Nanotube Stochastic Resonance: Noise–enhanced Detection of Subthreshold Signals

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ABSTRACT

Noise can help signal detection at the nano–level. Experiments on a single–walled carbon nanotube transistor confirm that a threshold exhibits stochastic resonance: a judicious amount of noise can help a threshold–like nanotube transistor detect subthreshold signals while large amounts of noise overwhelm the signals. The nanotube produced this stochastic–resonance effect using three types of *synchronized* discrete–time white noise and two performance measures: mutual information and input–output correlation. Experiments in a cryostatic vacuum chamber added Gaussian, uniform, and impulsive (Cauchy) electrical noise. The electrical noise corrupted a random digital (Bernoulli) voltage sequence that acted as the subthreshold input for the nanotube transistor. The noisy signal stimulated the transistor's gate and produced a sequence of random output (Bernoulli) current in the nanotube. Shannon's mutual information and simple correlation measured the nanotube system's performance gain by comparing the input and output sequences. Neither measure assumed any special nanotube structure. The observed nanotube SR effect was robust: it persisted even when infinite–variance Cauchy noise corrupted the signal stream. Such noise–enhanced signal processing at the nano–level promises applications to signal detection in wideband communication systems and biological and artificial neural networks.

Keywords: stochastic resonance, nanotube transistor, single-walled carbon nanotube, threshold system

Electrical noise can help carbon nanotube transistors detect subthreshold electrical signals by increasing the transistor's input–output mutual information or cross–correlation [1]. The threshold stochastic resonance (SR) occurs for various types of threshold units or neurons [2-7]. Nanotube experiments confirmed the specific SR prediction that simple memoryless threshold neurons exhibit SR for almost all finite–variance and infinite–variance noise types [8]. The experiments used three types of discrete–time additive white noise: Gaussian, uniform, and infinite–variance Cauchy electrical noise. White Gaussian noise gave the nonmonotonic signature of SR in figure 1. The modes of the mutual–information and cross–correlation curves occurred for nonzero noise strength with a standard deviation of at least 0.01.

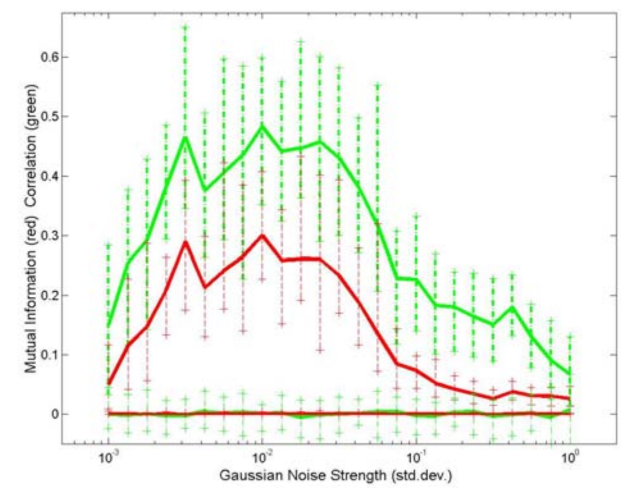

Figure 1: Stochastic resonance with additive white Gaussian noise (see reference [1]). The nanotube detector's mutual information (dark red curve) and zero-lag correlation (light green curve) increase for small amounts of noise and then decrease for larger amounts. The control experiments gave the flat non–SR mutual information (dark red line) and correlation (light green line) when no nanotube bridged the source and drain electrodes. The SR mode or optimal noise level was the same standard deviation value of 0.01 for both performance measures.

The experiments applied a nanotube transistor [9-14] whose threshold–like nonlinearity (figure 2) approximated a threshold detector. A semiconductor carbon nanotube acted as the nanometer–scale conduction channel of the transistor. A noise–corrupted voltage signal S or V stimulated the transistor's gate and produced a nano–amp scale output current Y or I for suprathreshold input voltages. The transconductance G related the drain–to–source current I to the gate voltage V and the threshold voltage V_T in a memoryless function: $I = G\ (V - V_T)$ if $V < V_T$ and zero otherwise. The nanotube transistor was undoped and exhibited p–type current–voltage characteristics so its transconductance G was negative.

The experiments produced the SR effect for the Shannon mutual information and the input–output correlation. Both measures compared the random input and output sequences. The detector performance computed the mutual information by subtracting the nanotube system's

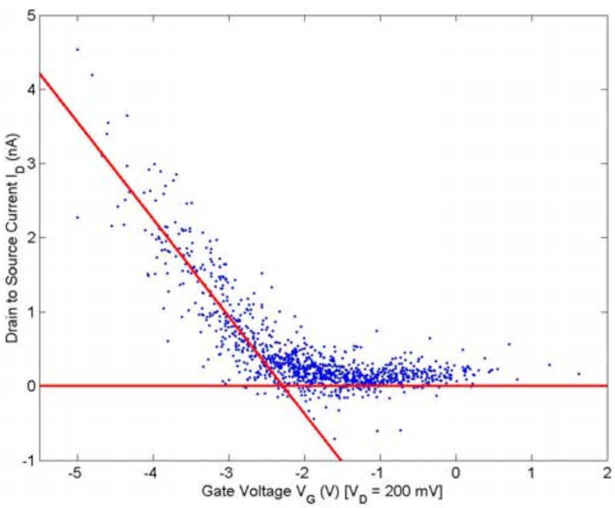

Figure 2 The nanotube transistor's threshold–like gate effect (see reference [1]). Each point shows the detector's response to a random input symbol. The experimental data showed that the nanotube detector behaved as a threshold in response to the noisy input signal stream. The gate effect showed little hysteresis.

output conditional entropy $H(Y|S)$ from its unconditional entropy $H(Y)$: $I(S, Y) = H(Y) - H(Y \mid S)$ where the input signal S was a random binary (Bernoulli) voltage and the output Y was a random transistor current. The detector performance computed the correlation measure [15] by removing the mean from the input and output sequences and normalizing the scalar zero–lag value of their cross–correlation sequence:

$$r_{SY}(l) = \sum_{k=1}^{N} s(k) y(k-l) \tag{1}$$

Each of the nanotube experiments applied 32 independent trials of 1000–symbol input sequences for 24 noise levels per type and over a range of gate voltages. The 24 sampled noise levels ranged from 0.001 to 1 standard deviation (dispersion for infinite–variance Cauchy) linearly in logarithmic scale. The noisy input was a *synchronized* Bernoulli sequence of independent random (subthreshold) ON/OFF values and additive white noise of three types. The discrete-time noise was white because the noise samples were uncorrelated in time. So the discrete-time Fourier transform was 2π–periodic and produced a flat noise power spectrum over the interval $[0, 2\pi]$ [16, 17]. Synchronization allows the nanotube systems to implement a variety of algorithms from signal processing and communications.

The ON/OFF values in figure 1 were ON = –1.6 volts and OFF = –1.4 volts. The input updated the symbols about once every 10 milliseconds. A 200–millivolt drain-source voltage biased the nanotube at room temperature in vacuum. The experiment measured and averaged 10 samples of the detector output at 100 kilo–symbols per second near the end of each symbol interval to estimate the output sequence.

The experiments made no assumptions about the nanotube structure. Nor did they impose a threshold scheme to interpret the detection. A histogram of the output sequences gave the discrete probability density function (pdf) $P(Y = Y_i) = p_i$ that computed the unconditional Shannon entropy:

$$H(Y) = -\sum_{i=1}^{N} p_i \ln p_i \tag{2}$$

for mutual information. A histogram of the sorted output sequences (based on the input symbols) gave the conditional output pdf $P_{Y|S}(Y = Y_i \mid S = S_j) = p_{ji} / p_j$ conditioned on the input symbols that computed the conditional entropy:

$$H(Y \mid S) = -\sum_{i=1}^{N} \sum_{j=1}^{N} p_{ji} \ln\left(\frac{p_{ji}}{p_j}\right) \tag{3}$$

The Shannon mutual information was the difference between the unconditional and the conditional entropies: $I(S, Y) = H(Y) - H(Y \mid S)$. A discrete correlation computed the cross–correlation sequence from the input and output symbol sequences. The zero–lag value of the cross–correlation sequence:

$$r_{SY}(0) = \sum_{k=1}^{N} s(k) y(k) \tag{4}$$

gave a scalar representation for the correlation measure. We converted the ON/OFF voltages to the bipolar values +1 and –1. This converted the input sequence into a nearly zero mean sequence. Subtracting the sample mean from the output sequence improved the match between similar input and output sequences. The normalized correlation measure divided the zero–lag cross correlation $r_{SY}(0)$ by the square root of the energy of the input and the output sequences:

$$C(S, Y) = \frac{\sum_{k=1}^{N} s(k) y(k)}{\sqrt{\sum_{k=1}^{N} s(k) s(k)} \sqrt{\sum_{k=1}^{N} y(k) y(k)}} \tag{5}$$

where the energy of a sequence is the same as the zero–lag value of its autocorrelation:

$$|x| = \sum_{k=1}^{N} x^2(k) = \sum_{k=1}^{N} x(k) x(k-l)\Big|_{l=0} = r_{XX}(0) \tag{6}$$

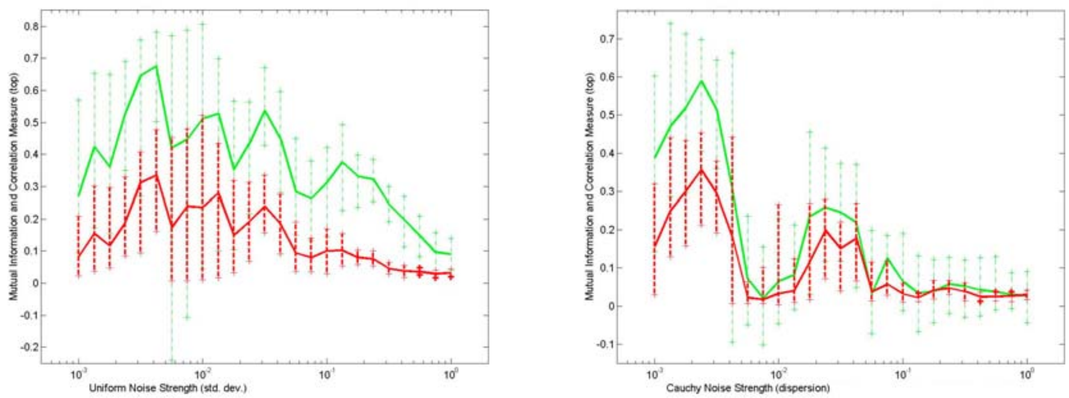

Figure 3: (a) Stochastic resonance with additive white uniform noise (see reference [1]). The noisy subthreshold input produced a clear SR response in both mutual information (dark red curve) and input-output correlation (light green curve) just as with additive white Gaussian noise. The figure shows the SR effect for the subthreshold signal ON = –1.8 V and OFF = –1.6 V. The SR mode is at 0.04 standard deviations. (b) Robust stochastic resonance with additive white Cauchy noise [1]. This highly impulsive noise has infinite variance and infinite higher-order moments. The figure shows an approximate SR effect for the subthreshold signal ON = –2 V and OFF = –1.8 V. The SR mode lies at about the 0.003 dispersion value. Several SR researchers have found multiple modes in the plot of system performance against noise strength [50-52].

The experiment found the SR effect for mutual information and correlation for Gaussian and uniform noise and for four combinations of binary symbols (–2.0, –1.8), (–1.8, –1.6), (–1.6, –1.4), and (–1.4, –1.2) volts. Figure 1 shows the SR effect for additive white Gaussian noise and the subthreshold signal pair ON = –1.6 V and OFF = –1.4 V. The SR mode of the mutual–information curve is six times the value at minimal noise. The SR mode of the correlation curve is three times the value at minimal noise. Figure 3a shows the SR effect for additive white uniform noise and the signal pair ON = –1.8 V and OFF = –1.6 V.

We also passed impulsive or infinite–variance white noise through the nanotube detector to test whether it was robust to occasional large noise spikes. We chose the highly impulsive Cauchy noise [2] for this task. This infinite–variance noise probability density function had the form:

$$p(n) = \left(\frac{1}{\pi}\right)\frac{\gamma}{n^2+\gamma^2} \qquad (7)$$

for zero location and finite dispersion γ. Figure 3b shows that a diminished SR effect still persists for Cauchy noise with subthreshold signal pair ON = –2.0 volts and OFF = –1.8 volts. Not all Cauchy experiments produced a measurable SR effect.

We note that the SR results occurred in a nanotube transistor that shows time–varying hysteresis [18-20] and sensitivity to adsorbed molecules [21-23].

These SR results suggest that nanotubes can exploit noise in other signal processing tasks if advances in nanotube device technology can overcome the problems of hysteresis and parasitic capacitance that affect logic circuits [24] and high-frequency signals [25]. The nanotube signal detectors might apply to broadband [26, 27] or optical communication systems [28] that use sub-micro amp currents and attenuated signals in noise because our nanotube detectors used nano-amp current and could distinguish between subthreshold binary symbols. The detectors might apply to parallel signal processing [29] at the nano level because they could have small minimum feature size [30] in vast parallel arrays of nanotubes. The parallel detectors could apply to spread spectrum communications: each nanotube can act as an antenna [31] that matches a separate frequency channel [32] in frequency hopping and perhaps in other types of spread spectrum communications [33]. A nanotube's length can code for a given frequency [34] while chemical adsorption can tune a nanotube's threshold [22, 23]. The detectors might apply to chemical detection and parallel field programming by tuning the threshold chemically. The nanotube detectors can also operate in a biological environment such as saline solution [35]. The nanotube detectors could interface with biological systems because an electrolyte can act as their gate [35, 36]. The nanotube detectors might also help implement pulse-train neural networks and exploit noise in biological [37-48] or robotic systems because the detectors are threshold devices similar to spiking neurons [49].

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