Piecewise perturbation method (PPM) simulation of electrostatically actuated beam with uncertain stiffness

J. Juillard, H. Baili, E. Colinet
Department of Measurement, Supélec,
91192, Gif-sur-Yvette CEDEX, France
jerome.juillard@supelec.fr

ABSTRACT

We present a new approach to the simulation of uncertainties in micro-electromechanical systems, based on the same principle as perturbation methods. This approach is valid for large variations of the uncertainties and requires much less simulations than a Monte-Carlo method. An implementation in the case of an electrostatically actuated beam with uncertain stiffness is presented and compared with obtained with Monte-Carlo.

Keywords: uncertainties, simulation, perturbation

1 INTRODUCTION

There are many difficulties to the simulation and modelling of micro-electromechanical systems : coexistence of many coupled physical phenomena, non-linearities and important uncertainties. They can also rise from simple lack of knowledge, for example, regarding the mechanical properties of a material. In fact, from an experimental point of view, such uncertainties are a major limiting factor to the credibility of MEMS models.

Taking them into account is most usually done via "stochastic" simulation methods, the most famous of which is the Monte-Carlo method [1]. This costly but robust approach requires an important number of realizations in order to obtain a correct statistical description of the system. It also relies on the ability to generate a random variable with a probability density function (pdf) that may be neither gaussian nor uniform. Other methods include the stochastic finite-element method [2], second-moment analysis [3] or resolution of the Fokker-Planck equation [4].

We propose in this paper a completely deterministic approach to the problem of simulating a possibly nonlinear system with one uncertain parameter, based on the principle of perturbation analysis and we illustrate this method with the case of a beam of uncertain stiffness undergoing electrostatic actuation. Results obtained with our method are then compared with Monte-Carlo simulations of the device.

2 CLASSICAL AND PIECEWISE PERTURBATION METHODS

2.1 Classical perturbation methods

Let us consider a system $(S)_\varepsilon$ with input x, output y and uncertain parameter ε, with zero mean and small variance. Let us consider also that the input-output relationship can be put in the form:

$$y = f(x,\varepsilon) \quad (1)$$

Supposing small perturbations of the system, an n[th]-order Taylor expansion of (1) can be made:

$$y \approx f(x,0) + \varepsilon \left.\frac{df}{d\varepsilon}\right|_{x,\varepsilon=0} + \ldots + \frac{\varepsilon^n}{n!}\left.\frac{d^n f}{d\varepsilon^n}\right|_{x,\varepsilon=0} = F_x(\varepsilon) \quad (2)$$

In the classical approach [3], the order of the expansion is usually limited to n=2. Relation (2) can then be used to determine the first and second moments of y, knowing those of ε. This approach requires the simulation of the original, unperturbed system plus n simulations for the higher-order terms of the series expansion.

Although this method is quite straightforward, there are some cases in which the moments do not hold much information and for which it is preferable to express the pdf of the system's output. This can be done using the following method.

Starting from (2), the pdf of the system's output can be calculated with the following formula [5]:

$$\mathrm{pdf}_y(y_0) = \sum_p \frac{\mathrm{pdf}_\varepsilon(\varepsilon_p)}{\left|\frac{dF_x}{d\varepsilon}\right|_{\varepsilon=\varepsilon_p}} \quad (3)$$

with ε_0 the real roots of $y_0 = F_x(\varepsilon)$.

Provided the degree of F_x is small enough, finding its roots is a relatively simple task. The probability density of the system's output can then be known, since the pdf of the uncertain parameter is supposedly known too (fig. 1).

This scheme can be extended to the case of several uncertain parameters: the multidimensional equivalent of (3) involves calculating convolution-type integrals of the uncertain parameters pdfs on domains depending on the degree of F_x [5]. This is only practical for n=1, which restricts the usefulness of this method to simple low-dimensional cases. Also, both methods are restricted to the case of small variations of the uncertain parameter(s), because the n[th]-degree Taylor expansion is only valid within a limited range. This drawback can be surmounted by using the following scheme, which is valid for arbitrarily large variations of the uncertain parameters.

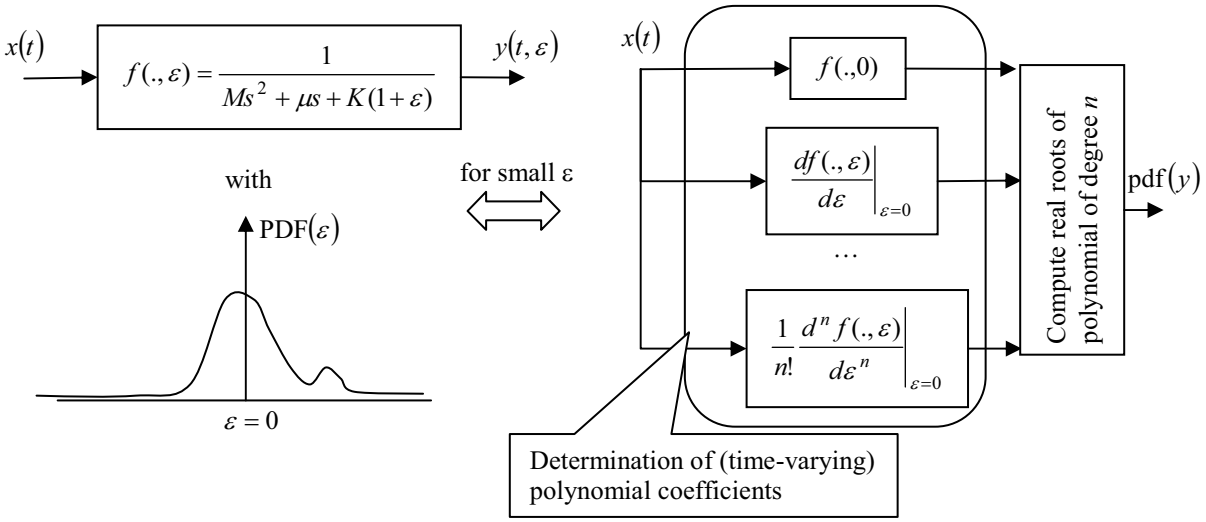

fig. 1: the implementation of the perturbation method requires the simulation of 1+*n* systems. Each of these systems corresponds to a derivative of the original system near the central value ε=0

2.2 Piecewise perturbation method

The piecewise perturbation method, whose algorithm is presented in fig. 2, consists in dividing the domain of the uncertain parameter in *P* sub-domains (which need not be the same size) with central values $\{\varepsilon_1,...,\varepsilon_P\}$ and applying to each of these sub-domains the pdf-based perturbation method described in the previous part.

For each sub-domain, the system and its derivatives are simulated using the corresponding central value and a truncated pdf that is zero outside the sub-domain: the *P* resulting partial pdfs can then be summed to yield the total pdf of the system's output.

It is clear that no matter how large the variation of the uncertain parameter, the sub-domain decomposition can be made fine enough to make the Taylor approximation valid. Another advantage of the method is that the size of the sub-domains can be adapted locally to account for discontinuities of the pdf of ε or for possible critical values of ε for which a bifurcation occurs in the system, as in the following case.

3 IMPLEMENTATION IN THE CASE OF A BEAM OF UNCERTAIN STIFFNESS

Let us consider a beam-mass system undergoing electrostatic actuation. Using modal analysis, it is a simple matter to deduce the position of the mass from the Bernouilli beam equation. Introducing mass *M*, damping coefficient *D*, stiffness *K*, voltage *V*, gap g_0, it is possible to approximate position y_ε as:

$$M\ddot{y}_\varepsilon + D\dot{y}_\varepsilon + K(1+\varepsilon)y_\varepsilon - \frac{\alpha V^2}{(g_0 - y_\varepsilon)^2} = 0 \quad (4)$$

In (4), ε is a random variable with mean 0 which is not necessarily Gaussian. Applying a first-order PPM, we write

$$y_\varepsilon = y_{e_k} + (\varepsilon - \varepsilon_k)y_{e_k}^{(1)} \quad (5)$$

Using a Taylor expansion of the non-linear term in (4), it is possible to show that, in the neighbourhood of ε_k, (4) is equivalent to the following system:

$$\begin{cases} M\ddot{y}_{\varepsilon_k} + D\dot{y}_{\varepsilon_k} + K(1+\varepsilon_k)y_{\varepsilon_k} = \dfrac{\alpha V^2}{(g_0 - y_{\varepsilon_k})^2} \\ M\ddot{y}^{(1)}_{\varepsilon_k} + D\dot{y}^{(1)}_{\varepsilon_k} + K(1+\varepsilon_k)y^{(1)}_{\varepsilon_k} = -Ky_{\varepsilon_k} + 2\dfrac{\alpha V^2}{(g_0 - y_{\varepsilon_k})^2}\dfrac{y^{(1)}_{\varepsilon_k}}{g_0 - y_{\varepsilon_k}} \end{cases} \quad (6)$$

Each of these equations must then be solved in turn for the different sub-domains of ε. The corresponding partial pdfs can then be calculated with (3) and (5) and then summed. It must be noted that there exists a certain value of ε below which the system is pulled-in.

4 RESULTS

To illustrate our method, we consider the case of a system with the following parameters: K=15 kg.s^{-2}, D=4.10^{-4} kg.s^{-1}, M=2.10^{-7} kg, V=2.5 V, g_0=2,5.10^{-6} m and α=5.10^{-18}.

For the sake of simplicity, we consider a Gaussian density for ε, with standard deviation 0,1. For the PPM, we split the interval [-0,4 0,4] into 40 sub-domains and obtain the results of fig. 3.

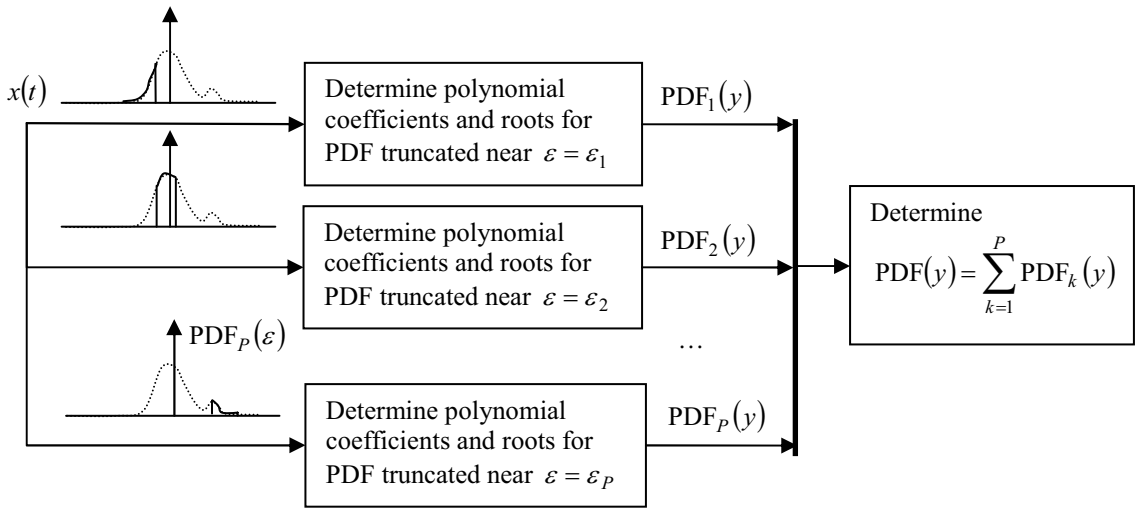

fig. 2 : schematic of the piecewise perturbation method. The total number of simulations is equal to the number of sub-domains (P) times the order of the Taylor expansion plus one (N+1).

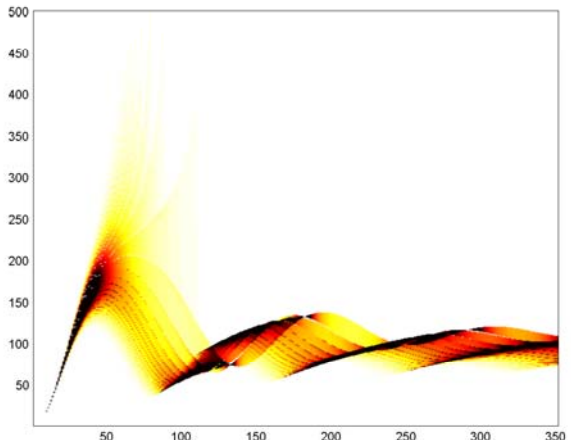

fig. 3: pdf(y) obtained with first-order PPM.

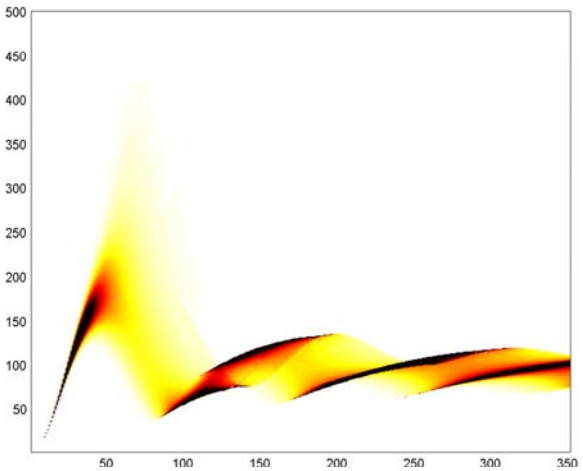

fig. 4: pdf(y) obtained with third-order PPM.

One can see clearly that the beam-mass system is pulled-in for certain values of ε, near which the sub-domain decomposition could be refined. An example of a higher-order PPM calculation is presented in fig. 4, with the same number of sub-domains as in fig. 3. We present in fig. 5 and 6 the comparison of these results with a Monte-Carlo simulation of the system with 10^5 realizations of ε.

These figures show that the PPM gives roughly the same results as the Monte-Carlo method for a much lower computational cost, even for low PPM orders.

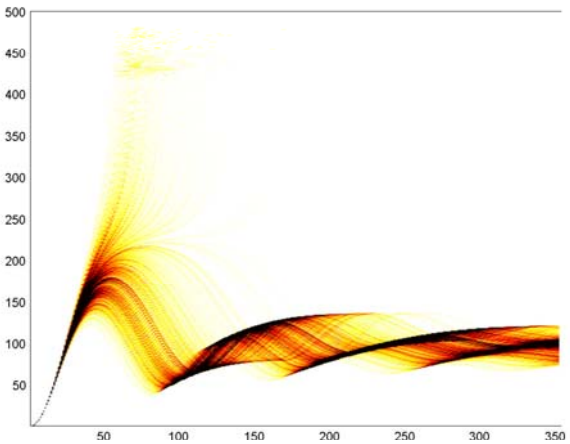

fig. 5: pdf(y) obtained with Monte-Carlo method.

Although the Monte-Carlo method is much more general and does not necessitate a Taylor expansion of the system with respect to the uncertain parameters, it is clear that the PPM also has advantages, such as the possibility to focus (i.e. to reduce the sub-domain size) locally in order to account for bifurcations in the solution and it has a good accuracy even for low expansion orders. It also makes it possible to account for large variations of the uncertain parameters, as opposed to classical perturbation methods. Finally, even though this approach is less immediate than the Monte-Carlo approach, it is much less costly: this should be even truer as the number of uncertain parameters increases.

5 CONCLUSION

We have introduced in this paper a novel approach to the simulation of uncertainties: this approach is based solely on deterministic simulations of the perturbed system and of its derivatives with respect to the uncertain parameter. As opposed to other perturbation methods, it is valid for arbitrarily large variations of the uncertain parameter. We have shown how this method applies in the case of an electrostatically actuated beam with uncertain stiffness. The results were compared with those obtained with a Monte-Carlo simulation of the system. This method is currently being extended to the case of many uncertain parameters.

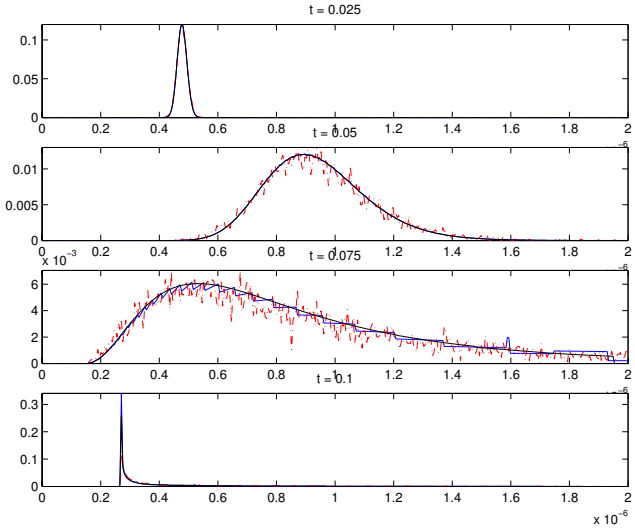

fig. 6: comparison of first-order PPM (40 sub-domains, blue line), third-order PPM (40 sub-domains, black line) and Monte-Carlo method (10^5 realizations, red line) at four different moments.

REFERENCES

[1] E. Vanmarcke, "Random fields, analysis and synthesis", MIT Press, Cambridge, 1984

[2] W. K. Liu, T. Belytschko, A. Mani, "Random field finite elements", International Journal of Numerical Methods in Engineering, vol. 23, pp. 1831-1845, 1986

[3] F. S. Wong, "First-order, second-moment methods", Computers and Structures, vol. 20, pp. 779-791, 1985

[4] H. Risken, "The Fokker-Planck equation: methods of solution and application", Springer-Verlag, Berlin, 1984

[5] P. Z. Peebles, "Probability, random variables and random signal principles", McGraw-Hill, New York, 1993