stress function, in this case $F(r) = h \cdot \phi(r)$.

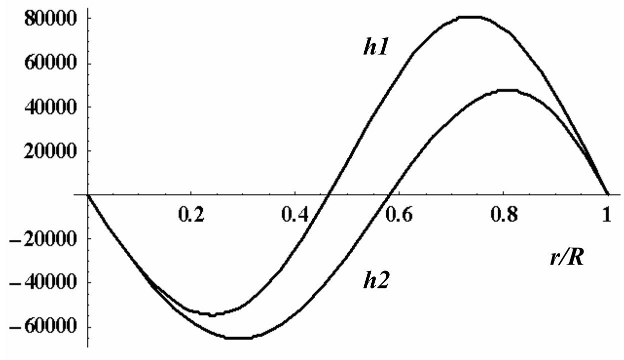

Figure 1 The $h_1 = \partial_r w \cdot \partial_{rr}\phi$ and $h_2 = \partial_r \phi \cdot \partial_{rr} w$ for $0 \le r \le R$ and clamped boundary conditions without build-in stress, $R = 50\,\mu m$, $h = 1.2\,\mu m$, $\nu = 0.24$, $E = 300\,\text{GPa}$, $P = 2\,\text{atm}$.

The simplified static model of the membrane for small and large deflection and its approximation using equivalent electric circuit has been presented in the paper [16], therefore in this paper will be proposed method suitable for semi-analytical analysis of membrane deflection. The nonlinear set of equations (1-2) can be numerically solved using orthogonal base $\{\psi_i\}_{i=1\ldots n}$, proposed for Ritz method [17]

$$\psi_i = \begin{cases} \left(1-(r/R)^2\right)^2 & \text{for} \quad i=1 \\ \left(1-(r/R)^2\right)^2 \cdot (r/R)^{2i-2} & \text{for} \quad i=2,\ldots,n \end{cases} \qquad (3)$$

where $\psi_i = \psi_i(r)$ - polynomial; n – approximation order.

We are proposing the alternative solution. It is based on Bibnickow-Galerkin approach [2][3] and simple iteration method. It is simple comparing to others approach. Therefore, the membrane deflection and Airy stress function can be approximated by the equations (4) and (5) respectively. Both of them satisfy clamped boundary conditions without build-in stress[2] ($w(\pm R)=0$, $\partial_r w|_{r=\pm R}=0$, $\phi(\pm R)=0$, $\partial_r \phi|_{r=\pm R}=0$).

$$w \cong \sum_{i=1}^{n} a_i \cdot \psi_i\,, \qquad \phi \cong \sum_{i=1}^{n} b_i \cdot \psi_i \qquad (4,5)$$

where $a_1,\ldots,a_n$, $b_1,\ldots,b_n$ - unknown coefficients

In the first iteration we have to estimate first preliminary solution of membrane deflection $w(r)$ solving linear equation (6), firstly can be assumed that $f = P/D$. This assumptions is equivalent to use the small deflection theory and for that reason we known that $a_1 = PR^4/(64D)$, $a_2 = 0$, $a_3 = 0$ … (compare with equation (8)).

$$\begin{bmatrix} (\Delta^2\psi_1,\psi_1) & \cdots & (\Delta^2\psi_n,\psi_1) \\ \vdots & \ddots & \vdots \\ (\Delta^2\psi_1,\psi_n) & \cdots & (\Delta^2\psi_n,\psi_n) \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \cong \begin{bmatrix} (f,\psi_1) \\ \vdots \\ (f,\psi_n) \end{bmatrix} \qquad (6)$$

where $f = P/D$ for first iteration and $f = \frac{P}{D} + \frac{h}{D \cdot r} \cdot \left(\partial_r\phi \cdot \partial_{rr} w + \partial_{rr}\phi \cdot \partial_r w\right)$ for the next ones; and $(g,\psi_i) = \int_0^R g \cdot \psi_i \cdot r\, dr$.

In the next step, Air stress function (represented by the coefficients $b_1,\ldots,b_n$) can be estimated from equation (7), taking into account the last obtained surface deflection ($a_1,\ldots,a_n$).

$$\begin{bmatrix} (\Delta^2\psi_1,\psi_1) & \cdots & (\Delta^2\psi_n,\psi_1) \\ \vdots & \ddots & \vdots \\ (\Delta^2\psi_1,\psi_n) & \cdots & (\Delta^2\psi_n,\psi_n) \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \cong \begin{bmatrix} \left(-\frac{E}{r} \cdot \partial_r w \cdot \partial_{rr} w, \psi_1\right) \\ \vdots \\ \left(-\frac{E}{r} \cdot \partial_r w \cdot \partial_{rr} w, \psi_n\right) \end{bmatrix}$$

(7)

The iteration process (calculation of equation (6,7)) is repeated several times in order to obtain sufficient accuracy, taking into account full equation form. Proposed algorithm has fast convergence to the final solution in the several iterations for $n = 5,\ldots,10,\ldots 20$ (see Figure 3). The order of proposed solution can be simply changed during the computation to obtain required accuracy, but the equations (6,7) can be ill-conditioned for large approximation order (e.g. $n \ge 10$). The obtained exemplary solution has been presented in Figure 2.

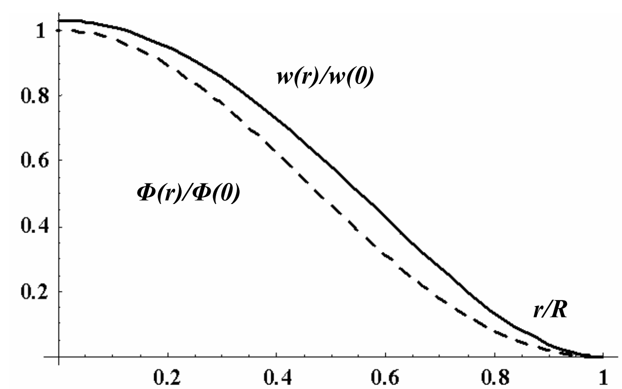

Figure 2 An exemplary normalized micro-membrane deflection ($w(r)/\lim_{r\to 0} w$, bolded line) and normalized Airy stress function ($\phi(r)/\lim_{r\to 0}\phi$, doted line) $\lim_{r\to 0} w = 0.409681\,\mu m$, $\lim_{r\to 0}\phi = -4.81\,\text{mm}$.

[2] The build-in residual stress ε_i can be introduced by the adding additional term in polynomial ϕ, which satisfy following boundary conditions $\partial_{rr}\phi - \nu\partial_r\phi/r = \varepsilon_i E$.

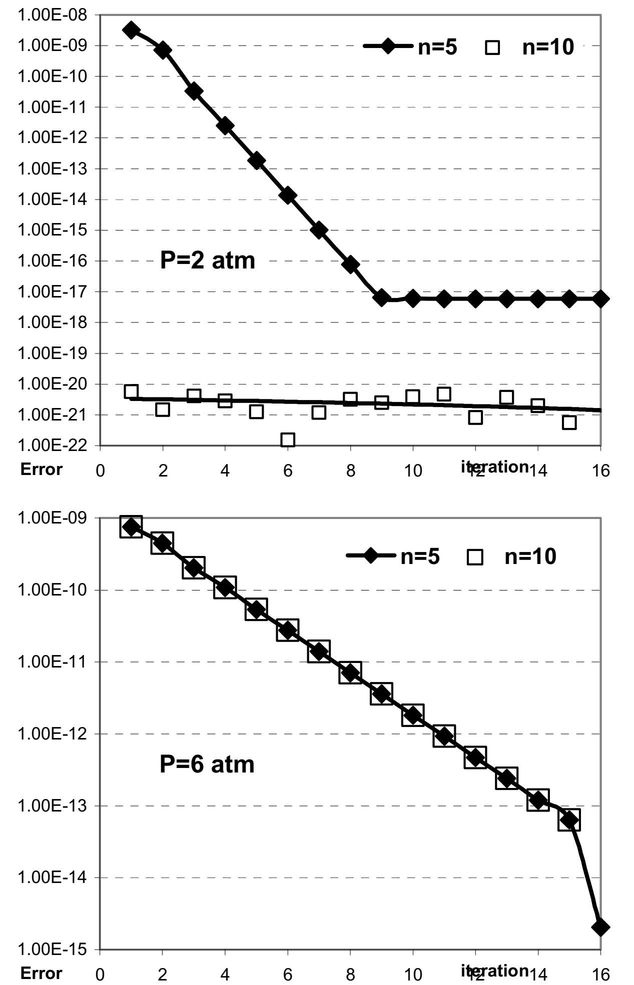

Figure 3 Estimated error of surface deflection for *n=5,10* and *P=2, 6* atm. The error has been calculated using following equation $\text{Error}(\text{itr}) = \sqrt{\int_0^R \left(w_{itr}(r) - w_{final}(r)\right)^2 dr}$, additionally $\int_0^R w(r)\,dr = 1.128\cdot 10^{-11}$ and $3.03\cdot 10^{-11}$ for *P=2* and *6* atm respectively.

3 ELECTRICAL DOMAIN

The main problem occurs in the electro-static phenomena model. The most of the authors divide the membrane plate for several parts and describe it as the parallel connected capacitances, assuming charge density invariability on the membrane surface (for each elementary capacitor).

This problem can be demonstrated for small deflected clamped circular membrane[3]. In the particular case the membrane deflection can be described by the following equation [10] (see Figure 2 and Figure 4):

$$w(r) = P\cdot\frac{R^4}{64\cdot D}\cdot\left(1-\frac{r^2}{R^2}\right)^2 \quad (8)$$

The charge density on the membrane surface $\rho_A(r,\varphi)$ is approximately inversely proportional to its curvature $k(r)$ (for simplification we assume that $h\to 0$ and $P>0$),

$$k(r) = \frac{\partial_{rr} w(r)}{\left(1+\left(\partial_r w(r)\right)^2\right)^{3/2}} \quad (9a)$$

therefore we obtain:

$$\rho_A(r,\varphi) \cong \lim_{h\to 0}\left(Q/\left|k(r)\right|\right) \quad (9b)$$

and finally

$$\rho_A(r,\varphi) \sim Q\cdot\frac{\left(256\cdot D^2 + r^2\cdot\left(r^2-R^2\right)^2\cdot P^2\right)^{3/2}}{256D^2\cdot\left|3\cdot r^2 - R^2\right|\cdot P} \quad (10)$$

where c – constant, φ - angle in the polar coordination system; Q – total plate charge.

Obtained equation allows for charge density estimation $\rho_A(r)$ using equation (4). The final shape of charge density has been presented in Figure 6 (see also Figure 5).

$$\rho_A(r) = \int_0^{2\pi} \rho_A(r,\varphi)\cdot r\,d\varphi \quad (11)$$

The charge density on the second capacitance sheet (electrode) is typically described by the equation $\rho_B(r) = -Q/R^2$, hence main assumption is wrong and some published results are erroneous.

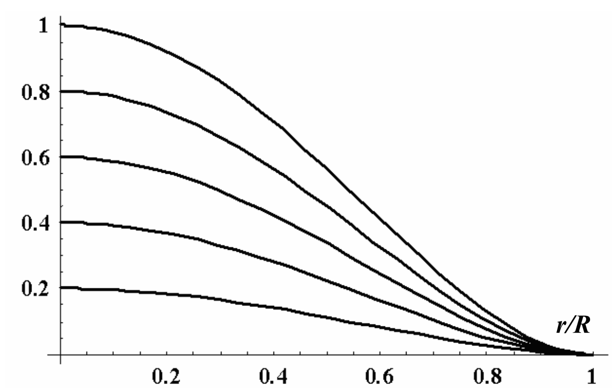

Figure 4 An exemplary normalized membrane deflection $w(r)/\left(w(0)\big|_{P=1atm}\right)$ derived from equation (8) for *P*=0.2, 0.4, 0.6, 0.8 and 1 atm (other parameters as in Figure 2 description).

[3] The similar consideration can be performed for the large deflected membrane presented in the previous section.

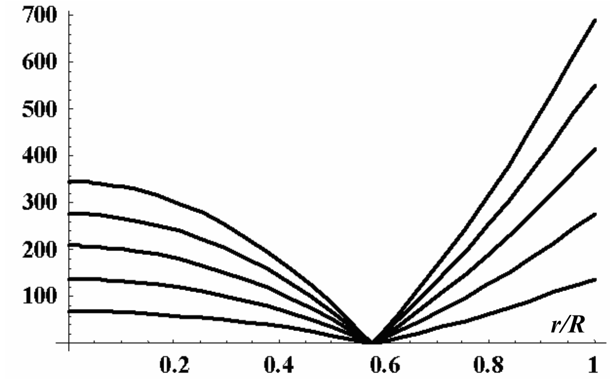

Figure 5 An exemplary distribution of $k(r)$ for *P*=0.2, 0.4, 0.6, 0.8 and 1 atm.

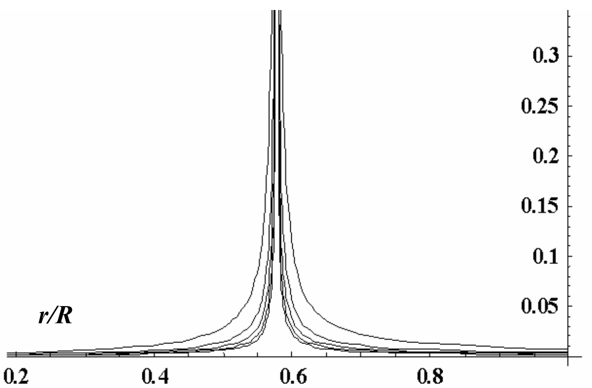

Figure 6 The exemplary distribution of $k^{-1}(r) \sim \rho(r)$ for *P*=0.2, 0.4, 0.6, 0.8 and 1 atm.

4 SUMMARY

As it was discussed in this paper, the mistakes can be introduced in electrical and mechanical domain and produce considerable error and model miss function. Taking into account the proper shape of charge density and membrane deflection can lead to obtain the fine accordance of the simulation with measured results.

5 ACKNOWLEGMENT

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REFERENCES

[1] Kármán Th.: “Enzyklopädie der mathematischen Wissenschaften”, t. IV, pp.349, 1910

[2] И.Г. Буънов. Отзыв о сочинениях профессора Тимошеико. Изд. Института инженеров путей сообщения, 1913 г

[3] Б.Г. Галеркин. Стержни и пластики. Ряды в некоторых вопросах упругого равновесия стержней и пластин. Вестник инженеров, No 19, 1915 г, 897-908

[4] G.H. Knightly. „An existence theorem for the von Kármán equations”. Arch. Rat. Mech. Anal. 27, (1967), pp. 65-78

[5] M.S.Berger, P.Fife. “On von Kármán equations and the bucking of a thin plate” Comm. Pure. Appl. Math. 21 (1968), pp. 227-241

[6] J. Brilla. “Stability problem in mathematical theory of viscoelasticity”. Proc. Internat Conf. Equadiff IV, Praha, J. Fabera ed., Springer Verlag, Berlin, 1979, pp. 46-53

[7] P.G. Ciarlet. A justification of the von Kármán equations”, Arch. Rat. Mech. Anal. 73 (1980), pp. 249-389

[8] P.G. Ciarlet, P. Rabier.”Les Équations de von Kármán”, Springer Verlag, Berlin 1980

[9] R.M. Christensen, “Theory of Viscoelastici”. Academic Press, New York 1982

[10] S. Timoshenko, S. Woinosky-Kreiger. "Theory of Plates and Shells", McGraw Hill Classic Textbook Reissue (1987).

[11] D. Fox, A. Raoul, J.C. Simo. “A justification of nonlinear property plate theories”. Arch. Rat. Mech. Anal. 124 (1993), pp. 157-199

[12] I. Block. “On the generalized von Kármán equations”. Z. Angew. Math. Mech. 76 (1996), pp. 559-571

[13] M. Zubert, M. Orlikowski, W. Wójciak, A. Napieralski, M. Janicki. "The Methodology of Multi-Domain Modelling Applied to the Thermopheumatic Micropump" in book: "Basic research for Microsystems Integration", CEPAD, Toulouse, France 1997, pp. 88-103.

[14] J.H. He. “Variational iteration method: a kind of nonlinear analytical technique: some examples”. Int. Nonlinear Mech. 34 (4) (1999) pp. 699-708.

[15] J.H. He. “Iteration perturbation technique depending upon an artificial parameter”. Int. Nonlinear Mech. 35 (2000) pp. 631-642.

[16] M. Zubert, A. Napieralski. “The revolution in multidomain simulations – Part I” MIEL 2000, Proceedings 22nd International Conference on Microelectronic and , Nis, Yugoslavia, pp. 549-553, vol 2.

[17] Ji-Huan He. “A Lagrangian for von Karman equations of large deflection problem of thin circular plate” Applied Mathematics and Computation 143 (2003) pp. 543-549.

[18] I. Block. “On the generalized von Karman system for the viscoelastic plate and its semidiscretization”