Transient Analysis of Cooling of Biological Tissues Using Semi-Infinite Approximation

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ABSTRACT

Thermal cooling of biological tissues due to flow of blood in micro-capillaries is useful for several biomedical applications [1]-[4]. The objective of the present study is to develop a better qualitative and quantitative understanding of the thermal cooling due to interaction between the blood flow in capillaries and the tissue environment. Analytical expression for one-dimensional temperature distribution in biological tissue is reported [5] without considering the bloodstream cooling. In this paper, an analytical modeling is reported to determine the transient temperature distribution inside a tissue due to flow of blood. The contact resistance between the tissues is neglected. Using the continuum approach the tissues are modeled as a single body and localized tissue is treated as a semi-infinite body. Finally the expression for heat flux is determined based on the transient temperature distribution.

Keywords: semi-infinite approximation, thermal cooling, biological tissues

1 INTRODUCTION

The significance of the thermal heat transfer can be realized from the fact that bioheat transfer process are encountered in different conditions like cancer hyperthermia, laser surgery, thermal comfort analysis, and tissue thermal parameter estimation [6]. The present work deals with determining an analytical expression for the steady-state and transient temperature distribution inside a tissue due to flow of blood around it. A linear profile for volumetric heat cooling inside the tissues is assumed. A second-order temperature profile is assumed taking into consideration the appropriate boundary conditions. First an expression for the extent of the cold wave travel or the penetration depth δ (t) is determined by solving the integral equation and thereafter this value is used to find the temperature profile. The equation related the steady and the transient temperature at any point inside the tissue with its thermal conductivity, density and the specific heat.

2 PROBLEM DEFINED

The flow of blood around the tissues has a transient effect on the thermal transport between its flow and the tissues. Figure 1 shows the transient problem in consideration. A linear volumetric cooling is assumed, represented by:

$$W = -AT$$

where A is a constant and T is the instantaneous temperature inside the tissue body and the negative sign signifies the cooling condition. Figure 1 shows the problem in concern.

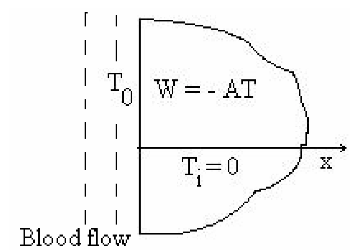

Figure 1: Transient Problem Conditions

Applying the energy conservation principle we can write:

$$\dot{S} = \dot{I} - \dot{O} \tag{1}$$

i.e. the storage rate is equal to the net difference between the inflow and the outflow heat flux. Now the storage rate can be expressed as:

$$\dot{S} = \rho\, c\, A_c \, \frac{d}{dt}\int_0^{\infty} \left(T - T_i\right)\, dx$$

where ρ, c and A_c represent the density, specific heat and the cross-section of the tissue and T_i is the initial temperature of the tissue. Here the integral limits signifies the extent of the heat wave travel. Let δ be the extent of heat wave travel inside the tissue. Since there is no heat wave travel after the point δ hence the above equation can be written as:

$$\dot{S} = \rho\, c\, A_c \, \frac{d}{dt}\int_0^{\delta} \left(T - T_i\right)\, dx$$

Input heat flux can be written as:

$$\dot{I} = -k\,A_c\,\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

k being the thermal conductivity of the tissue, and taking the negative sign of the cooling effect into account the output can be written as:

$$\dot{O} = \int_0^{\delta} A_c\,\left(T - T_i\right)\,dx$$

Now substituting the different values in the governing energy equation (1) and noting that the initial temperature is equal to 0, we get:

$$\rho\,c\,\frac{d}{dt}\int_0^{\delta} T\,dx = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} - \int_0^{\delta} T\,dx \qquad (2)$$

Now assuming a second degree polynomial for the temperature profile and using the boundary conditions as:

$$@\ x = 0,\ T = T_0,$$
$$@\ x = \delta,\ \frac{\partial T}{\partial x} = 0, and$$
$$@\ x = \delta,\ T = 0.$$

we get the temperature profile as:

$$\frac{T}{T_0} = \left(1 - \frac{x}{\delta}\right)^2$$

Now substituting the temperature profile in Eq. (2) and solving the integral equation, we get the following solvable differential equation:

$$\frac{d\delta^2}{dt} + \frac{2A}{\rho\,c}\delta^2 - 12\alpha = 0$$

Now substituting $\delta^2 = z$ and noting that z has two components, particular and homogeneous, the final solution takes the form:

$$z = De^{\left(-\frac{2At}{\rho c}\right)} + \frac{6k}{A} \qquad (3)$$

where D is a constant. The value of D can be found out by applying the boundary condition:

$$@\ t = 0,\ \delta = 0\ and\ u = 0$$
$$\Rightarrow D = -\frac{6k}{A}$$

Substituting the value of D in Eq. (3) and noting that $\delta^2 = z$, we get the final expression for the heat wave travel inside the tissue as:

$$\delta = \sqrt{\frac{6\,k}{A}\left[1 - e^{\left(-\frac{2At}{\rho c}\right)}\right]}$$

and hence the final expression for the temperature profile for the transient condition of blood flow around the tissues is:

$$T\left(x,t\right) = T_0\left\{1 - \frac{x}{\sqrt{\frac{6\,k}{A}\left[1 - e^{\left(-\frac{2At}{\rho c}\right)}\right]}}\right\}^2 \qquad (4)$$

The heat flux associated with the above temperature distribution can be determined from the heat diffusion equation:

$$q = -k\,A_c\left.\frac{\partial T}{\partial x}\right|_{x=0} \qquad (5)$$

Substituting the temperature profile represented by Eq. (4) in Eq. (5), we get:

$$q = \frac{2\,k\,A_c\,T_0}{\sqrt{\frac{6\,k}{A}\left[1 - e^{\left(-\frac{2At}{\rho c}\right)}\right]}}$$

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