Design and modeling of a 3-D micromachined accelerometer

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ABSTRACT

This paper presents the operation principles, modeling methods, design, and fabrication considerations of a 3-D micromachined accelerometer. MEMS technology in this work combines small size, low cost and low power consumption to create a sensor that is suitable for wide usage in different applications such as automotive industry and inertial navigation systems.

Keywords: accelerometer, inertial sensors, silicon sensors, micromachining, micro-fabrication technology.

1 INTRODUCTION

In several applications such as inertial navigation, vibration monitoring and robotics control, measurement of acceleration in 3-D space is required. In an integrated set of 1-D accelerometers, aligning the axes of sensors normal to each other is a difficult task. In these applications utilizing a single chip for measuring acceleration in three directions instead of three 1-D accelerometers, significantly increases the accuracy of measurement. In addition micromachining technology shrank the sensor size, reduce the fabrication cost, and allow its electronics to be integrated on the same silicon chip. Accelerometers generally consist of a proof mass which is suspended to a reference frame by anchored beams as spring elements. Acceleration causes a relative displacement of the proof mass with respect to the frame. The displacement of the mass, which can be measured by various methods, is proportional to driving acceleration.

2 PRINCIPLES OF ACCELERATION MEASUREMENT

Accelerometers are electro-mechanical transducers which change mechanical displacement to DC voltage. Figure 1 shows the lumped element model of an accelerometer. The accelerometer can be modeled by a second order mass-spring-damper system. External acceleration displaces the proof mass relative to the reference frame which changes the internal stress in suspension springs. Both the relative displacement of the proof mass and the beam stress can be used as a measure of external acceleration.

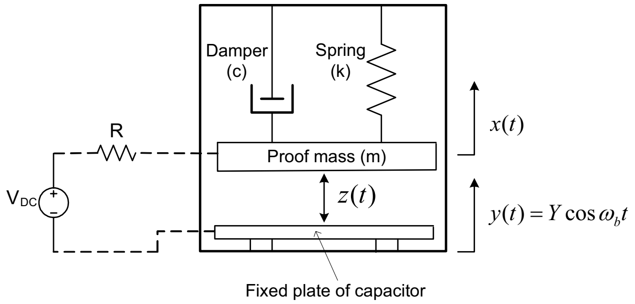

Figure 1: Lumped model of a capacitive accelerometer

The equation of the motion can be written in terms of relative displacement $z(t)$

$$\ddot{z}(t) + 2\xi\omega_n \dot{z}(t) + \omega_n^2 z(t) = -\ddot{y}(t) \qquad (1)$$

Where $\xi (= c/2m\omega_n)$ is the damping ratio, and $\omega_n (= \sqrt{k/m})$ is the natural frequency of the system. The solution of this expression can be obtained by applying a forced harmonic vibration analysis for this base excitation problem [1]. The steady state response of the system is shown to be

$$z(t) = \frac{-1}{\omega_n^2 \sqrt{\left(1-r^2\right)^2 + \left(2\xi r\right)^2}} \ddot{y}(t) \qquad (2)$$

Figure 2 shows the plot of coefficient of base acceleration versus the frequency ratio $(r = \omega_b / \omega_n)$. Within the region $0 < r < 0.1$, the coefficient of base acceleration is approximately equal to unity for a variety of values of ξ. As a result, this range defines the suitable bandwidth of the accelerometer within ten percent of its natural frequency.

Static response of an accelerometer regarding Newton's second law is equal to $z_{static} = (m\ddot{y}/k) = (\ddot{y}/\omega_n^2)$. These equations clearly illustrate the trade of between sensitivity and the sensor's bandwidth. For a high DC sensitivity, a low natural frequency is required which results in a short bandwidth.

Force feedback method is applied to eliminate this limitation in most accelerometers.

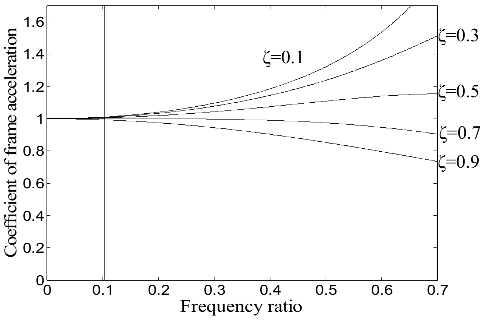

Figure 2: Proportionality of relative displacement of the proof mass and base acceleration.

2.1 Spring elements

Micromachined accelerometers mostly contain micro-springs in form of rectangular cross section beams. Spring elements are usually anchored to the ground layer in one end, and attached to a free moving mass at the other end. The configuration of the beams is extremely important to satisfy the off-axis sensitivity. This property, which defines the sensitivity of the sensor in different directions, plays an important role in the performance of the accelerometer. To minimize the off axis sensitivity the effect of residual strain in beams and their thermal expansion should be considered in designing the geometry of suspension structures.

Determining natural frequency of the beam is an essential requirement for designing accelerometers. One end or double end clamped beams are widely used as spring elements in inertial sensors. Solving forced response of a distributed-parameter beam results the natural frequency of the system. The deflection of the beam for a general load in the form of $Q(x,t) = f(x)q(t)$ by utilizing separation of variables technique $(w(x,t) = X(x)T(t))$ is equal to [2]

$$w(x,t) = \sum_{i=1}^{\infty} \frac{X_i}{\omega_n} \int_0^l f(x) X_i dx \int_0^t q(\tau) \sin \omega_n (t-\tau) d\tau \quad (3)$$

Table 1 summarizes the dynamic properties of one end and double end clamped beams.

Table 1

Parameters	Natural frequency	Beam stiffness	Effective mass
Cantilevered	$\frac{3.516}{2\pi}\sqrt{\frac{EI}{ml^3}}$	$\frac{3EI}{l^3}$	24%
Double clamped	$\frac{22.337}{2\pi}\sqrt{\frac{EI}{ml^3}}$	$\frac{192EI}{l^3}$	38%

A nonlinear characteristic of the electrostatic force between parallel plates of a capacitor creates a virtual spring. This effect causes a shift of natural frequency in the system. *Spring softening* would allow the possibility of controlling natural frequency of the sensor, which can actively control the sensitivity of the device to a specific range of excitation frequencies. This nonlinear phenomenon can be linearized in an equilibrium point and can be defined as a negative stiffness equal to [3]

$$k_e = -\frac{C_s}{z_0^2} V_{DC}^2 \quad (4)$$

Where C_s is equivalent capacity, and z_0 is initial gap between two plates of capacitor.

2.2 Viscous damping

Major sources of energy dissipation in an accelerometer are viscous damping of the moving proof mass in the surrounding fluid (usually air), and squeezed film damping between sensing fingers. Couette flow model can be used for estimating damping coefficient of steady viscous flow existing between moving proof mass parallel with ground layer [4]. The damping constant c can be obtained from $c = \eta A / h$ where, η is the viscosity of the surrounding fluid, A is the overlapped area and h is the air gap between moving mass and ground layer. The squeezed film damping happens when a gas fills the space between transverse moving parallel plates. This scenario usually happens for sensing fingers of a sensor. The following equation is an estimation of squeezed film damping factor [5].

$$c = n \frac{96 \eta l W^3}{\pi^4 h^3} \quad (5)$$

Where n is the number of fingers. As a result of micro scale effects, theoretically estimated c is 35 to 40% less than measured values.

2.3 Noise

MEMS sensor noise typically arises from the damping of fluid surrounding the proof mass, which is called "*Brownian motion*" noise. According to the thermodynamics the spectral density function of the force noise is $4k_B Tc$, where k_B is the Boltzman constant. An equivalent acceleration spectral density, which is called the Total-Noise-Equivalent-Acceleration, is commonly used to express the noise level of the system.

$$TNEA = \sqrt{\frac{4k_B T \omega_n}{Qm}} \quad (6)$$

It is obvious that to achieve low noise levels, a large proof mass and a high quality factor are required.

3 MODELING CONCEPTS

Accelerometers can be modeled at two different levels: system level and physical level. Finite element methods generally are used to simulate the device behavior in a 3D coupled field for physical level simulation. Regardless the geometry of the sensor its dynamic characteristic can be simulated in the system level as an energy transducer device. The main methods for system simulation are *multi port* device simulation and *equivalent circuit*. The goal of *multi port* theory is to develop the links between dependant and independent variables of the transducer. A set of differential equations can be obtained by applying energy methods and thermodynamics for defining the characteristics of the system state variables. Then these equations can be analyzed for various inputs and initial conditions by mathematical software. Since the capacitance transduction method utilized for the designed accelerometer, for instance we consider the capacitive accelerometer model shown in figure 1. Applying thermodynamics equilibrium for the stored energy through the mechanical and electrical ports of the transducer results the following governing equations of the capacitive accelerometer.

$$\frac{\partial q}{\partial t} = \frac{1}{R}\left(V_{in} - V\right) = \frac{1}{R}\left(V_{in} - \frac{qz}{\varepsilon A}\right) \qquad (7)$$

$$\frac{q^2}{2\varepsilon A} + m\frac{\partial^2 z}{\partial t^2} + c\frac{\partial z}{\partial t} + k(z - z_0) = \frac{\partial^2 y}{\partial t^2} \qquad (8)$$

The above set of nonlinear differential equations can be solved by means of mathematical software numerically.

The other quick way of getting insight into the dynamic behaviour of accelerometers is the *equivalent circuit* approach. In this method both electrical and mechanical elements of transducer are represented by electrical equivalents. In this analogy mechanical force plays the same role as voltage, the velocity as the current and the displacement as the charge. The mass in the mechanical system corresponds to the inductance, viscous damping to the resistance and the flexibility (k^{-1}) to capacitance. Also this analogy defines possible equivalent circuits, which represent electromechanical transducers [6].

4 FABRICATION AND PACKAGING

Manufacturability of an accelerometer should be considered as an important issue in the design of sensors. The first micromachined capacitive accelerometer used bulk micromachining and wafer bonding to fabricate a thick large proof mass in order to increase the sensitivity of the device. Although this design has the advantage of large proof mass, the wafer bonding process is required to realize an air gap which causes high damping factor. The thin structures fabricated using surface micromachining techniques contains array of holes to reduce damping. These devices can also be easily combined with their electronic circuitry.

There are various commercialized surface machining processes which provide design rules and guidelines to achieve a desired micromachined structure. In addition to the process design rules, there are critical manufacturing issues which should be considered in all micromachining processes to achieve defect free devices. The intrinsic film stress generated during the deposition process plays important role in the structural stability of the multi layer polysilicon sensors. Annealing will be required to release the residual stress created during fabrication process.

All accelerometers require an etch step to release the moving mass from the supporting layer or substrate. This process is usually a wet etch of a sacrificial layer. Release stiction typically occurs during the drying step when the surface tension forces in the liquid draw the micromachined structures into close contact due to van der Waals attraction. Super CO_2 drying can avoid the stiction failure. Controlling of etch properties and profile is the on-going challenge of etching technology. The variation of structural dimensions, which extremely impact design parameters, should be avoided by considering etching properties.

Accelerometers contain fragile components such as thin beams. Wafer level hermetic packages which provide electrical connections and mechanical support are suitable for protecting the MEMS structure of accelerometer.

5 MEMS 3-D ACCELEROMETER

The geometric design of the proposed open loop sensor is similar to the forced balanced accelerometer introduced by Lemkin [7]. Figure 3 shows the design layout of the 3-D accelerometer along with the FEM simulation of the out of plain resonance mode, and figure 4 shows the fabricated sensor using MUMPs®[1] foundry process.

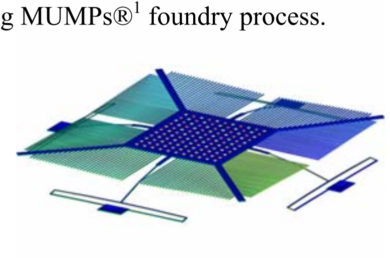

Figure 3: FEM stress and modal analysis of the 3-D accelerometer for out of plain acceleration.

The deposition of the first polysilicon layer creates the fixed electrode for sensing z-axis acceleration. Proof mass, sensing fingers and folded springs are patterned on the second 2μm-thick doped polysilicon. The nominal gap between capacitor plates and fingers is 2μm and the maximum length of overlap region is 150 μm. It is possible to assume sensing fingers as parallel plate capacitors to get the correct order of the at-rest total capacitance. The mass of the main electrode with attached sensing fingers including the effective mass of the springs is equal to 5.8×10^{-10} kg. The stiffness of the lateral motion including the spring softening effect of the applied voltage is 4.5 N/m. In addition by adding squeezed film damping with Couette flow effect, a quality factor equal to 5 can be obtained.

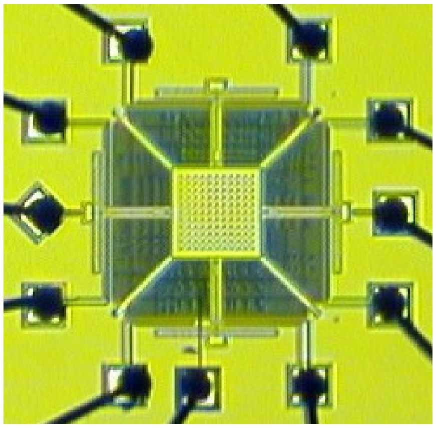

Figure 4: A 3-D micromachined accelerometer

Quad symmetry of the proof-mass about the z-axis minimizes sensitivity to off-axis accelerations. When a lateral acceleration is applied to the substrate, finger gaps change from their nominal 2μm value causing an imbalance in the capacitive sensing half bridge. The output voltage of the bridge is linearly proportional to the proof mass displacement. Under an applied z axis acceleration the proof mass moves out of plane causing a change in the parallel plate capacitance formed between the center of the proof mass and a bottom plate made from first polysilicon layer. Figure 5 shows the frequency response of the sensor in the range of 0 to 1200 Hz.

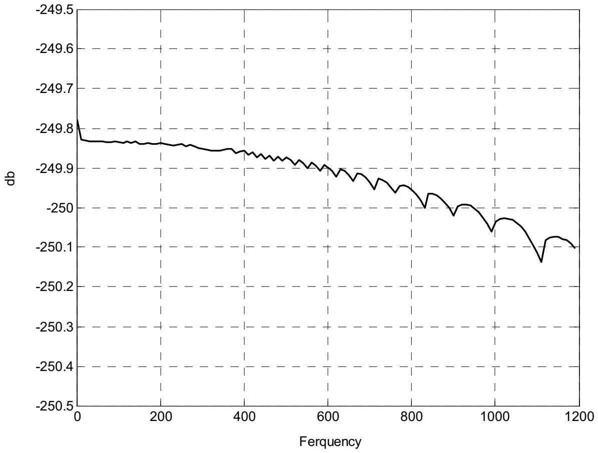

Figure 5: Frequency response of the 3-D accelerometer

Table 2 summarizes the other electrical and mechanical properties of the proposed accelerometer.

Parameter	x-axis	y-axis	z-axis
Natural frequency	15.2 kHz	15.2 kHz	4.6 kHz
Full scale range	±15g	±15g	±10g
Total sensing capacitance	116 fF	116 fF	242 fF
Noise level (TENA)	0.75 $mg/\sqrt{Hz}$	0.75 $mg/\sqrt{Hz}$	0.1 $mg/\sqrt{Hz}$

6 CONCLUSIONS

This article has outlined the modeling considerations of MEMS accelerometers and emphasized the important role of mechanical concepts in this area. A 3-D accelerometer has been designed and simulated using *multi port* theory. Simulation results show that the concept and the design are feasible and accurate. The process required to fabricate the proposed accelerometer, MUMPs®, is a commercialized micromachining process and a prototype has been fabricated to prove the concept and test the on-chip sensor.

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[1] MUMPs is a registered trademark of Cronos Integrated Microsystems, a JDS Uniphase Company.