Threshold-Voltage-Based Regional Modeling of MOSFETs with Symmetry and Continuity

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ABSTRACT

This paper presents a unified threshold-voltage-based \((V_t\)-based) MOSFET model, which maintains source–drain symmetry and allows accurate prediction of transconductance \((g_m)\) and drain conductance \((g_d)\) and their derivatives \((g_{m1}, g_{m2})\) with smooth transitions across regions of operation. This has been achieved based on our previous unified source-extrapolated \(V_t\)-based model but re-derived with bulk reference for the drain current \((I_d)\). The unified model combines \(V_t\)-based model in strong inversion with surface-potential-based \((\Psi_s\)-based) model in subthreshold with smooth transitions (in function as well as higher-order derivatives) across linear/saturation and weak/strong-inversion regions. It has been verified with the experimental data from a 0.18-\(\mu\)m CMOS shallow trench isolation (STI) technology wafer.

Keywords: Deep-submicron MOSFET, symmetry, threshold voltage, drain current, surface potential.

1 INTRODUCTION

Starting from the Meyer’s model [1], \(V_t\)-based models have been the standard for decades in MOSFET compact models (CMs) for circuit simulation. However, in recent CM developments, more attention is focused on mixed-signal and low-power applications in which requirements for smooth transitions across different regions of operation become increasingly important. This becomes the challenge for \(V_t\)-based models to continue to survive as a standard for MOSFETs. Furthermore, a major problem associated with \(V_t\)-based models is symmetry, as seen in the BSIM model [2], which has been attributed to being source referenced. However, as pointed out in [3], source-reference and bulk-reference are essentially equivalent. A \(V_t\)-based model, in which \(V_t\) is the source-extrapolated threshold voltage, does not necessarily mean asymmetry as long as the drain-current equation is derived with bulk as the reference; likewise, a bulk-referenced model does not necessarily guarantee symmetry if velocity-saturation and lateral-field mobility are not handled properly.

In this paper, we focus on further development of the unified \(V_t\)-based model based on our previous source-referenced model, Xsim [4], but re-derive with bulk reference for the drain current \((I_d)\). In Section 2, formulation of the bulk-referenced \(I_d\) and \(V_t\) models as well as parameter extraction is presented. Benchmark tests are carried out to verify the model continuity and symmetry, which are discussed in Section 3, together with experimental verification of the model.

2 MODEL FORMULATION AND PARAMETER EXTRACTION

The drain current equation based on charge-sheet approximation can be formulated as the drift current

\[
I_{\text{dr}}(y) = \mu_{\text{eff}} W_{\text{eff}} Q_s(y) \Psi_s(y) dy
\]

(1a)

and the diffusion current

\[
I_{\text{df}}(y) = \mu_{\text{eff}} W_{\text{eff}} v_d Q_s(y) dy
\]

(1b)

where \(\Psi_s(y)\) is the position-dependent surface potential, \(Q_s(y) = C_{\text{ox}} [V_{gb} - V_F - \Psi_s(y)] - Q_b(y)\)

(2a)

is the inversion charge along the channel, and

\[
Q_b(y) = C_{\text{ox}} \sqrt{\Psi_s(y)}
\]

(2b)

is the bulk charge. Rearranging (1a) and integrating across the channel from source \((y = 0)\) to drain \((y = L_{\text{eff}})\), with the boundary conditions \(\Psi_{s0} = 2\phi_s + V_{sb}\) and \(\Psi_{sL} = 2\phi_s + V_{ds}\):

\[
\int_0^{L_{\text{eff}}} I_d(y) dy = \mu_{\text{eff}} W_{\text{eff}} \int_0^{L_{\text{eff}}} Q_s(y) dy
\]

it can be shown [3] that the drift current is given by

\[
I_{\text{dr}} = \beta_s \left[ V_{gs} - V_t - \frac{1}{2} A_b \psi_{s0} - (A_b - 1) \psi_{sb} \right],
\]

(3)

in which

\[
V_t = V_F + \phi_s + \sqrt{\phi_{s0} - (A_b - 1) \psi_{sb}}
\]

(4)

\[
A_b = 1 + \sqrt{\frac{V_t}{2\phi_s}}
\]

(5)

for the bulk-referenced model, which are different in form from the source-referenced model. In the above equations, \(V_F\) is the flat-band voltage, \(\psi_{sb} = kT/\theta_{fb}\) is the thermal voltage, \(\phi_{s0} = 2\phi_s\) is the (“pinned”) surface potential at strong inversion, \(\gamma = (2q\theta_{fb}N_{i0})^{1/2}/C_{\text{ox}}\) is the body factor, and \(\beta_s = \mu_{\text{eff}} C_{\text{ox}} W_{\text{eff}} / L_{\text{eff}}\) is the gain factor.

Similarly, for the diffusion current, by rearranging (1b) and integrating from source to drain, and taking the source and drain end inversion charge as...
where respectively, the compact diffusion current can be derived:

\[ I_{\text{diff}} = \beta_n v_d \left( C_d / C_{\text{ox}} \right) e^{(V_{ds} - V_{th})/V_{th}} \left( 1 - e^{-V_{ds} / V_{th}} \right) \]  

(7)

where

\[ C_d = \gamma C_{\text{ox}} / 2 \sqrt{\phi_{\text{sub}}} \]  

(8)

\[ n = 1 + C_d / C_{\text{ox}} \]  

(9)

are the depletion capacitance and subthreshold slope, respectively. The bulk-referenced gate-bias-dependent subthreshold surface potential is given by

\[ \phi_{\text{sub}} = \frac{V}{2} + \sqrt{\frac{V^2}{4} + V_{gb} - V_{FB}} \]  

(10)

Eqs. (3) and (7) are regarded as the piece-wise regional models, in which subthreshold is \( V_{gb} \)-based whereas strong inversion is \( V_{gs} \)-based.

Since the bulk-referenced drain current (3) has the same form as the source-referenced model, our previously-developed analytical “drift + diffusion” model should still apply [4]:

\[ I_{ds} = \beta_n V_{gs} = \beta_n \left( V_{gs} + V_{gd} \right) \]  

(11)

that can be decomposed into the sum of the “drift” \( I_{\text{drift}} = \beta_n V_{gs} \) and “diffusion” \( I_{\text{diff}} = \beta_n V_{gd} \) currents, which approach the correct asymptotes in the strong-inversion and subthreshold regions, respectively. The key “smoothing” function is the “effective gate/drain voltage product” [5]:

\[ V_{geff} = \frac{2 n v_{th} \ln \left[ 1 + e^{(V_{gs} - V_{th})/(2 m_{mb})} \right]}{1 + V_{deff} 2 n \left( C_{ds} / C_{d} \right) e^{-V_{gs} - V_{th} - 2 V_{deff}} / (2 m_{mb})} \]  

(12)

which includes the effect of bulk charge

\[ V_{deff} = \left( 1 - \frac{A_{b}}{2} \right) V_{eff} \]  

(13)

\[ V_{gs} \] is given by \( V_{gs} = V_{gs}/W_{gs} \) to model the correct diffusion current in subthreshold without affecting drift current, where \( W_{gs} \) is derived to be [6]

\[ W_{gs} = \frac{n e^{(V_{gs} - V_{th})/(2 m_{mb})}}{A_{b} \left( 1 - e^{-V_{th} / V_{th}} \right)} \]  

(14)

In the above equations, \( V_{geff} \) is the effective gate overdrive (BSIM interpolation function [2]), given by

\[ V_{geff} = \frac{2 n v_{th} \ln \left[ 1 + e^{(V_{gs} - V_{th})/(2 m_{mb})} \right]}{1 + 2 n \left( C_{ds} / C_{d} \right) e^{-V_{gs} - V_{th} - 2 V_{deff}} / (2 m_{mb})} \]  

(15)

which approaches \( V_{gs} = V_{th} \) in strong inversion. \( V_{deff} \) is the BSIM smoothing function [2]

\[ V_{deff} = V_{dsat} - \frac{1}{2} \left( V_{dsat} - V_{ds} - \delta_s \right)^2 + 4 \delta_s V_{dsat} \]  

(16)

which approaches \( V_{ds} \) in linear region and

\[ V_{dsat} = E_{sat} L_{eff} \left( V_{gs} - V_{th} \right) \]  

(17)

in saturation region, where \( E_{sat} = 2 v_{sat} / \mu_b \) is the saturation field, and \( \delta_s \) is a fitting (smoothing) parameter.

To extract the model parameters, a 0.18-\( \mu \)m CMOS STI wafer is measured. Long-channel measured threshold voltages for different body biases have been extracted from the \( I_{ds} - V_{gs} \) curves at low drain bias based on the constant-current definition. The effective channel doping and the flat-band voltage (\( N_{ch}, V_{FB} \)) are calibrated to the \( V_{t} - V_{bs} \) data, and the semi-empirical mobility model [7] is then calibrated to the \( I_{ds} - V_{gs} \) data at low drain and body biases. The smoothing parameter \( \delta_s \) in (16), together with the channel-length modulation parameter (negligible for long channel), can be tuned to the \( I_{ds} - V_{ds} \) data at high gate and low body biases [4].
problem in symmetry, it is negligible for practical values of $I_{ds}$.

The circuit diagram used for the Gummel tree-top test [9], and the derivatives of drain current with respect to $V_{x}$, exhibiting smooth transition at $V_{x} = 0$.

3 RESULTS AND DISCUSSION

The unified $I_{ds}$ model has been calibrated to a 0.18-$\mu$m CMOS technology wafer, and the results are shown in Figs. 1–4 for the long-channel (10-$\mu$m) device (short-channel results are available but not shown in this paper). Figs. 1(a) and 1(b) show the fitted (and predicted for intermediate $I_{ds}$) $I_{dt} - V_{gb}$ characteristics in linear and saturation regions, respectively. Fig. 2 shows the circuit diagram [8] used for the Gummel symmetry test [9], and the first and second derivatives of drain current with respect to $V_{x}$. The smooth $I_{dt}'$ at $V_{x} = 0$ confirms model symmetry. The negligible “glitch” in $I_{dt}''$ at $V_{x} = 0$ may come from the non-unity slope of $V_{def}$ as $V_{ds}$ approaches zero. Although this is an intrinsic problem in symmetry, it is negligible for practical values of $\delta_{x}$. Fig. 3 shows the prediction of transconductance-to-current ratio versus $V_{gt}$ for $V_{bn} = -0.9$ V and $V_{ds} = 0.1$ V. The figure shows no discontinuity or kink effect between the weak and strong-inversion regions, which demonstrates model smoothness. The inset of Fig. 3 shows the same data versus $V_{gb}$ (left–bottom axes) and versus $I_{ds}$ (right–top axes), respectively. The $V_{gb}$-dependent $g_{m}/I_{ds}$ in the subthreshold region is a result of the surface-potential-based modeling ($\phi_{sub}$) in this region, which is consistent with the Gummel tree-top test [9].

Model prediction on higher-order derivatives of the drain current is shown in Fig. 4. Figs. 4(a) and 4(b) show the accurate prediction of the measured transconductance and output conductance versus gate and drain biases, respectively. Smooth transitions are observed between different regions [from weak to strong inversion in 4(a) and from linear to saturation region in 4(b)], even the second-order derivatives ($g_{m'}$ and $g_{ds'}$) are accurately predicted, as shown in the insets. The largest error occurs in $g_{m}$ near $V_{gs} \approx V_{t}$, which is attributed to the interpolation function ($I_{gt}$) as well as inaccuracies in $V_{t}$. The intrinsic disadvantage of the regional model (as compared to the $Y_{t}$-based model) will be traded off with other advantages such as scalability and simplicity. Model continuity across regions of operation is an important criterion for analog circuit design, and our unified regional model has demonstrated model smoothness with reasonable accuracy as well as symmetry.

4 CONCLUSION

In conclusion, our previous source-referenced $V_{t}$-based (long-channel) drain current model is revised with bulk reference to preserve model symmetry, which uses only 9 parameters to characterize the entire range of operation and is capable of accurately predicting drain current and its higher-order derivatives with smooth transitions across different regions. Contrary to the general belief, our regional $V_{t}$-based model has demonstrated symmetry and continuity. This development maintains simple MOSFET equations that are familiar to circuit designers while removing major problems associated with the regional
models. It serves as the starting point for the reconstruction of our previously-developed unified scalable drain current model, Xsim [4]–[7], including short-channel effects, consistent charge-based intrinsic capacitances, and poly-depletion effect, to be presented elsewhere [10].

REFERENCES