mor4ansys: Generating Compact Models Directly from ANSYS Models

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ABSTRACT

Model reduction of linear large-scale dynamic systems is already quite an established area [1]–[3]. In a number of papers (see references in [3]), the advantages of model reduction have been demonstrated. In the present paper, we describe a software tool to perform moment-matching model reduction via the Arnoldi algorithm directly to ANSYS finite element models. We discuss the application of the tool to a structural mechanical problem with a second order linear differential equation (ODE). Its successful application to the first order case of electro-thermal modeling is demonstrated elsewhere [4], [5].

Keywords: model order reduction, damped second order system, Rayleigh damping, Arnoldi process, Krylov subspace

1 INTRODUCTION

`mor4ansys` is a command-line tool built on top of the ANSYS-supplied library to read ANSYS binary files [7] and the TAUCS library for sparse linear algebra [8]. After a model is built and meshed in ANSYS, an ODE system is obtained. Consider for example the PDE for an elastic body

$$\boldsymbol{f}_I + \boldsymbol{f}_D + \boldsymbol{f}_S = \mathbf{b}\boldsymbol{u}(t), \tag{1}$$

with $\boldsymbol{f}_I$ the force caused by inertia, $\boldsymbol{f}_D$ the damping force, $\boldsymbol{f}_S$ the elastic force and $\mathbf{b}\boldsymbol{u}(t)$ external forces depending on user input and varying in time [6]. The *scatter matrix* $\mathbf{b}$ distributes the inputs $\boldsymbol{u}$ to the domain. By discretization with n spatial degrees of freedom x_i, $1 \le i \le n$, ANSYS transforms the PDE to n ordinary differential equations,

$$\boldsymbol{f}_S = \mathbf{K}\boldsymbol{x}, \qquad \boldsymbol{f}_D = \mathbf{C}\dot{\boldsymbol{x}}, \qquad \boldsymbol{f}_I = \mathbf{M}\ddot{\boldsymbol{x}} \tag{2}$$

$$\Longrightarrow \quad \mathbf{M}\ddot{\boldsymbol{x}}(t) + \mathbf{C}\dot{\boldsymbol{x}}(t) + \mathbf{K}\boldsymbol{x}(t) = \mathbf{B}\boldsymbol{u}(t), \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the system matrices, $\mathbf{B}\boldsymbol{u}(t)$ is the load vector, and $\boldsymbol{x}$ is a vector with unknown degree of freedoms, its dimension being routinely from 10 000 to 500 000. The outputs of the system can in principle be an arbitrary linear combination of states

$$\boldsymbol{y} = \mathbf{L}^T\boldsymbol{x}, \tag{4}$$

but usually $\mathbf{L}$ is only used to pick certain degrees of freedom.

The information from ANSYS is transferred as a file with element matrices (EMAT file) and lists for Dirichlet boundary conditions, nodal forces and output degrees of freedoms. The developed software uses these files as input and produces a reduced model by means of the Arnoldi algorithm. The user may choose a maximum order m for the reduced model. Because of the iterative nature of the Arnoldi algorithm, one obtains all possible reduced models with dimensions ranging from 1 to m as specified by the user. The postprocessing, that is, the solution of a reduced ODE system as well as computing its transfer function is currently performed in Mathematica. It is worth mentioning that it can be done in any other environment as the reduced model is stored as an ASCII file.

A conventional approach to model reduction is to find a low-dimensional subspace $\mathbf{V}$

$$\boldsymbol{x} = \mathbf{V}\boldsymbol{z} + \boldsymbol{\varepsilon} \tag{5}$$

that can well approximate the trajectory of the state vector and then project (3) on that subspace:

$$\mathbf{M}_r\ddot{\boldsymbol{z}} + \mathbf{C}_r\dot{\boldsymbol{z}} + \mathbf{K}_r\boldsymbol{z} = \boldsymbol{b}_r \tag{6}$$

where $\mathbf{M}_r = \mathbf{V}^T\mathbf{M}\mathbf{V}$, $\mathbf{C}_r = \mathbf{V}^T\mathbf{C}\mathbf{V}$, $\mathbf{K}_r = \mathbf{V}^T\mathbf{K}\mathbf{V}$, $\boldsymbol{b}_r = \mathbf{V}^T\boldsymbol{b}$. In mechanical engineering, the subspace $\mathbf{V}$ is usually chosen from the eigenstates of (3) or by the Guyan method [9].

Moment matching via Krylov subspaces is a new technique [1], [3], [10] that allows us to find a low-dimensional subspace with excellent approximating properites for relatively low computational effort. For example, the time for model reduction in `mor4ansys` is comparable with the time required for a stationary solution or for a single timestep during an ANSYS transient simulation process. Other advantages mentioned above are as follows:

1. User intervention is minimal: one has just to specify the maximum dimension for the reduced system. There is no selection of dominant eigenmodes, master degree of freedoms or the like.

2. Iterative nature of the algorithm: one can change the dimension of the reduced model without additional computations.

A straightforward application of Krylov subspace methods to second order ODEs produces a reduced system in the

form of a first order system of ordinary differential equations [1], [3], [10], and this is undesirable for structural mechanics. Su and Craig have suggested a modified version of the Arnoldi algorithm that preserves the second order in the reduced model [11]. In both cases, the damping matrix $\mathbf{C}$ takes part in the process of generation of the matrix $\mathbf{V}$.

The main difference of our approach with those in Refs [10], [11] is that the damping matrix is not employed at all during the generation of a low-dimensional basis $\mathbf{V}$, that is, the latter is built as the orthogonolized Krylov subspace $\mathcal{K}\left(\mathbf{K}^{-1}\mathbf{M}, \mathbf{K}^{-1}\boldsymbol{b}\right)$. Nevertheless, the reduced damping matrix has been computed as a projection in (5). Such an approach is based on the engineering intuition that the damping matrix should not play a major role in finding a good subspace $\mathbf{V}$ as the most essential information is contained within the mass and stiffness matrices. Unfortunately, we cannot prove this mathematically. From a pragmatic viewpoint, such an approach allows us a great deal of advantage in the most often encountered case in structural mechanics when a damping matrix is built up as a linear combination of mass and stiffness matrices [6], i.e., the damping is chosen as mode preserving Rayleigh damping

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}. \tag{7}$$

The unit of α is $1/\mathrm{s}$, the unit of β is $1\,\mathrm{s}$.

The motivation for this choice comes from the fact that by choosing $\mathbf{M}$ and $\mathbf{K}$ such that

$$\mathbf{C} = \mathbf{M}\sum_b a_b \left(\mathbf{M}^{-1}\mathbf{K}\right)^b, \tag{8}$$

we gain the following properties [6]:

- Damping orthogonality, thus the different modes of the system do not couple through the damping.
- The vibration mode shapes are the same for the damped and undamped system.
- The essential dynamic response is associated with the lowest few modal coordinates and thus suitable for reduction.

There are also intuitive interpretations of this form, basically saying that the damping contributions come from internal friction and the surrounding air, and it often happens that the resulting behavior is sufficiently accurate for many applications.

In this case, one can show that the reduced damping matrix $\mathbf{C}_r = \mathbf{V}^T\mathbf{C}\mathbf{V}$ can be computed directly from the reduced mass and stiffness matrices as

$$\mathbf{C}_r = \alpha\mathbf{M}_r + \beta\mathbf{K}_r, \tag{9}$$

that is, the parameters α and β remain as parameters during the model order reduction process.

1.1 Moment matching for second order systems

After Laplace-transformation of (3) and (4), the transfer function $\boldsymbol{H}(s) = \mathcal{L}(\boldsymbol{y}(s))/\mathcal{L}(\boldsymbol{U}(s))$ can be written as

$$\boldsymbol{H}(s) = \mathbf{L}^T\left(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}\right)^{-1}\mathbf{B}. \tag{10}$$

For simplicity, let us assume that we only have one output and one input terminal, so that $\boldsymbol{H}(s)$ becomes a scalar, $\mathbf{L}^T$ a vector $\boldsymbol{l}^T$ and $\mathbf{B}$ a vector $\boldsymbol{b}$. For our method, we also drop the damping term. We expand $H(s)$ by a Taylor series for s^2 at $s_0 = 0$

$$H(s) = \boldsymbol{l}^T\left(s^2\mathbf{M} + \mathbf{K}\right)^{-1}\boldsymbol{b} \tag{11a}$$

$$= \boldsymbol{l}^T\left(s^2\mathbf{K}^{-1}\mathbf{M} + \mathbf{I}\right)^{-1}\mathbf{K}^{-1}\boldsymbol{b} \tag{11b}$$

$$= \sum_{i=0}^{\infty} s^{2i}\boldsymbol{l}^T\left(\mathbf{K}^{-1}\mathbf{M}\right)^i\mathbf{K}^{-1}\boldsymbol{b} = \sum_{i=0}^{\infty} m_i s^{2i}. \tag{11c}$$

The m_i are called the moments of the transfer function. We now seek a projection $\mathbf{V}$ that provides a Padé approximation, i.e., that yields the same first q moments for the transfer function of the reduced system.

The Arnoldi algorithm reduces the $n \times n$ matrix $\mathbf{K}^{-1}\mathbf{M}$ to a small $q \times q$ block upper Hessenberg matrix $\mathbf{H}_q$ and during this transformation creates a matrix $\mathbf{V}$ such that

$$\mathrm{colspan}(\mathbf{V}) = \mathcal{K}_m(\mathbf{K}^{-1}\mathbf{M}, \mathbf{K}^{-1}\boldsymbol{b}) \tag{12a}$$

$$\mathbf{V}^T\mathbf{K}^{-1}\mathbf{M}\mathbf{V} = \mathbf{H}_q \tag{12b}$$

$$\mathbf{V}^T\mathbf{V} = \mathbf{I}_q \tag{12c}$$

It can be shown that by using these matrices the corresponding moments in the full and reduced system match up to the mth moment [2].

2 WIRE BOND MODEL

As a benchmark for the algorithm we use a model of a gold wire bond needed for the packaging of micro devices (see fig. 1). The material properties are listed in table 1. The application area for this benchmark is the design of wire bonds and the configuration of bonding machines.

The model was created in ANSYS and meshed with tetrahedral 10-node elements (SOLID187). It features 32877 degrees of freedom. A step load is applied to the first bend of the wire, with direction parallel to the z-axis.

Young's modulus	$E = 78\,\mathrm{GPa}$
Poisson's ratio	$\nu = 0.44$
Density	$\varrho = 19300\,\mathrm{kg/m^2}$

Table 1: Material properties for the bond wire.

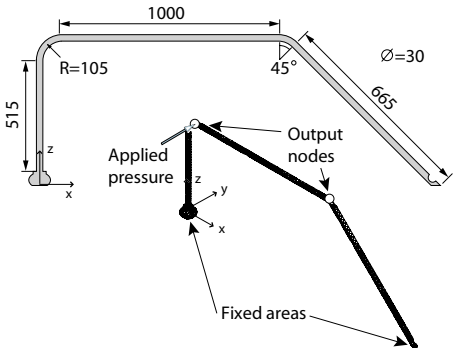

Figure 1: Sketch of a wire bond used for micro chip connection (all dimensions in μm). Inset: Mesh and applied loads for wire bond model.

3 RESULTS

The model was simulated in the time domain (transient simulation) and in the frequency domain (harmonic simulation). We investiged different settings for β, α was set to zero for all simulations.

ANSYS has a built-in model order reduction tool using the Guyan method [12]. This tool requires choosing "master nodes" from the complete set of degrees of freedom. These nodes can be chosen by hand, but since our aim is to provide a reduction method which should not need to rely on the experience of the designer, we only assigned the output nodes as master nodes and use the automatic master node selection from ANSYS. The model was order reduced using both the Guyan method and the Arnoldi method and the transfer functions were compared.

3.1 Transient simulation

Figure 2 shows the transient response at the output node marked with an arrow (fig. 1) to a step load for the ANSYS model (simulation performed in ANSYS) and two reduced models for a damping of $\alpha = 0$ and $\beta = 1\,\mu$s (fig. 2a) or $\beta = 0.01\,\mu$s (fig. 2c). It is remarkable that even a model with three (higher damping) or five (lower damping) degrees of freedom is able to catch the transient behavior almost perfectly. The curves for higher order reduced models are indistinguishable from the ANSYS curves for the chosen resolution of the graph.

Figures 2d, e show the same simulation with the Guyan method. To achieve similar results, the order of the reduced system needs to be considerably higher than for the Arnoldi method.

3.2 Harmonic simulation

The difference is also clearly visible in the frequency plot (fig. 3). The figures show the response of the output node for harmonic excitation of the beam at the arrow location. While the Guyan model of order 10 in fig. 3a leaves the curve of the full model near the second peak, the Arnoldi model of

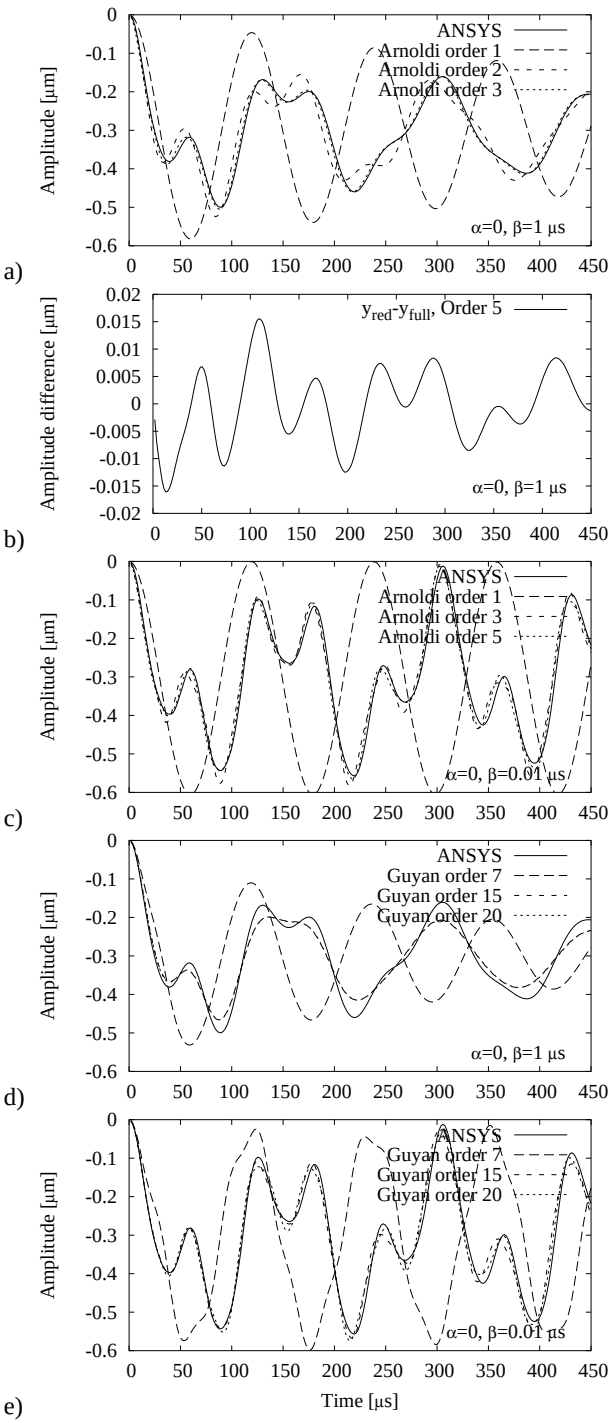

Figure 2: Transient simulation with full and reduced models of various order. a) Damping 1 μs, Arnoldi. b) Difference between reduced model of order 5 and full model. c) Damping 0.01 μs, Arnoldi. d) Damping 1 μs, Guyan. e) Damping 0.01 μs, Guyan

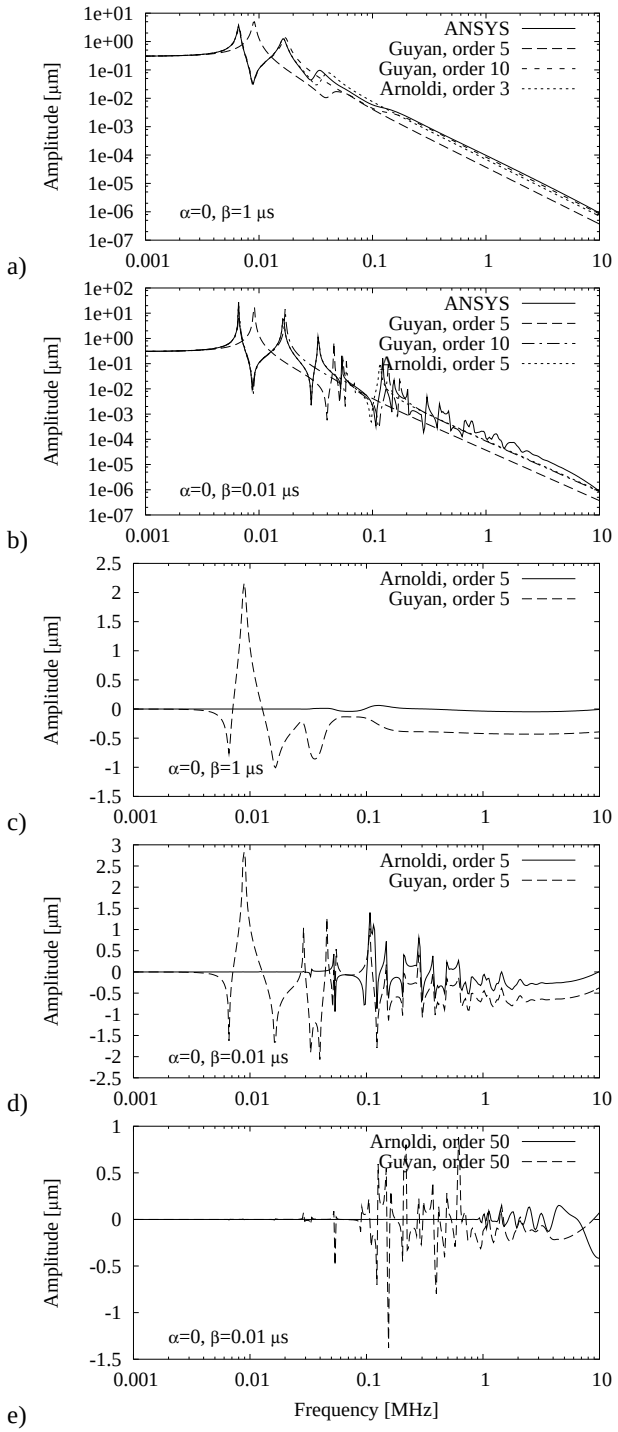

Figure 3: Harmonic spectrum for full and reduced models. a) Spectrum for Damping 1 µs. b) Spectrum for 0.01 µs. c) Logarithmic difference $\log_{10}|H_r(s)| - \log_{10}|H(s)|$ between reduced models and full model for damping 1 µs, d,e) for damping 0.01 µs.

order 3 matches the curve for a wide range of frequencies. The same behavior can be seen for lower damping. However, for lower damping, the deviations are usually larger than for lower damping because of the slower decay of high frequency modes.

4 CONCLUSIONS

We have presented a novel approach to compute reduced order models of second order damped systems. We showed that this approach works very well for systems where the damping matrix is a linear combination of the stiffness and mass matrix, and that it greatly outperforms the Guyan method.

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