Optimum Design of an Electrostatic Zipper Actuator

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ABSTRACT

We describe a method for predicting the optimal design of an electrostatic zipper actuator. The shape of the zipping beam and the electrode are optimized to maximize the applied force at a given stroke, while satisfying fabrication constraints, and the constraint that the pull-in instability occurs for a given applied voltage.

1 INTRODUCTION

This paper describes an algorithm for predicting the optimal design of an electrostatic zipper actuator[1], [2]. Such actuators operate [3] by charging up two electrodes (coated with thin dielectric layers) relative to each other. At least one of the electrodes is flexible, with a load attached to its end. When the charging voltage exceeds a critical value, the flexible electrode(s) "buckle" (the so-called "pull-in" instability), causing the two electrodes to come into contact (See Fig. 1). Further increase of the voltage causes the flexible electrode to "zip" along the other electrode, pushing against the applied load.

The force that an electrostatic actuator can exert depends on the applied voltage V, the thickness d of the dielectric layer (with dielectric constant ϵ_i), the beam thickness h, length L and extent b, and the elastic modulus E of the flexible electrode. In typical applications, the applied voltage is a modest ~ 100V; the force created by the actuation mechanism is used to deform silicon beams, with a bulk modulus $E \sim 10^{11}$Pa. Since the maximum energy density in the electric field $\epsilon_i V^2/d^2 \ll E$, such a small actuation voltage can only deform a silicon beam whose length L is much longer than its thickness. This constraint therefore sets the limit on the size of the device that can be successfully actuated.

Herein, we address the following optimization problem: what is the shape of the electrostatic actuator, undergoing the pull-in instability at a given voltage, and achieving a prespecified load, which has the smallest length L. The intuition that underlies our analysis is as follows: the force exerted by the zipper actuator clearly increases by increasing the thickness of the beam applying the force. However, increasing the thickness increases the threshold voltage for the pull-in instability. Therefore, the two requirements (achieving high force but remaining above the pull-in instability threshold) are competing constraints–designs which achieve pull-in at low applied voltages exert small forces; designs which exert large forces are hard to make pull-in. Our optimization strategy will aim to find the design which maximizes the force while satisfying a given pull-in voltage. Here we present the essential steps towards the solution of this problem for a flexible beam zipping on a straight electrode; a more complete presentation will include the generalization of the analysis to simultaneously compute the shapes of both the beam and the electrode.

2 THE MODEL

We model the electrostatic actuator as a laterally compliant beam (with moment of inertia $I(x) = 1/12bh^3$, where b is the breadth of the beam and h the thickness) above a fixed electrode, with a thin, uniform dielectric layer deposited on top of it. The beam and the electrode are held at a voltage V_0 relative to each other. If the deflection of the beam is $w(x)$, there is an electrostatic force pulling the beam and the electrode together of magnitude $0.5b\epsilon_0 V_0^2/(\epsilon_0 w + \epsilon_i d)^2$, where ϵ_0 (ϵ_i) is the dielectric constant of air (insulator), respectively. Hence the equation for the deflection of the beam is

$$(EIw'')'' = -\frac{b\epsilon_0}{2}\frac{V_0^2}{(\epsilon_0 w + \epsilon_i d)^2}. \qquad (1)$$

We nondimensionalize this equation by scaling w and S by the beam thickness h, and scaling all lateral length scales by the length of the electrode L. The spatial dependence of the beam thickness is accounted for by writing $I = bh^3/12\psi(x)$, so that

$$(\psi w'')'' = -\frac{\Gamma}{(w+\beta)^2}, \qquad (2)$$

where $\Gamma = \epsilon_0 (V_0/h)^2 (L/h)^4/E$ and $\beta = \epsilon_0 d(\epsilon_i h)^{-1}$. In these units, $w = w(y)$, where $0 \leq y = x/L \leq 1$.

We would now like to understand the solutions to (2) as the voltage is increased. We first assume that the end of the beam farthest from the electrode is held at fixed distance off the electrode so that $w(1) = w_0$; since there is no moment about the end we also have $w''(1) = 0$. When the electrostatic forces are small, the beam is essentially straight; above a critical Γ, there is a "pull-in" instability where the beam hugs the electrode.

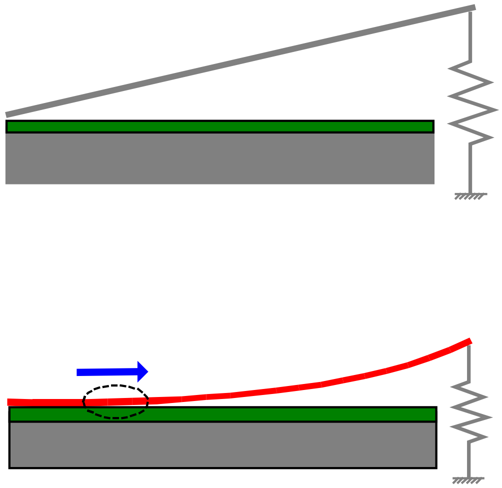

Figure 1: Schematic of the zipper actuator before (top) and after (bottom) the pull-in instability. The top electrode is charged to a voltage relative to the bottom electrode, which is coated by a thin dielectric layer.

Increasing the voltage further causes the beam to zip up along the electrode. In the first regime, before pull-in, the boundary conditions at the beginning of the beam depend on how the beam is held: i.e. $w(0) = w''(0) = 0$ if the end is free to rotate. In the second regime, after pull-in, the boundary conditions are $w(0) = 0; w'(0) = 0; w''(0) = 0$, reflecting that the beam is tangent to the electrode, and also the end is torque free. The last condition arises because if the end were not torque free then the beam would zip further up the electrode. In the second regime, there are more boundary conditions than allowed by the equation. The resolution to this is that the 'zipping' point is arbitrary, so that the left boundary condition is $w(s) = 0; w'(s) = 0; w''(s) = 0$, where s is the zipping point.

Analysis of this equation [4] demonstrates that the force-displacement relationship is given by

$$F_R \sim \left(\frac{\Gamma}{\beta}\right)^{3/4} \frac{1}{\sqrt{w_0}}, \qquad (3)$$

or in dimensional units

$$\mathrm{F} \sim E^{1/4}\left(\frac{\epsilon_i V_0^2}{d^2}\right)^{3/4} \frac{d^{3/4}h^{3/4}b}{\sqrt{\Delta}}, \qquad (4)$$

where Δ is the the gap distance between the right hand side of the zipper and the electrode. The force does not scale as V_0^2, as one might expect from the force between two parallel capacitor plates. The physical reason for the unusual dependence is that the force exerted on the zipper also depends on the voltage-dependent *shape* of the beam near the electrode. Numerical simulations demonstrate that these formulae quantitatively predict the force-displacement conditions over a wide parameter range.

This formula implies that when the thickness of the beam increases, the force increases like $h^{3/4}$; hence increasing the beam thickness increases the applied force.

3 PULL-IN INSTABILITY

The above analysis implies that increasing the force exerted by the zipper requires increasing the beam thickness. However, there is a critical beam thickness above which the pull-in instability does not occur for a given voltage. Ultimately, our optimization analysis will use a nonuniform beam shape ($\psi(x)$ not constant) to beat these constraints as much as possible.

First we introduce a method computing the pull-in voltage. Calculations of the pull-in voltage have been previously carried out [3], [5], using energy methods; our methodology for computing the pull-in voltage is easily amenable to the optimization described below.

To calculate when pull-in occurs, we note that before pull-in, the shape of the beam is determined by equation (2) with the boundary conditions $w(0) = w''(0) = w''(1) = 0$ and $w(1) = w_0$. Before pull-in the deflection from the straight beam shape is small, so $w = w_0 x + \zeta$, where $\zeta \ll w_0 x$. Plugging into equation (2) and linearizing in ζ implies

$$(\psi\zeta'')'' = -\frac{\Gamma}{(w_0 x + \beta)^2} + \frac{2\Gamma}{(w_0 x + \zeta + \beta)^3}\zeta, \qquad (5)$$

with the boundary conditions $\zeta(0) = \zeta''(0) = \zeta(1) = \zeta''(1) = 0$. When Γ is sufficiently small, the solution can be expanded in powers of Γ: $\zeta = \bar{\zeta} = \sum_{n=0}^{\infty} \Gamma^n \zeta_n$. At leading order we have

$$(\psi\zeta_1'')'' = -\frac{1}{(w_0 x + \beta)^2}.$$

In this regime, ζ depends continuously on Γ, so that small changes in Γ lead to small changes in the shape of the beam. At sufficiently large Γ corresponding to the pull-in instability, this expansion breaks down. If we write $\zeta = \bar{\zeta} + \xi$, this occurs when there is a nontrivial solution to

$$(\psi\xi'')'' = \frac{2\Gamma}{(w_0 x + \bar{\zeta} + \beta)^3}\xi, \qquad (6)$$

satisfying the boundary conditions $\xi(0) = \xi''(0) = \xi(1) = \xi''(1) = 0$. The existence of a nonzero ξ occurs at a critical value of $\Gamma = \Gamma^*$, which signals a discontinuous change in the solution, i.e. pull-in. Note that this is a nonlinear eigenvalue problem, since $\bar{\zeta} = \bar{\zeta}(\Gamma)$.

A good approximation to Γ^* can be obtained by using a truncated expansion for $\bar{\zeta}$ in equation (6). For example, a first approximation to Γ^* can be obtained by solving the linear eigenvalue problem

$$(\psi\xi'')'' = \frac{2\Gamma}{(w_0 x + \beta)^3}\xi \tag{7}$$

A next approximation can be obtained by solving the nonlinear eigenvalue problem

$$(\psi\xi'')'' = \frac{2\Gamma}{(w_0 x + \Gamma\zeta_1 + \beta)^3}\xi. \tag{8}$$

This nonlinear eigenvalue problem (8) can be solved by iteration. Our numerical calculations show that the solution to the nonlinear problem differs from the solution to the linear problem by about a factor of two; In the limit that w_0 is large, the pullin voltage obeys the approximate law $\Gamma^* \approx \left(\frac{\pi}{4}\right)^4 w_0^3$. In dimensional units, this corresponds to the critical voltage

$$\epsilon_0 V^2 \approx E\frac{\Delta^3 h^3}{12L^4}\left(\frac{\pi}{4}\right)^4. \tag{9}$$

The critical voltage increases with increasing gap thickness Δ and beam thickness, and decreases with increasing length of the beam.

4 UNIFORM ACTUATOR

The optimal actuator design is the device of smallest length L which achieves a given load. First we assume that that both the beam thickness does not vary along the actuator, so the optimization involves finding the the beam thickness h allowing the actuator to have smallest length.

To determine the optimum choice of (h, L), we require first that the pull-in instability occurs, and second that the actuator force $F_{actuator}$ is larger than the requisite applied load. The pull-in requirement is

$$L^4 > h^6 \frac{E}{12\epsilon_0 V^2}\left(\frac{w_0}{h}\right)^3\left(\frac{\pi}{4}\right)^4. \tag{10}$$

For the force requirement, the force applied by the actuator must exceed the applied load. The load typically requires deforming an elastic element, the size of which is comparable to the actuator itself: this is because the load force is minimized by making it as large as possible, and the size of the device is not compromised if the load is as large as an actuator. For definiteness, in the present analysis we assume the load is a relay switch [8], for which the force requirement implies

$$h > \frac{E}{12\epsilon_0 V^2}\left(800 h_{switch}^3 d\right)^{4/3}\frac{w_0^{2/3}}{L^4}, \tag{11}$$

where I_{switch} and h_{switch} are the moment of inertia and thickness of the switch. The two requirements are summarized in figure 2: Below the dashed line are the (h, L) combinations that will undergo pullin; above the solid line are the combinations that satisfy the force constraint.

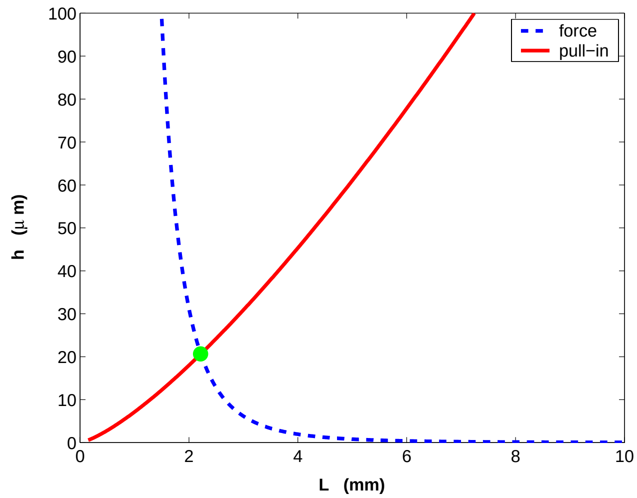

Figure 2: Phase diagram showing the allowed uniform zipper configurations. Below the dashed line are the (h, L) combinations that will undergo pullin; above the solid line are the combinations that satisfy the force constraint. Calculations assume $V = 100$Volts, $w_0 = 10\mu$m, and bulk modulus E corresponding to silicon.

The intersection point labelled in the figure shows the size of the smallest device satisfying the force and pull in constraints. This is the optimal device with a uniform thickness beam.

5 NONUNIFORM ACTUATOR

We now ask: can the length L of the device be made smaller by allowing the zipping beam to have nonuniform thickness? Intuitively, the reason this should be possible is as follows: the force can be increased by increasing the beam thickness at the pull in point; in contrast, the pull-in instability relies on an average beam thickness being thin. Therefore, perhaps it is possible to only thicken the beam in one region, which would increase the force but not affect the pull in instability.

In order to find the optimal device, we need several pieces of information: first, we must have a formula for how the force exerted by the actuator (at a given stroke w_0) changes upon changing the beam shape, $\psi \to \psi + \delta\psi$. Such a formula can be derived using the adjoint-method of optimal control theory[6]; the formula is of the form

$$\delta F = \int \delta\psi \lambda'' w'', \tag{12}$$

where λ is a function that solves an adjoint equation $(\lambda''\psi)'' - \Gamma(2x/(w+\beta)^3 + 2\lambda/(w+\beta)^3) = 0$ with appropriate boundary conditions.

Second, we need a formula for how the pull-in voltage Γ changes on changing ψ. A straightforward analysis of the pullin equation (7) (analogous to [7], [8]) gives that

$$\delta\Gamma = C \int \delta\psi \zeta''^2, \quad (13)$$

where $C = \int \zeta^2/(w_0 x + \beta)^3$.

Now, for any change in the beam shape $\delta\psi$, there will be a change in both the pullin voltage and the applied Force. Since the goal of the optimization is to produce a device with smaller extent L, we will use any increase in the applied force and decrease in the pull in voltage that we obtain to decrease L. In order to maximize our effort, we would like the decrease in L that is achievable from increasing the applied force to be *exactly* the same as that from decreasing the pull in voltage: Given the scalings $\Gamma \sim L^4$ and $F \sim L^{-3}$, this requirement translates into the constraint

$$-\frac{\delta F}{3F} + \frac{\delta\Gamma}{4\Gamma} = \int \delta\psi \left(-\frac{\lambda'' w''}{3} + C\frac{\zeta'^2}{3} \right) = 0. \quad (14)$$

The optimal device can now be found by carrying out an iterative procedure in which we choose $\delta\psi$ to increase the force, subject to the constraint equation (14). Figure 3 demonstrates the results of the iteration, assuming initial beam thickness $h = 20\mu$m, length $L = 2.2$mm; dielectric layer thickness $h_0 = 2\mu$m, stroke $w_0 = 65\mu$m, and voltage $V = 100$ volts. The top panel shows the change in the shape of the beam with iteration; the bottom shows the evolution of the force on the beam and the pull in voltage. The algorithm achieves an approximately 40 percent decrease in pull in voltage and a corresponding increase in the force: this is achieved by thickening the beam *near the zip point*, and thinning it throughout the rest of the beam. The average thickness of the beam therefore decreases (lowering the pull in voltage) , while the thickness of the beam at the touchdown point increases (raising the force). The optimal design for a given situation is chosen by the fabrication constraints.

Note that the above calculation shows the improved design allows an increase (decrease) in force (pull in voltage) for fixed L; in practice, we will keep the applied load and pull in voltage the same and decrease L.

To conclude, we have presented an algorithm for optimizing the force exerted by an electrostatic actuator while satisfying the pull in constraints, and shown that modest changes in the shape of an electrostatic actuator can lead to dramatic enhancement of the force. Larger enhancements can be obtained by simultaneously solving for the optimal beam and electrode shapes, a topic that we will present in detail elsewhere.

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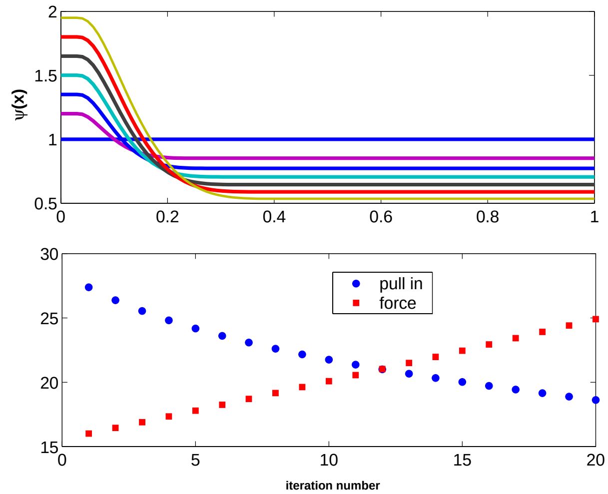

Figure 3: Evolution of the zipper profile upon optimization. Upper panel shows the evolution of the beam shape, and lower panel shows the corresponding decrease in pull in voltage and increase in applied force.

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