

# Numerical Modeling of a Piezoelectric Micropump

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## ABSTRACT

An accurate description and understanding of any pumping method critical, especially on the micro scale. With the existence of a comprehensive and adaptable model, accurate preproduction predictions of performance are realized. A three-dimensional FEA approach for parametric design and optimization of a piezoelectrically actuated membrane micropump is presented. The model includes the piezoelectric material, membrane, pumping chamber, and cantilever valves. This numerical representation includes electro-mechanical coupling for piezoelectric actuation as well as consideration of fluid-structural interaction. Transient consideration of electrical, mechanical, and fluidic effects is included. The effects of independent factors such as component geometry, backpressure, and excitation voltage and frequency are each evaluated. Outputs include membrane deflection, flow pattern and velocities, and volumetric flow rate.

**Keywords:** finite element analysis, simulation, micropump, piezoelectric, modeling

## 1 INTRODUCTION

Research on micropumps began in 1980 [1] with significant advances in both pumping effectiveness and modeling capabilities resulting from this and subsequent work. Efforts continue to develop both analytical [2] and numerical models [3]. The valves on the pump in this work are based on those found in literature [4]. In general, efforts to characterize microscale pumps and their valve counterparts have been diverse and less than comprehensive. Complete understanding of the system is only possible when such factors as the component dimensions, backpressure, driving frequency, membrane deflection, and fluid flow are each included in and able to be analyzed by the model. The work described in this paper focuses on the development of a three dimensional numerical model using a commercially available finite element code (ANSYS 7.0) for the simulation of the transient behavior of a piezoelectrically actuated microscale membrane pump with cantilever valves. The development of this model is significant for several reasons. First, it allows the complete pump geometry to be contained in a single simulation environment, avoiding complex external

iterative procedures and facilitating design optimization. This includes the electro-mechanical coupling required for piezoelectric simulations as well as the fluid-structure interactions (FSI) of a membrane pump. Next, numerical solutions allow for great design flexibility and analysis which is not constrained by known analytical equations. This model considers pump and valve geometry, ambient pressures, excitation voltage shape (sinusoidal, square, etc.), and magnitude, as well as material and fluid properties. Significant outputs include (but are not limited to) membrane deflection profile, operating pressure, and flow rate. In the sample geometry in Figure 1, the top (purple) region is the piezoelectric material, the next (dark blue) layer is the silicon membrane, and the remaining volumes (light blue) are the fluid. The fluid region includes the inlet (left) and outlet (right) valves. The overall goal of this project was to develop a comprehensive, self-contained, means of micropump evaluation.

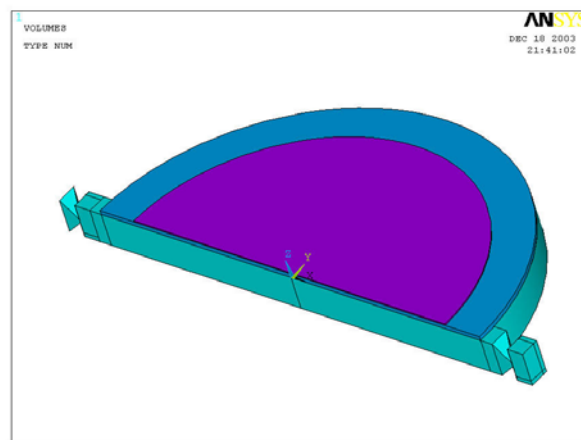


Figure 1: Isometric view of complete piezoelectrically actuated micropump with cantilever valves

## 2 THEORETICAL CONSIDERATIONS

Due to the extremely coupled nature of this device, care must be taken to consider all relevant physics. The applied voltage excites a structural deformation in the membrane, which causes a movement in the fluid resulting in a net fluid flow. The significant governing equations for the analysis are presented below, in order of input to output.

## 2.1 Electromechanical Coupling

First, the voltage is applied to the piezoelectric material resulting in a deformation. The electromechanical coupling is described by the following equations [5]:

$$\{\sigma\} = [c]\{\varepsilon_s\} - [e]\{E\} \quad (1)$$

$$\{D\} = [e]^T \{\varepsilon_s\} + [\varepsilon_{di}]\{E\} \quad (2)$$

where  $\{\sigma\}$  is the stress vector,  $[c]$  is the elasticity matrix,  $\{\varepsilon_s\}$  is the strain vector,  $[e]$  is the piezoelectric stress matrix, and  $\{E\}$  is the electric field vector. Additionally in equation (2) we define the electric flux density vector  $\{D\}$ , and the dielectric matrix  $[\varepsilon_{di}]$ . Application of finite element procedures to equations (1) and (2) results in the global matrix equation of:

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{U}\} \\ \{\dot{V}\} \end{Bmatrix} + \begin{bmatrix} [C] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{V\} \end{Bmatrix} + \begin{bmatrix} [K] & [K^Z] \\ [K^Z]^T & [K^d] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{V\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{L\} \end{Bmatrix} \quad (3)$$

where the  $[M]$ ,  $[C]$ ,  $[K]$ ,  $[K^d]$ , and  $[K^Z]$  are the structural mass, structural damping, structural stiffness, dielectric conductivity, and piezoelectric coupling matrix, respectively. Also,  $\{U\}$  is the nodal displacement vector,  $\{V\}$  the nodal electric potential vector,  $\{F\}$  is the vector of nodal, surface, and body forces, and  $\{L\}$  is the nodal charge vector. The single and double dot represent the first and second time derivatives, respectively.

## 2.2 Fluid-Structure Interaction

The boundary between the diaphragm and the fluid, as well as that between the flaps of the valves and the surrounding fluid is a significant factor in the analysis of the pump. The moving membrane (whose motion is described above) exerts a pressure on the fluid which moves in response to this stimulus. Similarly, the fluid exerts a force on the structure which affects its movement. The global system of finite element equations for this interaction is as follows:

$$\begin{bmatrix} [M_s] & [0] \\ \rho_f [R]^T & [M_f] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{P}\} \end{Bmatrix} + \begin{bmatrix} [K_s] & -[R] \\ [0] & [K_f] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{F_s\} \\ \{F_f\} \end{Bmatrix} \quad (4)$$

Here the subscripts s and f denote solid and fluid, respectively.  $\{P\}$  is the pressure vector. The matrix  $[R]$  is a coupling matrix which accounts for the normal direction

of each face. The density of the fluid is denoted  $\rho_f$ .  $[M]$ ,  $[K]$ ,  $\{U\}$ , and  $\{F\}$  remain as described in section 2.1.

## 2.3 Fluidic

This model assumes an incompressible fluid. Also, because of the small length scales we can be assured of small Reynolds numbers, which supports the laminar flow assumption. The fluid flow is solved by a segregate sequential solver. The element matrix formulations are solved for each degree of freedom separately. In this problem, the velocities in the x, y, and z directions are solved respectively. From these results, the pressure is computed and the velocities are recalculated. If the solution has not converged, the algorithm proceeds to the next global iteration.

## 3 NUMERICAL MODELING

The model used the following elements to capture the various physical processes present in the pump. The entire fluid region of the model is meshed with the 3-D fluid element FLUID142. The pumping chamber and valves were meshed using 16500 swept hex elements. The piezoelectric material was meshed with 1620 tetrahedral SOLID98 coupled field solid elements; the pump membrane was meshed with 1720 tetrahedral SOLID92 elements. The flaps of the valves were meshed using 1350 structural SOLID45 elements. Additionally, the upstream face of the valves was meshed with the contact element CONTA174. This element (and its target element counterpart TARGE170), allows the modeling of contact between moving three dimensional bodies. In this case, the contact is to allow the flap of the valve to shut when a negative pressure gradient is applied across it. The entire mesh is shown in Figure 2 with an expanded view of the valve in Figure 3.

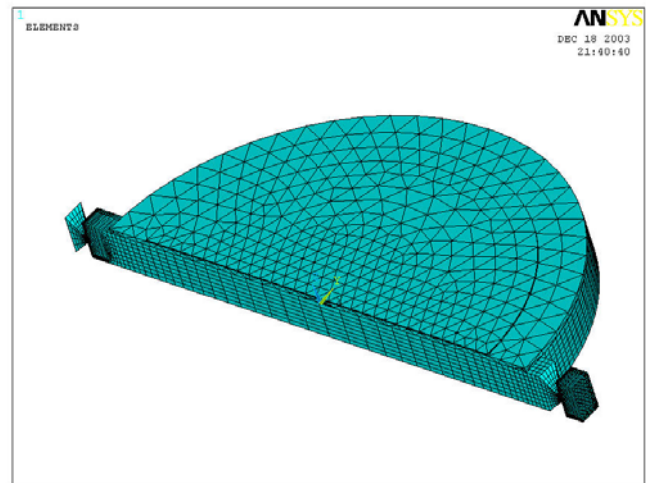


Figure 2: Isometric view of meshed pump

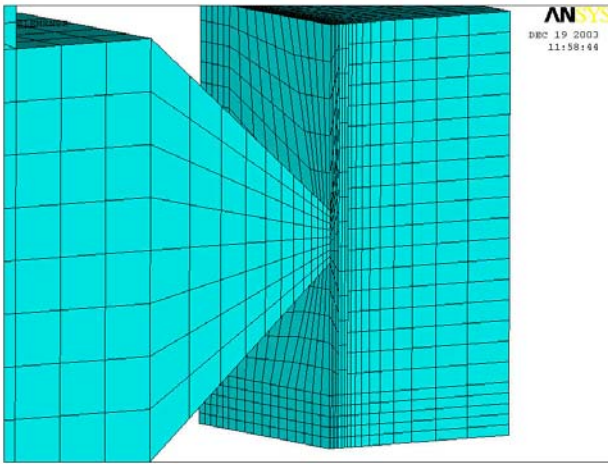


Figure 3: Expanded view of valve mesh

## 4 DESIGN PARAMETERS

### 4.1 Pump Geometry

The pumping chamber itself is circular, modeled as a semicircle due to symmetry considerations. The model consists of a fluid chamber and a piezoelectric actuator separated by a thin membrane. Common materials were used for the simulations presented here, namely, water, PZT-4, and silicon, respectively. The inlet and outlet ports are located on opposite sides of the chamber and each has a width of  $800 \mu\text{m}$ . The pump radius (both the fluid and pump membrane) is  $5 \text{ mm}$ ; the membrane has a thickness of  $100 \mu\text{m}$ . The diameter was arbitrarily chosen to approximate existing pumps, the thickness is both a common membrane thickness and a value near the optimal value predicted by [2]. The PZT has a radius of  $4.1 \text{ mm}$  and a thickness of  $30 \mu\text{m}$ . This size of actuator was chosen to reflect the results of optimization of the piezo/membrane system without fluidic effects. The radius and thickness were optimized simultaneously using both the ANSYS optimization algorithms and manual techniques.

The key pump parameters and the final dimensions are shown in Figure 4 and Table 1.

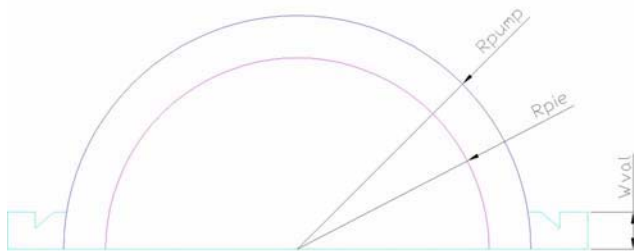


Figure 4: Pump parameters

$R_{\text{mem}}$	$5000 \mu\text{m}$
$T_{\text{mem}}$	$100 \mu\text{m}$
$R_{\text{pie}}$	$4100 \mu\text{m}$
$T_{\text{pie}}$	$30 \mu\text{m}$
$W_{\text{val}}$	$800 \mu\text{m}$

Table 1: Pump dimensions

### 4.2 Valve Geometry

The valves incorporated into this model are simple cantilever-style valves. As in the valve demonstrated by [4] the channel area is reduced before the fluid encounters a movable flap. This style of valve allows flow in the forward direction while restricting it in the opposite. This causes a net forward flow as fluid is forced out of the pumping chamber. The valve parameters and the chosen dimensions for the analysis are shown in Figure 5 and Table 2 respectively.

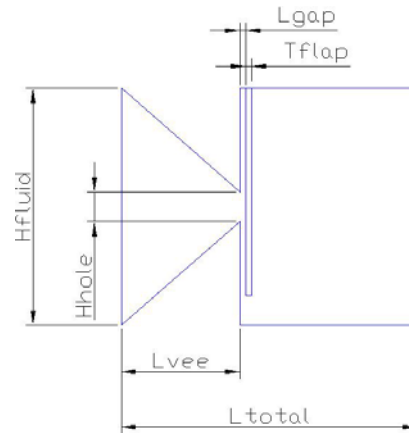


Figure 5: Valve parameters

$H_{\text{fluid}}$	$800 \mu\text{m}$
$T_{\text{flap}}$	$4 \mu\text{m}$
$L_{\text{gap}}$	$1 \mu\text{m}$
$L_{\text{total}}$	$1000 \mu\text{m}$
$L_{\text{vee}}$	$400 \mu\text{m}$
$H_{\text{hole}}$	$100 \mu\text{m}$

Table 2: Valve dimensions

### 4.3 Operating Parameters

In addition to the physical size and shape, several other factors have significant roles in determining the output characteristics of a pump. The magnitude, frequency, and shape (sinusoidal, square, etc.) of the actuation voltage are critical. This model uses a sinusoidal excitation of  $300V_{0-P}$  with a frequency of 100Hz.

## 5 RESULTS

The results garnered from the complete model thus far are of varying degrees of success. The inherent problem with the model is that it does not solve using fluids of realistic compressibility. The model failed to converge for fluids of stiffness (defined by ANSYS as  $\delta P/\delta \rho$ ) higher than  $100 \text{ m}^2/\text{s}^2$ . Pump operation using highly compressible fluids resulted in compression of the fluid rather than a mechanically forced fluid flow. Also, the model will solve when the geometry is altered to minimize the occurrence of thin, plate-like elements. This, too, however will not yield realistic results. Investigations into this alternative are continuing.

On a positive note, when solved using a lower compressibility, the trends and motion of the model certainly followed the predicted values and patterns. This was to be expected based on previous success in modeling the individual components of the pump. A sinusoidal input voltage generated a (slightly lagging) sinusoidal membrane displacement. A pressure difference across the valves was accompanied by both flap displacement and fluid flow. Additionally, the components interacted correctly. For example, the timing of the valve flap displacement corresponded well with the physical device though the value was several orders of magnitude lower than expected.

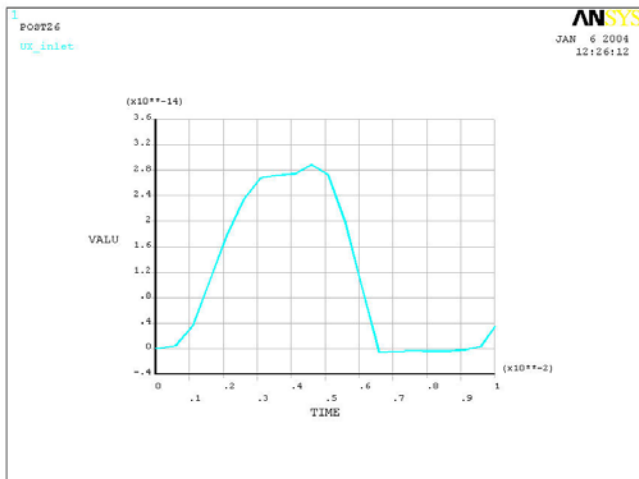


Figure 6: Inlet valve displacement (m) as pump draws fluid into the chamber (time 0-.005s) then expels it (time .005-.01s)

Note in Figure 6 that the flap opens (positive values of displacement) as the flow is drawn into the pump chamber; then, as the fluid is pressed out, the flap returns to its original closed position.

One means of understanding the compressibility effects is to view the velocity of the fluid in the x direction (from inlet to outlet). As can be seen from Figure 7 the fluid is not forced out of the valve but simply compressed; specifically, the flow has a high velocity in the pump chamber and a low velocity in (through) the valves.

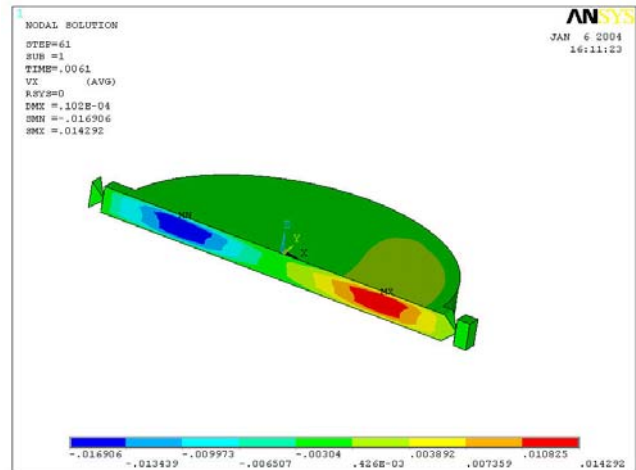


Figure 7: Velocity profile (m/s) in the x direction showing effects of compressibility

Work is continuing to be done in an effort to solve the current problems and provide a useful model for micropump optimization and characterization.

## REFERENCES

- [1] Gravesen, P., Jens Branebjerg, Ole Sondergard Jensen (1993). "Microfluidics - a review." *Journal of Micromechanics and Microengineering* 3(1993): 168-182.
- [2] Li, S., and Shaochen Chen (2003). "Analytical analysis of a circular PZT actuator for valveless micropumps." *Sensors and Actuators A* 104: 151-161.
- [3] Morris, C. J., Fred K Forster (2000). "Optimization of a circular piezoelectric bimorph for a micropump driver." *Journal of Micromechanics and Microengineering* 10: 459-465.
- [4] Koch, M., A G R Evans and A Brunnschweiler (1996). "Coupled FEM simulation for the characterization of the fluid flow within a micromachined cantilever valve." *Journal of Micromechanics and Microengineering* 6(1996): 112-114.
- [5] ANSYS Theory Reference v7.0