Electro-Static Membrane Model in CAD

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ABSTRACT

In this paper the analysis of mistakes often emerging in the micromembrane sensors modeling in mechanical and electrical domain will be presented. Additionally the application of Bibnickow-Galerkin and simple iteration method and its convergence has been discussed.

Keywords: multidomain modelling, MEMS membrane, pressure sensor, von Kármán equation, electric charge density distribution

1 INTRODUCTION

The model simplification is one of the most important problems in the design process. Very often this approach tends to the physical phenomena violation, exceeding assumed domains or using the models (and commercial software) without taking into consideration their fundamental limitations. Unfortunately, there are a lot of publications confirming this problem.

The paper will present the frequently appearing mistakes introduced in the modeling of micromachined pressure sensor based on silicon membrane and taking into account its electrostatic phenomena. Both mechanical and electrostatic phenomena can be wrongly described but the electrical one had the most important modeling consequences since the introduced errors lead to the improper description of the membrane behaviour.

2 MECHANICAL DOMAIN

The mechanical model of micromachined sensors can be described using small and large deflection theory of the clamped thin plate under uniform pressure and with (or without) additional build-in stress [10][13]. Unfortunately, several published solutions of large deflection include some minors’ mistakes. In most of cases it can be visible in the simulation result when comparing with FEA. The membrane behaviour can be described using von Kármán set of equations [1], with basic assumptions: quasistationary state, small plate thickness – considered element is a plate which surface deflection is similar to deflection of its central plane:

\[ \Delta^2 w = \frac{P}{D} + \frac{h}{D_r} (\partial_r \phi \cdot \partial_r w + \partial_r w \cdot \partial_r \phi) \]  
\[ \Delta^2 \phi = -\frac{E}{r} \frac{\partial}{\partial r} \frac{\partial^3 w}{\partial r^4} \]

where \( w = w(r) \) – membrane deflection \( w(r) = w(-r) \); \( r \) – radial coordinate, \( 0 \leq r \leq R \); \( R \) – membrane radius; \( \phi = \phi(r) \) – Airy stress function; \( P \) – pressure differences between membrane surfaces; \( D \) – flexural rigidity \( D = E \cdot h^4 / (12 \cdot (1 - \nu^2)) \); \( E, h, \nu \) – Young’s modulus, plate thickness and Poisson’s ratio, respectively; \( \Delta^2 \) – biharmonic operator; \( \partial_r, f = \partial f / \partial r \), \( \partial_r, f = \partial^2 f / \partial r^2 \), ..., – differential operators.

This nonlinear equation has been extensively investigated by Berger, Fife [5], Kinghtly [4], Ciarlet [7][8], Fox, Raoult, Simo [11], Block [12], Brilla [6], Christensen [9] and others. Unfortunately the micromembrane behaviour and introduced assumptions presented in the MEMSSs literature is often not free from mistakes. The knowledge about the source of errors can improve the future simulation results and released CAD software quality. The main mistake introduced in the publications is taking into account model without taking into consideration the basic assumptions e.g.: apply equation (1) for the membrane with considerable plate thickness, taking into account not enough number of approximation series and base order, ignoring bucking or large deflection effects for operating area where it is required. The another group of awkward is associated with too high simplification level e.g. assuming in equation (1) that term \( \partial_r w \cdot \partial_r \phi \) can be neglected – unfortunately this simplification is improper because of \( \partial_r w \cdot \partial_r \phi \) value is comparable with \( \partial_r \phi \cdot \partial_r w \) depending on membrane place (see Figure 1), therefore cannot be take into consideration for whole membrane surface and lead to additional error in electrical domain.

1 In some publication \( F(r) \) is used instead of \( \phi(r) \) Airy stress function, in this case \( F(r) = h \cdot \phi(r) \).
The simplified static model of the membrane for small and large deflection and its approximation using equivalent electric circuit has been presented in the paper [16], therefore in this paper will be proposed method suitable for semi-analytical analysis of membrane deflection. The nonlinear set of equations (1-2) can be numerically solved using orthogonal base \( \{\psi_i\}_{i=1}^n \), proposed for Ritz method [17]

\[
\psi_i = \begin{cases} 
1 - (r/R)^2 & \text{for } i = 1 \\
1 - (r/R)^2 \cdot (r/R)^{2i-2} & \text{for } i = 2, \ldots, n 
\end{cases}
\]  

(3)  

where \( \psi_i = \psi_i(r) \) - polynomial; \( n \) – approximation order.

We are proposing the alternative solution. It is based on Bibnickow-Galerkin approach [2][3] and simple iteration method. It is simple comparing to others approach. Therefore, the membrane deflection and Airy stress function can be approximated by the equations (4) and (5) respectively. Both of them satisfy clamped boundary conditions without build-in stress\(^2\) \( (w(\pm R) = 0, \phi(\pm R) = 0, \partial_r \phi)_{r=\pm R} = 0 \).

\[
w = \sum_{i=1}^{n} a_i \psi_i, \quad \phi = \sum_{i=1}^{n} b_i \psi_i
\]  

(4,5)  

where \( a_1, \ldots, a_n, b_1, \ldots, b_n \) - unknown coefficients

In the first iteration we have to estimate first preliminary solution of membrane deflection \( w(r) \) solving linear equation (6), firstly can be assumed that \( f = P/D \).

This assumptions is equivalent to use the small deflection theory and for that reason we known that \( a_1 = PR^2/(64D) \), \( a_2 = 0 \), \( a_3 = 0 \) … (compare with equation (8)).

\[
\begin{bmatrix}
(\Delta^2 \psi_1, \psi_1) & \cdots & (\Delta^2 \psi_n, \psi_1) \\
\vdots & \ddots & \vdots \\
(\Delta^2 \psi_1, \psi_n) & \cdots & (\Delta^2 \psi_n, \psi_n)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
\vdots \\
a_n
\end{bmatrix}
= 
\begin{bmatrix}
(\psi, \psi_1) \\
\vdots \\
(\psi, \psi_n)
\end{bmatrix}
\]  

(6)  

where \( f = P/D \) for first iteration and

\[
f = \frac{P}{D} + \frac{h}{D \cdot r} (\partial_r \phi \cdot \partial_r w + \partial_r \phi \cdot \partial_\phi)\text{ for the next ones;}
\]

and \( (g, \psi_i) = \int g \cdot \psi_i \cdot r \, dr \).

In the next step, Air stress function (represented by the coefficients \( b_1, \ldots, b_n \)) can be estimated from equation (7), taking into account the last obtained surface deflection \( (a_1, \ldots, a_n) \).

\[
\begin{bmatrix}
(\Delta^2 \psi_1, \psi_1) & \cdots & (\Delta^2 \psi_n, \psi_1) \\
\vdots & \ddots & \vdots \\
(\Delta^2 \psi_1, \psi_n) & \cdots & (\Delta^2 \psi_n, \psi_n)
\end{bmatrix}
\begin{bmatrix}
b_1 \\
\vdots \\
b_n
\end{bmatrix}
= 
\begin{bmatrix}
-E_r \cdot \partial_r w \cdot \partial_\phi w, \psi_1 \\
\vdots \\
-E_r \cdot \partial_r w \cdot \partial_\phi w, \psi_n
\end{bmatrix}
\]  

(7)  

The iteration process (calculation of equation (6,7)) is repeated several times in order to obtain sufficient accuracy, taking into account full equation form. Proposed algorithm has fast convergence to the final solution in the several iterations for \( n = 5, \ldots, 10, \ldots, 20 \) (see Figure 3). The order of proposed solution can be simply changed during the computation to obtain required accuracy, but the equations (6,7) can be ill-conditioned for large approximation order (e.g. \( n \geq 10 \)). The obtained exemplary solution has been presented in Figure 2.
The charge density on the membrane surface $\rho_A(r, \varphi)$ is approximately inversely proportional to its curvature $k(r)$ (for simplification we assume that $h \to 0$ and $P > 0$),

$$k(r) = \frac{\partial \varphi}{\partial r}$$

(9a)

therefore we obtain:

$$\rho_A(r, \varphi) \approx \lim_{k \to 0} \left( \frac{Q}{|k(r)|} \right)$$

(9b)

and finally

$$\rho_A(r, \varphi) \sim Q \left( \frac{256 D^2 + r^2 \left( r^2 - R^2 \right)^2}{256 D^2 - 3 r^2 - R^2 - P} \right)^{3/2}$$

(10)

where $c$ – constant, $\varphi$ - angle in the polar coordination system; $Q$ – total plate charge.

Obtained equation allows for charge density estimation $\rho_A(r)$ using equation (4). The final shape of charge density has been presented in Figure 6 (see also Figure 5).

$$\rho_A(r) = \int_0^{2\pi} \rho_A(r, \varphi) \cdot r d\varphi$$

(11)

The charge density on the second capacitance sheet (electrode) is typically described by the equation $\rho_B(r) = -Q / R^2$, hence main assumption is wrong and some published results are erroneous.
Figure 5 An exemplary distribution of $k(r)$ for $P=0.2, 0.4, 0.6, 0.8$ and $1$ atm.

Figure 6 The exemplary distribution of $k^{-1}(r) \sim \rho(r)$ for $P=0.2, 0.4, 0.6, 0.8$ and $1$ atm.

4 SUMMARY

As it was discussed in this paper, the mistakes can be introduced in electrical and mechanical domain and produce considerable error and model miss function. Taking into account the proper shape of charge density and membrane deflection can lead to obtain the fine accordance of the simulation with measured results.

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