Vector potential equivalent circuit for efficient modeling of interconnect inductance*

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ABSTRACT

We present a compact topology for inductive parasitics, using the vector potential as a state variable. The model is local, i.e., only coupling between neighboring wires is explicitly modeled. However, the topology accounts for long-range coupling by propagating the vector potential from one wire to the next. Examples of rule-based generation and model reduction are presented for a digital bus.

Keywords: Interconnects; Inductance; Vector potential; Model reduction

1 INTRODUCTION

Modeling of high-speed interconnects must account for the full range of electrostatic, magnetostatic, and possibly electromagnetic effects in a large, complex, three-dimensional structure. While resistance and capacitance tend to be local, inductance is global and gives rise to dense interaction matrices. Modeling of inductance in interconnects is usually accomplished by defining a dense matrix of interaction coefficients between all wires, or partial inductance matrix (PIM) [1]:

$$V_i = \sum_j L_{ij} \frac{dI_j}{dt}, \tag{1}$$

where I_j is the current flowing through wire j and V_i is the resulting electromotive force on wire i. The main problem with this approach is that the number of coefficients, and circuit elements, grows with the square of the number of wires, so that circuit simulation rapidly becomes unmanageable. Even for a relatively small circuit, in the range of thousands of devices, millions of coupling elements result, which dominate the simulation time. On the other hand, the arbitrary removal of inductive couplings can lead to severe loss of accuracy and even loss of stability of the resulting network.

Existing methods to sparsify the inductance matrix can be divided into two general groups:

A) Partitioning. The circuit is subdivided into blocks, and interactions between blocks are neglected, while interactions within each block remain dense. In its most basic form, this method is very easy to implement, as it simply consists in zeroing entire blocks of the PIM. While the resulting, block-diagonal PIM is always stable, the loss of accuracy due to the neglect of long-range interactions may be unacceptable. One way of achieving the partitioning is to identify return paths which isolate areas of the circuit [2], [3]. In the shell (shift-and-truncate) method, fictitious currents are added to cancel couplings beyond a given radius [4]–[6]. Possibly, return loops can be enumerated for each wire, so that only loop inductance, rather than partial inductance, is modeled within each block [2]. In the limit of one wire per block, only wire self-inductances are modeled, neglecting inductive coupling [7]. At the other extreme, long-range couplings can be reintroduced by means of hierarchical models [8].

B) Inversion. In this most recent approach, the model is recast in an inverse form: i.e., we replace Eq. (1) by

$$\frac{dI_i}{dt} = \sum_j K_{ij} V_j, \tag{2}$$

where $K = L^{-1}$. It has been shown that the inverse matrix K has very attractive properties, such as better sparsity, and remains stable if truncated [9], [10]. It is not necessary to invert the entire matrix, because the circuit can be partitioned into small blocks and the PIM inverted for each of them. The resulting inverse matrices are then 'stitched' together.

In this work we present a new approach, based on the vector-potential formalism which describes the magnetostatic problem on the same grounds of electrostatics [11]. The model is loosely related to the inversion method, in that it recasts the problem in a different form which leads to better locality properties. We will show that the new topology has good properties in terms of stability, accuracy-complexity tradeoff, and physical interpretation.

2 VECTOR POTENTIAL EQUIVALENT CIRCUIT TOPOLOGY

In order to derive a local equivalent circuit for the vector potential, we start from the basic equation of magnetostatics:

$$\nabla^2 A_i + \mu J_i = 0, \qquad i = x, y, z, \tag{3}$$

where $\mathbf{A}$ is the vector potential in the Coulomb gauge:

$$\nabla \times \mathbf{A} = \mathbf{B}, \qquad \nabla \cdot \mathbf{A} = 0.$$

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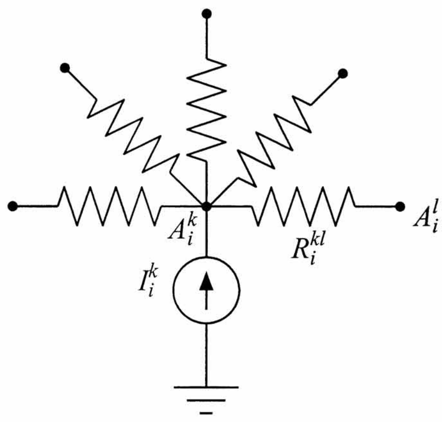

Figure 1: Equivalent circuit for the i-component of the vector potential **A**. *Voltages A_i^k and A_i^l correspond to the vector potentials in two neighboring control volumes Ω^k and Ω^l.*

We now decompose the entire domain of interest into discrete *control volumes* of arbitrary shape, that we denote as Ω^k [12], [13]. Typically, each control volume will surround a single wire. We also define S^{kl} as the surface separating volume Ω^k from Ω^l. Integrating Eq. (3) over volume Ω^k and applying the Gauss theorem, we obtain

$$\sum_l \int_{S^{kl}} d\mathbf{S} \cdot \nabla A_i + \mu \int_{\Omega^k} d\Omega J_i = 0, \quad (4)$$

where the sum over l includes all nearest neighbors of Ω^k. The surface integrals in Eq. (4) are proportional to the gradient of the vector potential on the surfaces S^{kl}. This is similar to the expression for conduction current flow through a surface S:

$$I = -\sigma \int_S d\mathbf{S} \cdot \nabla \phi,$$

where σ is the conductivity, and ϕ is the electrostatic (scalar) potential. By analogy with Ohmic transport, we model the flux of the gradient of the vector potential as a current flow through resistors R_i^{kl}. Eq. (4) is then rewritten as

$$\sum_l \frac{A_i^l - A_i^k}{R_i^{kl}} + I_i^k = 0 \quad (5)$$

where the node voltages A_i^k and A_i^l represent the (suitably averaged) i-component of the vector potential in volumes Ω^k and Ω^l. Eq. (5) can be seen as the Kirchhoff current law for the *vector potential equivalent circuit* (VPEC) of Fig. 1.

To complete the model, one needs current sources to feed the VPEC:

$$I_i^k = F_i^k I^k$$

and voltage sources to represent the electromotive force V_{em}^k:

$$V_{em}^k = -\sum_{i=x,y,z} E_i^k \frac{\partial A_i^k}{\partial t}.$$

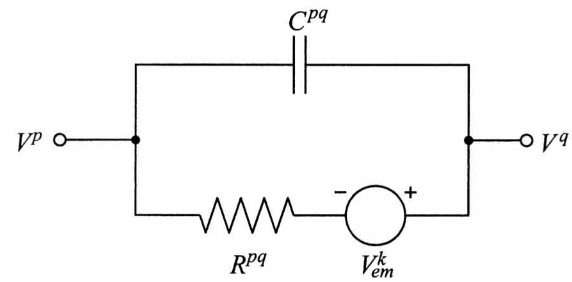

Figure 2: Equivalent circuit connecting two voltage nodes, V^p and V^q, including ohmic resistance R^{pq}, dielectric capacitance C^{pq}, and electromotive voltage source V_{em}^k.

The quantity F_i^k is a 'current gain' which maps the conduction current I^k flowing in wire k into *vector-potential current* I_i^k. The quantity E_i^k is a 'voltage gain' which maps variations of vector potential into the electromotive force affecting mobile charge. For narrow, rectilinear wires, the current and voltage gains are given by

$$F_i^k = \mu \lambda_k \mathbf{u}_k \cdot \mathbf{u}_i, \qquad E_i^k = \lambda_k \mathbf{u}_k \cdot \mathbf{u}_i$$

where λ_k is the length of wire k, and $\mathbf{u}_i$ and $\mathbf{u}_k$ are the direction vectors of coordinate i and wire k, respectively.

As in partial-element equivalent circuit (PEEC) models [14], we adopt two complementary discretization schemes, one for the electrical quantities (scalar potential) and another for the magnetic quantities (vector potential). When obtaining circuit equations, the edges of the electrical network will correspond to nodes for the magnetic network, and vice versa. The topology of the electrical circuit is shown in Fig. 2. An ohmic resistance R^{pq} and a capacitance C^{pq} are shown, as well as the electromotive force V_{em}^k.

The VPEC topology offers a number of peculiar advantages over other methods. First of all, although the model is *local,* it allows for *nonlocal* effects, by letting the vector potential 'enter' and 'exit' discrete regions in space. Second, the model is physically sound, because the vector potential is used as a state variable instead of inductor currents as in the PIM approach. By extending the electrostatic paradigm, inductance is modeled similarly to capacitance and resistance, by extracting coupling elements between neighboring circuit nodes. Furthermore, the model can be directly used in a Spice simulation, without changes to the model or the simulator itself. Finally, it can be shown that the VPEC topology is stable (i.e., the resulting PIM is positive definite) as long as all resistors in the vector-potential circuit are positive [11].

3 CIRCUIT EXTRACTION

By analogy with capacitance extraction, the VPEC element values can be obtained by rule-based extraction, or by direct extraction from numerical simulations. In the first case, we use a technique described in [12] to compute element values from exact analytical solutions [11]. We start from an arbitrary distribution of current density, which we call a 'trial' and denote by $\overline{\mathbf{J}}$. The vector-potential resistance R_i^{kl} is then obtained as

$$R_i^{kl} = \frac{\overline{A}_i^l - \overline{A}_i^k}{\int_{S^{kl}} d\mathbf{S} \cdot \nabla \overline{A}_i}, \quad (6)$$

where $\overline{\mathbf{A}}$ is the vector potential computed from the trial current distribution $\overline{\mathbf{J}}$, and $\overline{A}_i^k$ is its average value over Ω^k. Note that in principle, we do not need the current distribution $\overline{\mathbf{J}}$ to satisfy any particular property. In the case of narrow rectilinear conductors, one can simply assume a uniform current density, and force a unit current through one of the regions involved (Ω^k or Ω^l), leaving zero current in all other regions. If the conductors are rectilinear, the surface integral in Eq. (6) can be analytically evaluated, or well approximated, in many important cases (e.g., flat and cylindrical region boundaries).

The direct, rule-based model generation has the advantage of great simplicity and efficiency. However, the accuracy is tightly linked to the choice of the trial current. Also, the circuit topology is limited to nearest-neighbor couplings. Both limitations can be overcome if one uses the VPEC as a topological template, fitting the model parameters to a given PIM. The PIM can be computed from textbook analytical formulas [15], or from standard numerical tools such as FastHenry [16]. The inductance matrix can be written as [11]

$$L^{lk} = \sum_{i=x,y,z} E_i^l Z_i^{lk} F_i^k, \quad (7)$$

where the quantity Z_i^{lk} is an entry in the impedance matrix of the resistive network for the i-component of the vector potential, i.e., the vector potential at node l when a unit current is injected at node k:

$$Z_i^{lk} = \frac{A_i^l}{I_i^k}.$$

From any given wire geometry and PIM, we can immediately determine the VPEC impedance matrix. The VPEC generation problem is then reformulated as *finding the resistors R_i^{kl} to reproduce a given impedance matrix Z_i*. This problem can be solved analytically for simple cases, e.g., a one-dimensional bus topology. In the general case, the VPEC extraction is in principle extremely difficult, as it involves solving a large system of nonlinear equations. However, the problem can be brought down to a manageable size by a block iteration which reduces it to a smaller, linear problem [17]. Note that using this approach, it is not necessary to compute all the elements of the PIM, but only those to which the VPEC elements are to be fitted. This makes the method attractive from a computational point of view. For this technique we speak loosely of *model reduction,* even though the order of the model (i.e., the number of state variables) remains the same. The complexity reduction is achieved by making the matrix formulation of the problem more sparse.

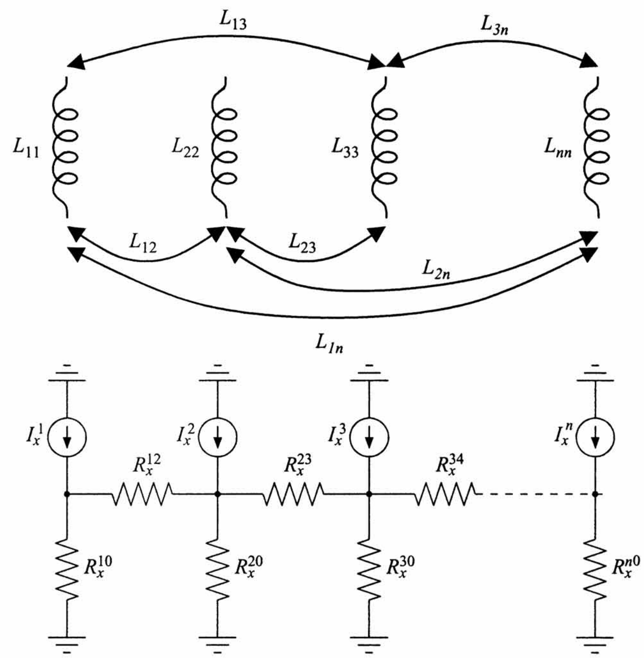

Figure 3: Partial-inductance representation (top) and simple nearest-neighbor vector-potential representation (bottom) for an n-line bus.

However, the resulting PIM from Eq. (7) remains dense (ideally, it remains identical to the original one!)

We apply the two methods to a simple 32-line bus, with four return lines. Each line is $1000\,\mu$m long, $2\,\mu$m thick and $1\,\mu$m wide, with a spacing of $1\,\mu$m. Values of wire resistances and wire-to-wire capacitances were extracted from the wire geometry and material properties. The PIM and VPEC circuit topologies are shown in Fig. 3. For direct, rule-based extraction, only nearest-neighbor resistors are included in the model. For model reduction, any number of neighbors can be included. Fig. 4 compares results from the rule-based method, and model reduction with two and four neighbors. A step signal of 1 V, with rise time of 10 ps, is applied to the first line, and the voltage transient is shown at the far end of the first (aggressor) line and the second, third, and fourth (victim) lines. The extraction and two-neighbor model reduction give comparable results. Adding two extra neighbors improves the accuracy for long-range inductive couplings, at the expense of model complexity.

4 CONCLUSIONS

Model complexity is a barrier to effective modeling of global inductive coupling in integrated circuits. Model reduction techniques must face issues of stability, loss of accuracy, computational complexity, and ease of implementation. We have presented an intrinsically local circuit topology, where the number of coupling coefficients is of the same order of the number of wires in the circuit, rather than quadratic as in the partial-inductance-matrix approach. We have reported

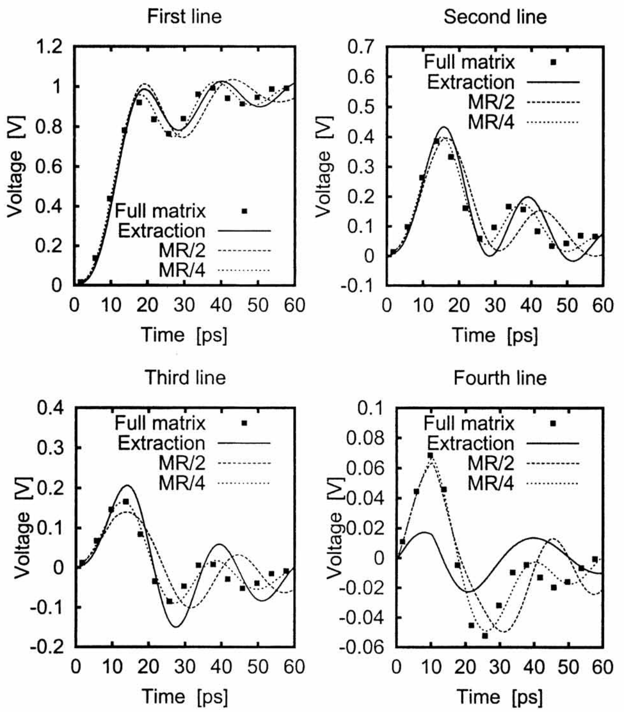

Figure 4: Comparison of rule-based and model-reduction VPEC model generation, for a 32-line bus. The voltage transient at the far end of the first (aggressor) line and the second, third, and fourth (victim) lines are shown, for the cases of full PIM (symbols), direct extraction (solid line), and model reduction (MR) with increasing complexity (connectivity to 2 and 4 neighbors).

examples of both rule-based extraction and model reduction of a precomputed partial-inductance matrix, showing that the method allows great flexibility in the accuracy-complexity tradeoff while keeping an intuitive physical interpretation.

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