Characterization and Modeling of Silicon Tapered Inductors


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ABSTRACT

The characteristics of tapered inductors and standard inductors have been compared, and an improved compact model for tapered inductors is presented. The measured data of spiral inductors shows that the tapered inductor has higher quality factor (Q) than standard ones above 2.3GHz and higher frequency in which the maximum Q occurs. Because the parasitic capacitances, such as parallel capacitance and oxide capacitance, have lower values for tapered inductors, the real part of the input impedance is smaller, leading to higher Q value. For accurately modeling the behavior of tapered inductors, we propose an improved compact model, which adds a branch to model the frequency-dependent resistance. This model has the advantage of easily acquiring a relative equation because of the simple parallel structure of the skin effect model. The modeled and measured results have excellent agreement.

Keywords: tapered inductor, quality factor, skin effect, frequency-dependent resistance

1 INTRODUCTION

Spiral inductor technologies on Si substrate have been widely studied for integrating RF circuits into Si IC technologies. However, the lossy Si substrate contributes to the low quality factor (Q) of on-chip inductor. To enhance the quality factor, spiral inductors with width-tapered structure (as shown in Fig. 1), which metal width of the inductor is increasing from the center to the outer, have been proposed recently [1], [2]. It was presented that the width-tapered inductor has higher quality factor and higher frequency in which the maximum Q occurs [1], [2].

![Geometrical structure of a tapered inductor](image)

Figure 1: Geometrical structure of a tapered inductor

In this paper, we compared the measured data of the width-tapered inductors with various geometries in Section 2. By analyzing these measurement results, some guidelines for designing the layout geometries of tapered inductors have been obtained. Besides, we also present an improved inductor equivalent model for tapered inductors by adding additional branches to model the skin effect and substrate loss. The skin effect causes the series conductor resistance has frequency-dependent characteristic [3]. The equivalent single-π inductor model and extraction procedure will be described in Section 3. Experimental results show that the improved model can predict the frequency-dependent resistance, quality factor and inductance behaviors.

2 MEASURED RESULTS

To verify the advantage of tapered inductors, we compared the measured data of spiral inductors with metal space ($S$) = 3μm, inner diameter ($ID$) = 60μm, number of turns ($N$) = 4.5, but varied metal width. The results are listed in Table 1, including the geometric structures of a standard inductor and tapered inductors. We found that the tapered_1 inductor has the best performance of Q and inductance than the other tapered inductors. However the characteristics of tapered_2 and tapered_3 inductors do not have the improvement as expected. Because the size of the contact via is 4x4μm² in our layout design, the tapered_2 and tapered_3 inductors have only one contact via connecting the center metal with underpass line due to narrower center metal. It would increase the contact resistance and thus degrade the performance of tapered_2 and tapered_3 inductors.

<table>
<thead>
<tr>
<th>Inductor List</th>
<th>Metal Width Center to Outer (μm)</th>
<th>Q max</th>
<th>L (nH)</th>
<th>Frequency @max Q (GHz)</th>
<th>Via NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>15/15/15/15/15</td>
<td>4.427</td>
<td>3.816</td>
<td>2.3</td>
<td>2x2</td>
</tr>
<tr>
<td>tapered_1</td>
<td>13/14/15/16/17</td>
<td>4.516</td>
<td>3.961</td>
<td>2.7</td>
<td>2x2</td>
</tr>
<tr>
<td>tapered_2</td>
<td>11/13/15/17/19</td>
<td>4.483</td>
<td>3.747</td>
<td>3</td>
<td>1x1</td>
</tr>
<tr>
<td>tapered_3</td>
<td>9/12/15/18/21</td>
<td>4.349</td>
<td>3.589</td>
<td>3</td>
<td>1x1</td>
</tr>
</tbody>
</table>

Table 1: Comparison of inductors with same metal space but different metal widths.
Figure 2 compares the characteristics of standard and tapered_1 inductors. As shown in Fig. 2(a), the tapered_1 inductor has higher Q than the standard one above 2.3GHz. Also, the frequency at maximum Q shifts to a higher value. To understand the reason of Q-enhancement, the inductance and resistance are analyzed respectively.

Figure 2(b) illustrates the tapered_1 inductor has higher inductance and self-resonant frequency (f_{SR}). The thinner center metal width can naturally increase the inductance, but decrease the f_{SR}. The result of f_{SR} is different with Fig. 2(b), which means the tapered_1 inductor must have other reasons to increasing the f_{SR}.

Since the width-tapered inductors have narrower center metal, their underpass lines are thinner than standard inductors, which could effectively reduce the parasitic capacitance. It leads to higher f_{SR} for tapered-inductor in spite of higher inductance as compared with standard inductor.

The parasitic capacitances not only affect f_{SR}, but also raise the slope of series resistance. Figure 2(c) illustrates the influence of parasitic capacitances to the resistance. Standard inductor has lower resistance at low frequency, but higher resistance above 2.5G due to higher rising slope of resistance than tapered one.

With analyzing these measurement results of inductors, we propose some guidelines to enhance Q in the tapered structure for the RFIC designers: (1) Reducing parasitic capacitance, such as parallel capacitance (C_P) and oxide capacitance (C_{Ox}), will cause slower rising of Re(Z_{in}) than standard device before resonance and enhance the Q value. (2) Contact via area may be reduced with narrowing center metal width; this is why the performance of tapered_2 and tapered_3 devices is not as good as expectation.

3 IMPROVED MODELING

3.1 Model Description

A conventional 9-element model for planar spiral inductors is extensively used in circuit simulation due to the advantage of easy parameters extraction due to the nature of equivalent single-π circuit structure. Additional elements are still needed to model some physical effects and to enhance the accuracy prediction of inductor behavior.

For accurately modeling the behavior of tapered inductors, we propose an improved compact model, as shown in Fig 3. Since the tapered inductors have apparent skin effect, we added a branch (L_{S2} and R_{S2}) to model the skin effect. Otherwise, a resistance R_{P} is series connected with the capacitance C_p. This takes conductor loss into account [4].
The other element parameters: \( R_{s1} \) and \( L_{s1} \) denote the main resistance and inductance of the spiral inductor, respectively; \( C_p \) represents the direct capacitive coupling among the parallel turns and the coupling of the superimposition between the spiral and the underpass lines; \( C_{\text{sub}} \) and \( R_{\text{sub}} \) signify the capacitance and the resistance of the lossy substrate; and \( C_{\text{OX}} \) depicts the oxide capacitance between the metal line and silicon substrate.

### 3.2 Extraction Procedure

Because this model based on the equivalent single- \( \pi \) circuit structure is shown in Fig. 4, we can use this simple structure to extract the element parameters rather than the other model methods with complex skin effect components.

At low frequency, the branches with series capacitance can be neglected and the series inductance can be regarded as a short circuit, which leads to easy acquisition of the value of \( R_{s1}/R_{s2} \)

\[
Y = \begin{bmatrix} Y_A & Y_B \\ Y_B & Y_C \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_B \\ -Y_B & Y_B + Y_C \end{bmatrix}
\]

**Figure 4:** The single- \( \pi \) circuit structure and its relative Y parameters.

1. De-embed two port S-parameters
2. Calculate \( R_{s1}/R_{s2} = \text{Real}(1/Y_{12}) \) and \( (L_{s1}/R_{s1})^2 + (L_{s2}/R_{s2})^2 = \text{Imag}(Y_{12}) \) at lowest frequency, respectively.
3. Evaluate \( R_{s1}, R_{s2}, L_{s1} \) and \( L_{s2} \) from the above extraction values and curve fitting of \( \text{Real}(Y_{12}) \) and \( \text{Imag}(Y_{12}) \) before 6GHz.
4. Extract \( C_p \) and \( R_p \) from the resonance frequency of \( Y_{12} \) and \( \text{Real}(Y_{12}) \).
5. Extract the \( C_{\text{ox}}, C_{\text{sub}}, \) and \( R_{\text{sub}} \) using the method of [5].

### 3.3 Simulation Results

The improved model is mainly for modeling the frequency-dependence resistance. We make comparisons in terms of the real part of \( Z_{12} \) up to 6GHz between the simulation of improved and conventional model with measured data in Fig. 6. It can be shown that the improved model has an appreciable improvement in the frequency-dependence resistance. The improved model also has good prediction of Q and inductance, which is shown in Fig. 7

**Figure 6:** Simulated results of \( \text{Re}(Z_{12}) \) for a spiral inductor with conventional and our improved model. The measured data is also shown for comparison.
Figure 7: Measured and simulated (a) quality factor and (b) inductance for a spiral inductor using our improved model.

4 CONCLUSION

In this paper, we prove that spiral inductors with width-tapered structure can enhance the quality factor and find the quality factor of tapered inductors is dominated by via contact area and parasitic capacitances. The parasitic capacitance, such as parallel capacitance and oxide capacitance, will cause slower rising of Re(Z_in) than standard device before resonance and enhance the Q value.

An improved inductor model is proposed in this paper. These model parameters can be extracted based on a single-π model. This model can predict the frequency-dependent behavior of series resistance accurately. Further, we also can accurately model the quality factor and inductance of spiral inductors over a wide-band frequency.

REFERENCES