

# Analytical Surface Potential Model with Polysilicon Gate Depletion Effect for NMOS

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## ABSTRACT

Different modeling approaches for the sub-100nm MOSFET are discussed in [11] and the surface potential description model is reported to be promising, [9, 11]. Surface potential changes impact gate capacitance and current-voltage (I-V) characteristics of the MOS device at thin gate oxides (below 4nm.) The surface potential model is usually obtained by solving the Poisson equation with boundary conditions. The major drawback of this approach is that the surface potential is given by an implicit relation and can only be solved iteratively, requiring expensive computational time. For circuit analysis application, analytical, explicit solutions are preferred because of their simplicity and computational speed. Here we present an analytical model for the polysilicon (poly) gate depletion effect on the surface potential of the NMOS device. The reduction of the channel surface potential due to poly depletion is expressed in terms of the doping concentration on both sides of the gate oxide and the oxide thickness. The surface potential model is derived by directly solving the Poisson equation on the poly and silicon sides using asymptotic methods,[4], and the final analytical model exhibits an excellent fit with numerical data (see Figures 2 and 3.) We believe that these models will be very useful in improving SPICE circuit simulations in advanced VLSI since the gate depletion effect is significant in nanoscale MOSFET devices.

**Keywords:** Device modeling, MOSFET, Polydepletion, SPICE, Surface potential.

## 1 INTRODUCTION

Asymptotic methods to solve the semiconductor equations have been shown successful, e.g. see [1-6]. Here we extend the asymptotic method to solve the semiconductor equations for both the silicon and poly sides

of the NMOS device. The model is applicable for deep submicron MOSFETs with thin gate oxide, < 4nm, and high level of channel doping. In NMOS device modeling, for  $N_d \gg N_a$ , potential changes in the poly are relatively small, and there is no serious penalty in neglecting them. For very small devices poly depletion effects can no longer be ignored, and their neglect may cause non-physical model parameter extraction results, [7]. Current CMOS technology requires ultra-thin gate dielectrics and higher levels of channel doping in order to maximize the drive current of the transistor. For a given poly doping, an increase in channel doping and a decrease in the oxide thickness have a direct effect on poly depletion, [8]. Consequent MOSFET performance degradation of reduced channel current and gate capacitance, [8], is of major concern. A heavily doped poly gate mimics a metal gate. However, for the very small devices the electric potential changes in the poly are large enough to cause device performance degradation when the device is operating at strong inversion. This performance degradation is due to the voltage drop across the poly gate as a result of the formation of a depletion layer near the poly/silicon-oxide interface. Moreover the derivative of the electric potential at the poly/silicon-oxide and silicon/silicon-oxide interfaces is equal.

The effect of poly depletion on the channel surface potential and I-V characteristics is well established [8, 10]. As the C-V plots indicate, this effect is significant far from the threshold voltage. Adjusting the threshold voltage does not capture the whole effect, and detailed examination of the poly depletion effect at a fundamental level is required. Here we present a surface-potential compact model with the poly depletion effect suitable for SPICE application.

## 2 POLYDEPLETION MODEL

We apply the one-dimensional Poisson equation for

the potential  $\psi(x)$ , where  $x$  is the perpendicular distance from the gate to silicon substrate. In this model, both the poly and the silicon doping are considered to be uniform and separated by a thin oxide layer.

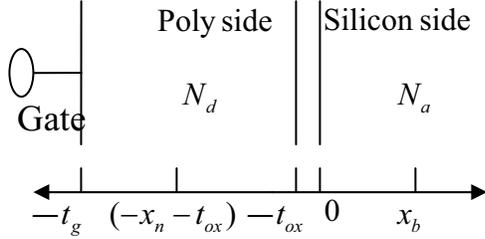


Figure 1: Poly/oxide and Silicon/oxide interfaces

Thus, the Poisson equation for the two regions become

$$\psi'' = \frac{q}{\epsilon_s} \begin{cases} n - p - N_d & x \leq -t_{ox} \\ n - p + N_a & x \geq 0 \end{cases} \quad (1)$$

$$n = n_i e^{(\psi - \phi_n)/V_{th}}$$

$$p = n_i e^{-(\psi - \phi_p)/V_{th}}$$

The boundary conditions consist of the continuity of electric potential,  $\psi$ , and the electric displacement,  $\epsilon \partial \psi / \partial x$ , at the oxide interfaces  $x = 0, -t_{ox}$ .

The assumption here is that the doping density  $N_d \gg N_a$  in the poly and  $N_a \gg N_d$  in the silicon, where  $N_d$  and  $N_a$  are the donor and acceptor doping densities respectively. The parameter  $q$  represents electron charge,  $\epsilon_s$  semiconductor permittivity,  $\epsilon_i$  oxide permittivity,  $n$  electron density,  $p$  hole density,  $n_i$  intrinsic density,  $V_{fb}$  flat band voltage,  $V_{gs}$  applied gate voltage,  $V_d$  drain voltage,  $V_s$  source voltage,  $t_{ox}$  oxide thickness,  $K_b$  Boltzmann constant,  $T$  temperature,  $V_{th} = K_b T / q$ .

We use the following parameter scaling for the gate voltage, electric potential, quasi-Fermi potential, drain/source voltage, gate oxide capacitance respectively, and we non-dimensionalise lengths using the length scale  $L_d \sqrt{\ln \lambda / \lambda}$ .

$$[V_{gs}, (V_{gs} - V_{fb}), \psi, \phi] = [v_g, v_g^*, w, \phi] V_{th} \ln \lambda;$$

$$V_{ds} = (V_d - V_s) = v_{ds} V_{th}; \quad C_{ox} = \frac{c_{ox} \epsilon_s}{L_d} = \frac{\epsilon_i}{t_{ox}}; \quad (3)$$

$$L_d^2 = \epsilon_s V_{th} / q n_i$$

The parameter  $\lambda = N_a / n_i$  ranges from  $10^6$  to  $10^9$  and it is the size of this parameter (or more accurately its logarithm) that allows asymptotic approximations to be effective. We use the parameter  $\beta = N_a / N_d$  to indicate a doping reference. Typically  $\beta$  is order  $10^{-3}$ , and it is sometimes useful to make approximations for  $\beta \ll 1$ .

In scaled variables equation (1) becomes

$$w'' = \begin{cases} \frac{1}{\beta} e^{(w - v_g^*) \ln \lambda} - \beta e^{-(w + 2 - v_g^*) \ln \lambda} - 1/\beta & x \leq -t_{ox} \\ e^{(w - 1 - \phi) \ln \lambda} - e^{-(w + 1 - \phi) \ln \lambda} + 1 & x \geq 0 \end{cases} \quad (4)$$

where at thermal equilibrium,  $\phi_n = \phi_p = \phi$ .

The applied gate voltage generates a difference between the levels of the quasi-Fermi potentials at the extremities of the device. The scaled quasi-Fermi potential is taken to be unity in the silicon substrate ( $\phi = 1$ , so that  $w \rightarrow 0$  at large  $x$ ), and it is  $1 + v_g$  at the poly gate. The Fermi-potential varies from  $v_s / \ln \lambda$  to  $v_d / \ln \lambda$  locally in the silicon channel. The boundary condition on the scaled electrostatic potential at  $x = -x_n - t_{ox}$  is then the applied gate voltage minus the flat band voltage, where the flat band voltage represents the built-in potential or work functions across the oxide interfaces, [12].

The poly is considered to be in depletion at strong inversion of the channel and the solution for the poly side is

$$w = v_g^* - (x + x_n + t_{ox})^2 / 2\beta \quad (5)$$

$$\text{for } (-x_n - t_{ox}) \leq x \leq -t_{ox}$$

Using the continuity of electric displacement, the boundary conditions at  $x = -t_{ox}$  and  $x = 0$  become

$$\left. \frac{dw}{dx} \right|_{x=-t_{ox}} = \left. \frac{dw}{dx} \right|_{x=0} = c_{ox} \sqrt{\ln \lambda / \lambda} (w_s - w_t) \quad (6)$$

$$\text{where } w_s = w(0) \text{ and } w_t = w(-t_{ox})$$

The poly depletion width has two possible solutions that can be obtained from the boundary condition (6) and equation (5) at  $x = -t_{ox}$ , and we choose the physically valid one. The scaled poly depletion depth is

$$x_n = -\sqrt{\lambda / \ln \lambda} / c_{ox} + \sqrt{\left(\frac{1}{c_{ox}}\right)^2 \frac{\lambda}{\ln \lambda} + 2\beta(v_g^* - w_s)} \quad (7)$$

The boundary condition at  $x = 0$  in (6) gives equation (8) below. It replaces equation (23) of [4] to account for the poly depletion effect

$$\left. \frac{dw}{dx} \right|_{x=0} = \frac{1}{\beta} \left\{ \frac{\sqrt{\lambda / \ln \lambda} / c_{ox} - \sqrt{\left(\frac{1}{c_{ox}}\right)^2 \frac{\lambda}{\ln \lambda} + 2\beta(v_g^* - w_s)}}{\left(\frac{1}{c_{ox}}\right)^2 \frac{\lambda}{\ln \lambda} + 2\beta(v_g^* - w_s)} \right\} \quad (8)$$

Methods to solve the silicon side of the equation are well developed, and a detailed asymptotic analysis of the equation is done in [3, 4 and 6.]. For the silicon side,

$$\frac{1}{2} \left( \left. \frac{dw}{dx} \right|_{x=0} \right)^2 = \frac{1}{\lambda \ln \lambda} \{ \exp[(w_s - \varphi - 1) \ln \lambda] + \exp[-(w_s + 1) \ln \lambda] \} + w_s + 1 \quad (9)$$

The scaled surface potential can be solved from equation (8) and (9) using the first order Newton iteration technique. The numerical iteration converges pretty quickly (less than five iterations):

$$(w_s)_{n+1} = (w_s)_n - F[(w_s)_n] / F'[(w_s)_n] \quad (10)$$

$n = 0, 1, 2, \dots$

where

$$F(w_s) = \frac{1}{\beta} \left\{ \frac{\sqrt{\lambda / c_{ox}^2 \ln \lambda}}{\left(\frac{1}{c_{ox}}\right)^2 \frac{\lambda}{\ln \lambda} + 2\beta(v_g^* - w_s)} \right\} + \sqrt{2\{\exp[(w_s - \varphi - 1) \ln \lambda] + \exp[-(w_s + 1) \ln \lambda]\} / \lambda \ln \lambda + 2w_s + 2}$$

$F'$  is the derivative of  $F$  with respect to the surface potential. The second iterative solution  $(w_s)_1$  is a good analytical approximation and its comparison with the full numerical, equation (10), is shown in Figure 2 and 3.

$$(w_s)_1 = (w_s)_0 - F[(w_s)_0] / F'[(w_s)_0] \quad (11)$$

where

$$(w_s)_0 = 1 + \varphi + \frac{2}{\ln \lambda} [\ln(\ln \lambda) + \ln z_0] \quad (12)$$

$$z_0 = \frac{c}{\sqrt{2}} (\varphi_p - \varphi), c = c_{ox} / \sqrt{\lambda}$$

The pinch-off potential is given by

$$\varphi_p = v_g^* - 1 + \left\{ 1 - \sqrt{1 + \ln(\lambda) 2c^2 (1 + v_g^* - 1 / \ln \lambda)} \right\} / c^2 \ln \lambda \quad (13)$$

Equation (12) is a good approximation if there is no poly depletion and detailed asymptotic analysis and comparison with the numerical solution is done in [4].

For  $\beta \ll 1$  the analytical solution with poly depletion can further approximated by

$$\psi_s = w_s V_{th} \ln \lambda$$

$$w_s = 1 + \varphi + 2 \frac{\ln(\ln \lambda)}{\ln \lambda} + 2 \frac{\ln Z_1}{\ln \lambda} \quad (14)$$

$$Z_1 = \frac{c}{\sqrt{2}} (\varphi_p - \varphi) - \frac{\beta c^3}{2\sqrt{2}} (\varphi_p - \varphi)^2 \ln \lambda$$

Comparison of Equation (14) with the full numerical result is shown in Figures 4 and 5.

### 3 RESULTS AND CONCLUSION

The main purpose of this work is to present a new poly depletion analytic surface potential model that is going to be used to derive a MOSFET I-V model by applying semiconductor device physics. The model is derived from the Poisson equation using asymptotic methods, and it is intended for application in SPICE circuit simulators.

We are able to achieve analytic models for the poly depletion effect for the channel surface potential at strong inversion,  $\varphi_p > \varphi$ . Our final model comparison, with and without poly depletion confirms that one of the main sources for poly depletion effect and MOSFET performance degradation is an increase in the channel to poly doping ratio,  $\beta$ .

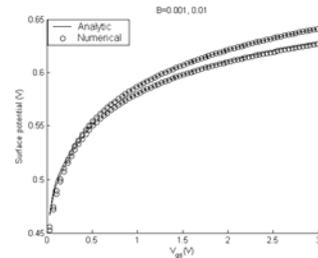


Figure 2: Surface potential versus gate voltage for 3.5 nm gate oxide with different channel to poly doping ratios,  $N_A/N_D = \beta = 0.001, 0.01$  from top to bottom, source/drain voltage is zero, and analytical result is determined from equation (11).

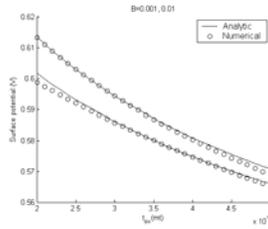


Figure 3: Surface potential versus oxide thickness for a gate voltage of 1V with different channel to poly doping ratios,  $\beta=0.001, 0.01$ , top to bottom. Source/drain voltage is zero and analytical result is determined from equation (11).

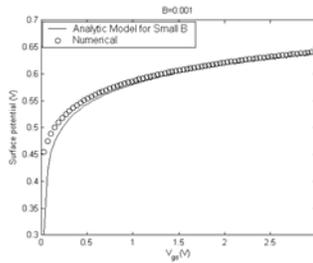


Figure 4: Surface potential versus gate voltage for  $t_{ox} = 3.5nm$ ,  $\phi = 0$ ,  $\beta = 0.001$  and  $N_a = 10^{17} cm^{-3}$  using equation (14).

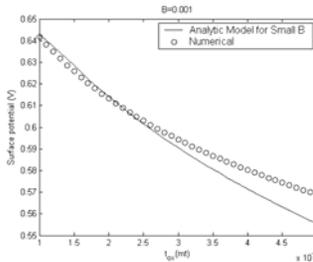


Figure 5: Surface potential versus oxide thickness for  $V_{gs} = 1V$ ,  $\phi = 0$ ,  $\beta = 0.001$  and  $N_a = 10^{17} cm^{-3}$  using equation (14).

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