Linear Cofactor Difference Extrema of MOSFET's Drain Current and Their Application in Parameter Extraction

Jin He, Xuemei Xi, Mansun Chan*, Ali Niknejad, and Chenming Hu

Department of EECS, University of California at Berkeley, CA, 94720 jinhe@eecs.berkeley.edu
*Department of EEE, HKUST, Clear Water Bay, Kowloon, Hong Kong

ABSTRACT

The linear cofactor difference extrema of metal-oxide-semiconductor field effect transistor (MOSFET) drain current are presented in this paper and their application to extract MOSFET parameters is demonstrated. The extrema of the characteristic drain current are obtained by the applying the linear cofactor difference operator to the drain current versus gate voltage curve in the linear region. These extrema can be directly used to find the threshold voltage and mobility of a MOSFET. The method has been tested with experimentally fabricated MOSFETs and simulation data obtained by the device simulator DESSIS-ISE. The results agree well with those obtained with the standard second-derivative method, which demonstrates the validity of the method presented.

Keywords: Drain current linear cofactor difference, MOSFET, threshold voltage, parameter extraction.

1 INTRODUCTION

Threshold voltage and mobility are among the most important physical parameters in the analysis and modeling of metal-oxide-semiconductor field effect transistor (MOSFET) characteristics. The precise extraction of these parameters plays an important role in the accuracy of MOSFET models to predict the terminal currents in circuit simulations [1-3]. As a result, a number of methods have been developed in the attempt to provide a simple and accurate procedure to determine the values of these parameters [4-8]. However, simplicity and accuracy are difficult to achieve together, and most techniques are subjected to tradeoffs between the two extremes. Methods like linear extrapolation are simple, but do not provide enough accuracy. On the other hand, high precision methods like the regressive algorithm are very complex and time-consuming.

In this paper, a method to extract the threshold voltage and effective mobility of a MOSFET from the linear cofactor difference drain current of MOSFETs is proposed. By applying a newly developed linear cofactor difference operator (LCDO) on the transfer characteristics of a MOSFET measured in the linear region, the characteristic linear cofactor difference drain current extrema are obtained. Based on these extrema, the threshold voltage and the effective mobility can be directly determined. This method has been tested on fabricated and simulated n-channel MOSFETs. Very good agreement from the extraction result using the LCDO method and the standard second-derivative method is observed. Compared with other reported methods, the LCDO approach is straightforward and can easily be extended to other complex MOSFET models.

2 FORMULATION OF LCDO METHOD

In this section, the application of the linear cofactor difference operator method to obtain the corresponding extrama of drain current linear cofactor difference is described. If a function $f(x)$ is strictly monotonic, non-linear, continuous over $(x_0,\ x_1)$ and differentiable on $[x_0, x_1]$, then there exists a point x_p belongs to $(x_0,\ x_1)$ such that

$$G'(x_p) = \frac{\partial G}{\partial x}\Big|_{x=x_p} = 0 \tag{1}$$

where

$$G(x) = \Delta f(x) \equiv b + Kx - f(x) \tag{2}$$

is the linear cofactor difference of $f(x)$ and $\Delta f(x)$ is the linear cofactor difference operator (LCDO). b and K are the intercept and linear factor respectively.

The constants b and K can be determined by the end points of the interval (x_0, x_1) where the LCDO is applied, via the equations

$$G(x_1) = b + Kx_1 - f(x_1) = 0 \tag{3}$$

$$G(x_0) = b + Kx_0 - f(x_0) = 0 \tag{4}$$

The value of K are governed by $K > 0$ if $f(x)$ is a monotonically increasing function and $K \le 0$ otherwise. The above statement can be easily proven by using the Rolle's theorem [9]. We would like to emphasize that it is the choice of the LCDO region $(x_0,\ x_1)$ that determines the value of K and b.

The drain current of an n-channel MOSFET in the linear region is usually modeled as [4]

$$\frac{V_{ds}}{I_{ds}} = R_s + R_d + \frac{1}{\beta\mu_{eff}(V_{gs} - V_{th})} \tag{5}$$

with $\beta = \frac{WC_{ox}\mu_0}{L}$, $\mu_{eff} = \frac{\mu_0}{1+\theta_0\left(V_{gs}-V_{th}\right)}$.

In equation (5), R_s and R_d are the source and drain parasitic resistances and θ_0 is the intrinsic mobility attenuation coefficient.

If we define $R_t = R_s + R_d$ and $\theta = \theta_0 + R_t\beta$, R_t carries the meaning of total parasitic source/drain series resistance, and θ represents the effective attenuation coefficient that includes the effects of the parasitic source/drain series resistance. Eq.(5) can then be simplified into a common expression

$$I_{ds} = \frac{\beta\left(V_{gs}-V_{th}\right)V_{ds}}{1+\theta\left(V_{gs}-V_{th}\right)} \qquad (6)$$

When we apply the LCDO to Eq. (6), the following equation is obtained

$$\Delta I_{ds}\left(V_{gs}\right) = \frac{\beta\left(V_{gs}-V_{th}\right)V_{ds}}{1+\theta\left(V_{gs}-V_{th}\right)} - \left(KV_{gs}+b\right) \qquad (7)$$

The region to apply the LCDO has to be carefully chosen to include the extrema of Eq (7). Afterward, *K* and *b* can be simultaneously determined from Eqs. (3) and (4).

To proceed, we defines

$$\frac{\partial \Delta I_{ds}\left(V_{gs}\right)}{\partial V_{gs}}\Big|_{V_{gs}=V_{GP}} = 0 \qquad (8)$$

where V_{GP} is the critical gate voltage that correspond to the extrema of the drain current linear cofactor difference $\Delta I_{ds}\left(v_{GP}\right)$ for a particular value of *K*. The extrema of the linear cofactor difference drain current versus the gate voltage can then be solved for different value of *K* and the result is shown in Fig.1. Substituting Eq.(8) into Eq.(7) at the critical gate voltage V_{GP}, we obtain

$$\frac{1}{1+\theta\left(V_{GP}-V_{th}\right)} = \sqrt{\frac{K}{\beta V_{ds}}} \qquad (9)$$

Substituting Eq.(9) into Eq.(6) gives

$$I_{ds}\left(V_{GP}\right) = \sqrt{Kv_{ds}\beta}\left(V_{GP}-V_{th}\right) \qquad (10)$$

By using two different K_1, K_2 corresponding to 2 different LCDO, Eq. (10) can be directly used to calculate the transistor gain β, and the result is a close form expression given by:

$$\beta = \frac{1}{V_{ds}}\left[\frac{\frac{I_{ds}\left(V_{GP1}\right)}{\sqrt{K_1}} - \frac{I_{ds}\left(V_{GP2}\right)}{\sqrt{K_2}}}{V_{GP1}-V_{GP2}}\right]^2 \qquad (11)$$

After evaluating β, the low field mobility μ_0 can be found with given gate oxide thickness, effective channel *L* and width *W*. V_{th} can then be determined using Eq. (10).

It is worth emphasizing that a significant feature of the method presented can be observed by studying Eqs. (11) and (12). The derived V_{th} and β are independent of θ, which eliminated the effects of source and drain series resistance. It is in general not the case in other previously reported methods. The knowledge of V_{th} and μ_0 can in turn be used to calculate θ. Form Eq.(9), which we get

$$\theta = \frac{\sqrt{\beta V_{ds}}-\sqrt{K}}{\sqrt{K}\left[V_{GP}-V_{th}\right]} \qquad (12)$$

The intrinsic attenuation coefficient θ_0 can thus be deduced if the series resistance R_{sd} is known from a separate measurement, for instance, using the method described in [5].

3 RESULTS AND DISCUSSION

We first apply the LCDO method to output data from a fabricated n-channel MOSFET experimentally measured with the HP-4156B parameter analyzer. The experimental data are shown in the inset in Fig.1. The measured device was fabricated in the CMOS laboratory belongs to the Institute of Microelectronics, Peking University. The substrate doping concentration of the device is 1×10^{16} cm^{-3} and the gate oxide thickness is 16nm. The threshold voltage of the MOSFET was adjusted by ion-implant. The measured effective channel length and width are 5 μm and 4 μm respectively. Since we are interested in applying the LCDO to the linear region of the MOSFETs, the gate voltage has to be limited to $[x_0 \geq 1.2V, x_1 = 6V]$ as shown in Fig. 1. As a result, we get $K \geq 4\cdot10^{-6}$ and $b = 2\cdot10^{-6}$ for the I_{ds} versus V_{gs} curve with V_{ds}=0.1V.

Fig.1 presents a plot of the drain current linear cofactor difference versus V_{gs} with *K* assumed four different values of 4×10^{-6}, 4.2×10^{-6}, 4.4×10^{-6} and 4.6×10^{-6}. The same value of $b = 2\cdot10^{-6}$ is used in all 4 cases. As shown in the figure, the extrema of the drain current linear cofactor difference can be located for any of four the 4 different *K* values. The corresponding critical gate voltages are listed in Table I. From the result of Fig.1 and Table I, we obtain $\beta = 114.17\mu A/V^2$ and the threshold voltage V_{th} =1.5469V using Eqs. (11) and (10) with K_1 $=4.0\times10^{-6}$ and K_2=4.2×10^{-6}. From the value of β, we calculate the low field mobility and obtain μ_0=413.28 $cm^2/V\cdot s$. This result is consistent with previously measured inverse layer electron mobility [10]. The effective mobility attenuation coefficient is then give by $\theta = 0.2985V^{-1}$ after extraction using any one of the two different *K*, together with β. As the series resistance R_{sd} of our devices is measured to be about $R_s + R_d = 2\cdot810\,\Omega$, θ_0 is found to be around 0.1153V^1.

In order to investigate the sensitivity of the extracted parameters on the choice of the LCDO factor pairs of K_1 and K_2, six groups of calculated data using different combinations of K_1 and K_2 are used to examine the accuracy of the extracted values of V_{th}, β, μ_0 and θ. All the results are shown in Fig.2, which indicates the consistency of the parameter values despite the use of different combination of K_1 and K_2. Thus, the extracted parameters are relatively invariant on the choice of K_1 and K_2, indicating viability of the LCDO method. For instance, the extracted threshold voltage is 1.5469V by using the pair of $K_1 = 4.0\times10^{-6}$ and $K_2 = 4.2\times10^{-6}$, and V_{th}= 1.5479V for the combination of $K_1 = 4.0\times10^{-6}$ and $K_3 = 4.4\times10^{-6}$. In fact, decreasing the incremental step of gate voltage in the measurement can further reduce the difference in extracted threshold voltage between the choices of K pairs.

The LCDO method has been further verified MOSFETs characteristics data generated by the semiconductor device simulator DESSES-ISE [11]. The simulated device has a substrate doping concentration of 5×10^{16} cm^{-3}, a source/drain doping concentration of 10^{20} cm^{-3}, a junction depth 0.5 μm, and the gate oxide thickness is 20nm. In the simulations, the effective channel length is chosen to be 10 μm.

In order to obtain the extrema in both the subthreshold and the linear regions, the LCDO is applied to a range of operation regions covering the cut-off region, through the subthreshold region, and all the way to the linear region. The gate voltage is varied form zero to 6V to provide sufficient coverage in the simulation. As the absolute magnitude of the drain current at zero gate voltage is extremely small in this case, the value of the LCDO intercept, *b*, can be set to zero. The value of *K* is chosen to make the minimum and the maximum values of the gate voltage V_{gs} to approach the zero and 6V, respectively.

Fig.3 shows the plot the results, obtained by applying *LCDO* to the drain current, versus V_{gs} using three different values of *K* (1×10^{-6}, 1.05×10^{-6} and 1.1×10^{-6}). The original transfer characteristic of the MOSFET is also included in this figure. As shown in Fig.3, two extrema are observed, one locating in the subthreshold region and the other in the linear region, with the *K* values used. From the values of the extrema in the linear region, we obtain $\beta = 8.1908\times10^{-2}\,\mu A/V^2$ from Eq. (11). The V_{th} is then extracted from any one of the determined *K* together with the value of β, V_{GP} and $I_{ds}(V_{GP})$. We once again observe the consistency of the extracted threshold voltage using the LCDO method even with significantly different LCDO extrema values. The extracted threshold voltage of 0.8449V matches well with the value of V_{th} = 0.8459V extracted separately using a conventional second-derivative method. It should be noted that the extrema in the subthreshold region can also be used to study the subthreshold conduction of a MOSFET [12].

Using the same technique, linear cofactor difference operator with a different set of K values is also applied to the simulated n-channel MOSFETs with the same structure but different effective channel lengths. Fig.4 shows the threshold voltage obtained by the LCDO method for four devices with the effective channel length of 0.5, 1, 5 and 10 μm. As a control experiment, the second-derivative method has also been used to extract the threshold voltages of the devices, and the results are also included in the figure. Very good matching is observed between the results obtained using the 2 different methods, indicating the validity of the LCDO method to capture the threshold voltage roll-off with channel length scaling. Similarly, the room temperature low field electron mobility and its effective attenuation coefficient are also extracted from the transistor with different channel lengths. The results are shown in Fig.5. As observed from the figure, all mobility curves converges to a particular value of μ_0=478 $cm^2/V\cdot s$ at low V_{gs}. It is reasonable from the general understanding of the device physics. The data obtained from LCDO method correctly indicates the trend that the shorter the channel length of a MOSFET, the larger is the mobility degradation factor.

4 CONCLUSION

The use of the extreme in the drain current linear cofactor difference of a MOSFET to extract the threshold voltage and effective mobility of MOSFEFs has been presented in this paper. The principle of this extraction method is based on the application of linear cofactor difference operator to produce extrema in the current versus the gate voltage function after the transformation. Based on the extrema, different parameters from n-MOSFETs with the different channel length from simulation and experiment data have been extracted. The results are in good agreement with that obtained from the standard second-derivative method, showing the correctness of the method presented.

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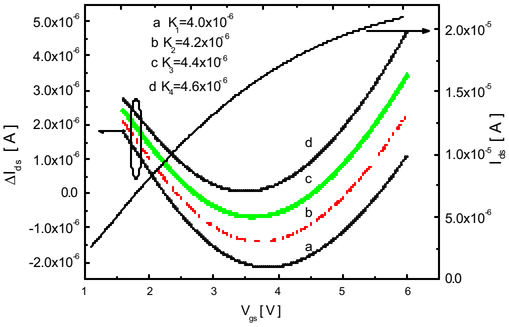

Fig.1 The drain current linear cofactor difference versus gate voltage for different value of *K*. The LCDO method is applied to the linear region I-V characteristics of a fabricated n-channel MOSFET.

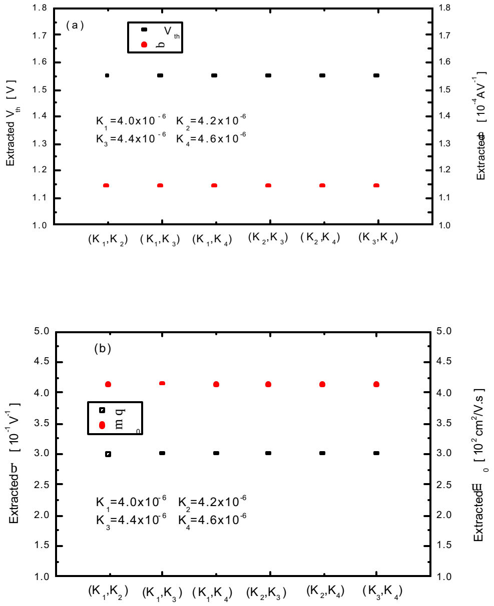

Fig.2 The dependence of the extracted parameters on the choice of the values of K_i and K_j: (a) Threshold voltage and transistor gain factor extracted for the different value of *K* and (b) Low field mobility and its degradation factor extracted for the different value of *K*.

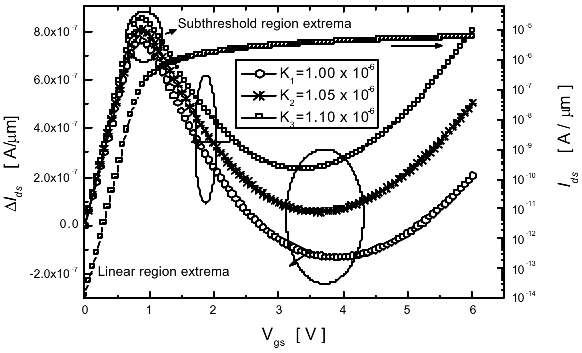

Fig.3 The drain current linear cofactor difference versus gate voltage for different values of *K* in the subthreshold and linear region of a simulated n-channel MOSFET.

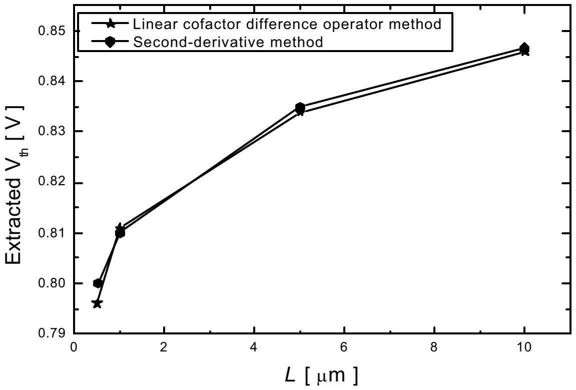

Fig.4 Comparison of threshold voltage variation obtained by LCDO the standard second-derivative methods.

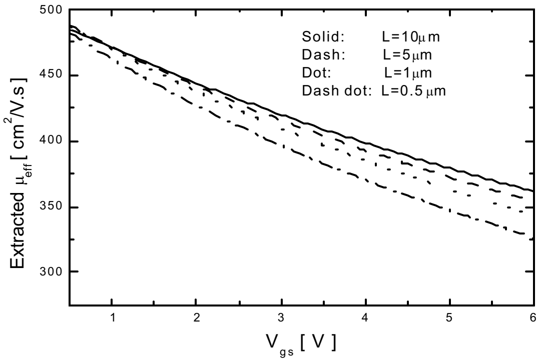

Fig.5 Mobility degradation characteristics of the simulated MOSFETs with the different channel length obtained by the linear cofactor difference operator method.