Finite element validation of an inverse approach to the design of an electrostatic actuator

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ABSTRACT

We present an approach to the design of electrostatically-actuated micro-structures and discuss its implementation in a software tool called IDEA. The main advantage of this approach is that it considerably reduces problems associated to coupling and to large-displacement non-linearities. The results obtained with IDEA are then compared with ANSYS simulations of an electrostatic micro-mirror.

Keywords: MEMS design, optimization, coupled-field problems

1 INTRODUCTION

From simple RF micro-switches to high-precision adaptive micro-mirrors [1], electrostatic actuation is one of the most common principles in the field of MEMS. However, the fact that the pressure-voltage and displacement-pressure relationships are non-linear and the strong coupling between these relationships, makes such MEMS very difficult to design accurately. For example, in one of the most elaborate of these models [4], the deformable membrane of the MEMS under consideration can be described by two purely linear models corresponding to whether bending stresses or tensile stresses are dominant. This paper, based on the results of [6], aims at validating an approach to the design of electrostatic MEMS and of deformable micro-mirrors in particular, where both kinds of stresses may be present.

After showing how the inverse mechanical problem, yielding a pressure distribution as a function of prescribed displacements, can be described by the equations of Von Karman and how these can be treated as a set of two equations, the more complex of which is a linear partial differential equation (PDE), the electrostatic problem is addressed: starting from the calculated ideal pressure distribution and from a given set of electrodes, we use a linearized model of the membrane to compute the voltages that give out the smallest residual deformation from the desired shape. The main advantage of this approach is that electromechanical coupling can be neglected inside the optimization loop, leading to efficient and simple calculations.

We then go on to discuss the implementation of this approach into a graphical tool using Matlab. Finally, in the last part of this paper, we validate the approach by comparing the results obtained with IDEA in the case of an electrostatic deformable micro-mirror and those obtained with fully non-linear ANSYS simulations of the device.

2 DIRECT AND INVERSE APPROACH TO THE DESIGN OF ELECTROSTATIC ACTUATORS

Let us consider the problem of imposing a displacement to a thin deformable structure with a given set of electrodes, such as can be found in micro-relays, micro-pumps or micro-mirrors. It is particularly difficult to solve this problem accurately because of the electromechanical coupling and also because of the non-linearity arising from large displacements (i.e. the prescribed displacement is not small compared to the thickness of the structure).

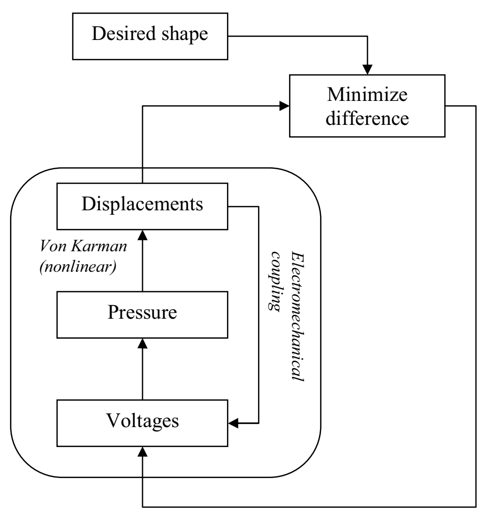

fig. 1: classical optimization approach. The optimization loop contains complex non-linear calculations which can only be solved iteratively.

A typical approach is illustrated in fig. 1. Starting from an initial guess of the voltages, the corresponding displacements are computed through a non-linear coupled-field simulation; they can then be compared to the desired shape and the initial guess can be modified accordingly. The main difficulty with this approach is that the optimization loop contains a non-linear coupled-field problem which must be solved at every iteration.

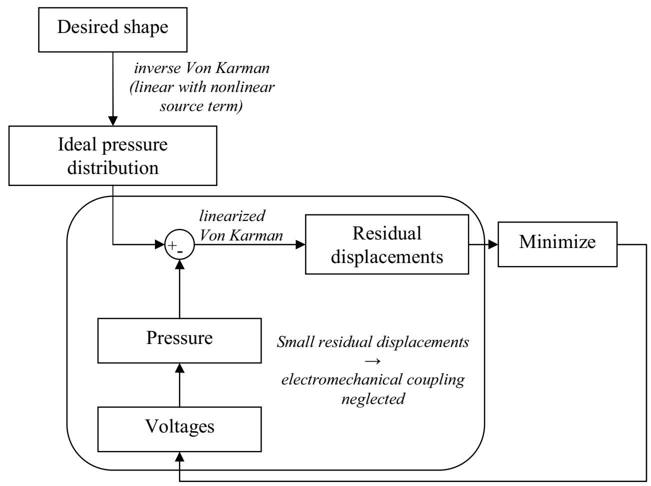

fig. 2 : inverse optimization approach. The only delicate calculation is placed outside the optimization loop. Since residual displacements are supposedly small, electro-mechanical coupling involves no iteration within the optimization loop.

We have proposed in [6] another approach to this problem, which consists in (fig. 2):

- solving the non-linear mechanical inverse problem to calculate the ideal pressure distribution corresponding to the desired displacements,
- for a given set of electrodes, solving the quadratic programming problem of finding the voltages that give the best approximation to this ideal pressure distribution.

This can be translated into a more mathematical language; let us consider a thin two-dimensional structure described by the equations of Von Karman:

$$\begin{cases} \dfrac{Eh^3}{12(1-\nu^2)}\Delta^2 w \\ -h\left(\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 F}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}-2\dfrac{\partial^2 F}{\partial x\partial y}\dfrac{\partial^2 w}{\partial x\partial y}\right)=P(x,y) \\ \Delta^2 F + E\left(\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}-\left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2\right)=0 \end{cases} \quad (1)$$

As shown in [6], it is possible to solve these equations for the pressure *P(x,y)* knowing the displacements *w(x,y)*. For simple membrane shapes, it is even possible to find analytical solutions. For example, for a circular membrane, the quasi-parabolic shape:

$$w(r)=W_0\left(1-\frac{r^2}{R_0^2}\right)^2 \quad (2)$$

can be obtained with the following pressure distribution:

$$P(r)=\frac{16}{3(1-\nu^2)}\frac{Eh^3W_0}{R_0^4} + \frac{4}{3}\frac{EhW_0^3}{R_0^4}\left(\frac{5-3\nu}{1-\nu}-\frac{22-18\nu}{1-\nu}\frac{r^2}{R_0^2}+30\frac{r^4}{R_0^4}-20\frac{r^6}{R_0^6}+5\frac{r^8}{R_0^8}\right) \quad (3)$$

Once the ideal pressure distribution corresponding to *w(x,y)* is known, it can be injected into the optimization loop. Assuming small residual displacements $\delta w(x,y)$ and a plane capacitance approximation, one may neglect part of the electro-mechanical coupling by writing the electrostatic pressure created by the k^{th} electrode with potential V_k as:

$$P_k=\frac{\varepsilon_0}{2}\frac{\tilde{V}_k^2}{(g-w-\delta w)^2}\approx\frac{\varepsilon_0}{2}\frac{\tilde{V}_k^2}{(g-w)^2} \quad (4)$$

Finding the voltages that yield the best approximation to the desired shape can then be re-formulated into a quadratic programming problem with non-negativity constraints, using (4) and the first equation of Von Karman linearized in the neighborhood of *w(x,y)*. This problem can then be solved using Matlab's *quadprog* function or any other quadratic programming algorithm.

There are two advantages to this approach :

- part of the mechanical problem is linearized (the inverse Von Karman equations are linear with non-linear source terms) and placed outside the loop. For some simple shapes, it also has analytical solutions.
- the optimization loop is initialized very close to the solution. It is thus possible to linearize the inner loop calculations and, assuming small residual displacements, to neglect electro-mechanical coupling.

This method has been implemented using Matlab into a graphical tool called IDEA (for Inverse Design of Electrostatic Actuators). Symbolic calculations are used to compute the ideal pressure distribution corresponding to the desired displacements. Then, for a given electrode set, the voltages can be computed using either a Galerkin method (with the membrane mode shapes as basis functions) or a collocation method: using a collocation method rather than a Galerkin method makes it possible to minimize the residual displacements locally rather than globally. It is also possible to solve the regularized problem and look for an approximate solution giving out small residual displacements together with low actuation voltages.

3 FINITE ELEMENT VALIDATION

To illustrate our approach, we consider, as in [6], a clamped micro-mirror with radius 1cm and thickness 3µm and a set of electrodes of increasing outer radii [1.5, 3, 4.5, 6, 7.5, 9]mm. The electrostatic gap is 50µm and the spacing between neighbouring electrodes is 100 µm.

An equivalent ANSYS model of the device is built using *membrane51* elements (axisymmetric shell) and *plane121* elements. A fine regular mesh is chosen for the air region with a typical element size of 10 µm. The principle of the validation is the following: starting from a given shape of the membrane, the corresponding optimal voltage distribution is taken from IDEA and used as an input of the ANSYS model. The resulting deflection is then computed with ANSYS under large deflection/stress-stiffening hypotheses and compared to the objective displacement. As long as the residual displacements computed with IDEA remain small, the two models compare very well (fig. 3 and fig. 4).

There is more discrepancy when IDEA is used to compute the voltage distribution corresponding to an 'impossible' shape, such as a parabola – which is 'impossible' considering the mirror is supposedly clamped on its edges (fig. 5).

This points out the necessity to take bending stresses into account even when large displacements are involved – in the case of fig. 5, the mirror's maximum deflection is 20 µm, which must be compared to the mirror's thickness of 3 µm. However, this problem may be solved by finding first an approximation to the 'impossible' shape which verifies the boundary conditions and then using IDEA to compute the voltage distribution corresponding to the approximation.

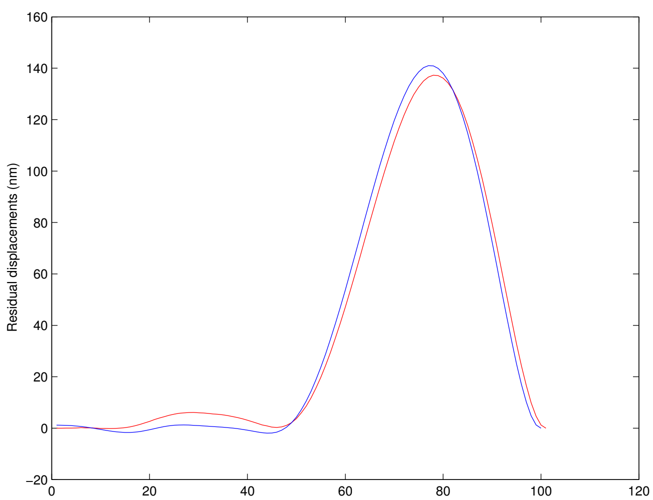

fig. 3 : residual displacements of the membrane obtained with IDEA (blue curve) and ANSYS (red curve) for the quasi-parabolic shape (eq. 2) with maximum deflection W_0=10 µm.

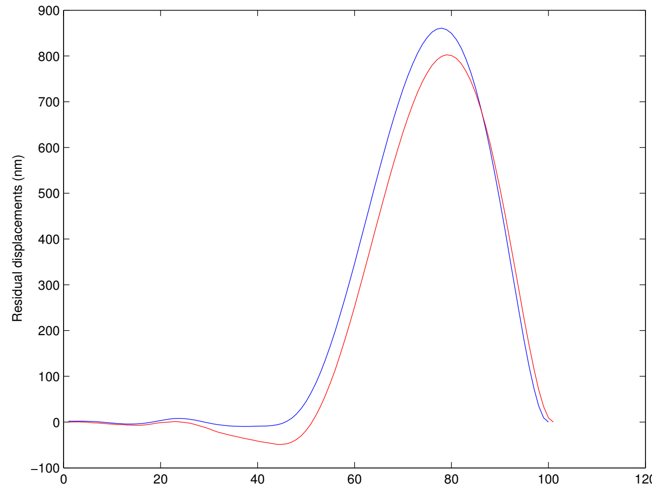

fig. 4 : residual displacements of the membrane obtained with IDEA (blue curve) and ANSYS (red curve) for the quasi-parabolic shape with maximum deflection W_0=20 µm.

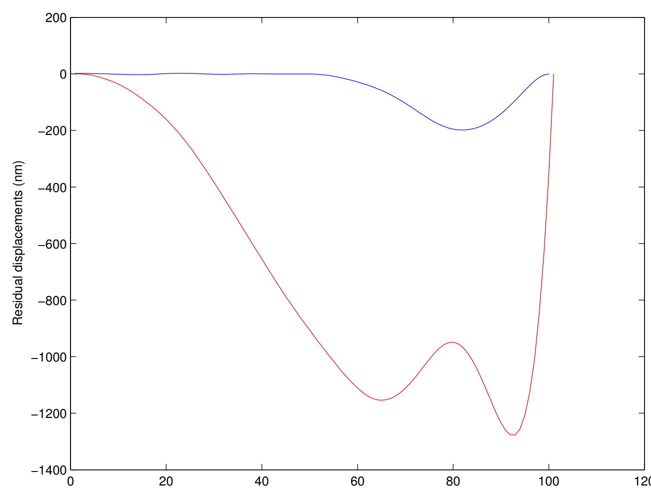

fig. 5 : residual displacements of the membrane obtained with IDEA (blue curve) and ANSYS (red curve) for the perfect parabolic shape with maximum deflection W_0=20 µm.

4 CONCLUSION

We have given the outline of a novel approach to the design of electrostatic actuators for thin structures undergoing large displacements and we have discussed its implementation in a software tool called IDEA. We have then given results concerning the actuation of a micro-mirror and compared them with ANSYS simulations of the device. Both models compare well as long as the desired mirror shape is a possible solution of the Von Karman equations with the good boundary conditions. If this is not the case, an approximation to the desired shape must first be found prior to using IDEA.

IDEA is currently being extended to two-dimensional problems and to dynamical problems – an advantage of the inverse approach being that, assuming small residual displacements, the strong coupling typical of non-linear damping phenomena, such as squeeze-film damping, can also be neglected.

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