Modeling of charge and collector field in Si-based bipolar transistors

H. Tran and M. Schroter
Chair for Electron Devices and Integrated Circuits, Dresden University of Technology
Mommsenstr. 13, 01062 Dresden, Germany, mschroter@ieee.org

ABSTRACT

An analytical formulation for the voltage and current dependent electric field in the collector of a bipolar transistor is presented. The new field expression is then employed for calculating the base-collector depletion capacitance and the field related transit time components. Comparison to device simulation results show good agreement.

1 Introduction

Rapidly increasing mask cost and reduced design cycles are putting increasing pressure on EDA tool capabilities, including compact models. Furthermore, Si/SiGe bipolar technology development has accelerated tremendously over the past couple of years, leading to an increasing boost for advanced compact bipolar transistor models and modeling methodologies. For instance, the capability for predictive and statistical modeling (and design) has become an important requirement from design houses for many foundries. Such a capability in turn requires physics-based compact models and extraction strategies.

The electrical behavior of bipolar transistors, regardless whether they include a heterojunction at the collector or not, is strongly determined by the conditions in the collector region, especially at medium and high current densities, i.e. at peak f_T and beyond. One of the key variables here is the electric field in the collector, in particular at the base-collector (BC) junction, which is linked to the BC capacitance, minority carrier density in base and collector, and the avalanche breakdown. In III-V HBTs, and possibly also in future SiGe HBTs, velocity overshoot occurs in the collector region, causing the conventional model formulations for the above mentioned quantities and effects to become inaccurate. In addition, certain parameter extraction methods such as determining the transit time from $1/(2\pi f_T)$ vs $1/I_C$ cannot be easily applied anymore [1].

In this work, a first version of a bias dependent analytical model for the electric field in the collector is presented. Based on this formulation, the current dependent BC depletion capacitance and transit time are described. The resulting model equations are compared to 1D device simulations in order to verify the fundamental suitability and accuracy, and to possibly identify areas of improvement.

2 Investigated technology

The SiGe HBT under investigation contains a "conventional" doping profile as shown in Fig. 1, with a high emitter and moderate base concentration. The collector doping corresponds to that of a "power" or high-voltage transistor type in such processes. The peak transit frequency of this transistor is about 35GHz at V_{BC}=0V.

Since the investigations in this paper are based on 1D device simulations, currents, charges and capacitances are normalized to the unit area $A_E=1\mu m^2$.

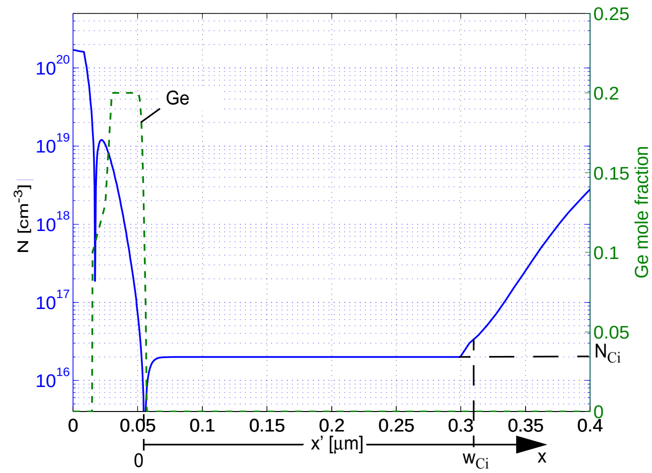

Fig. 1 Doping profile of the investigated transistor. The dashed line indicates the Ge profile.

3 Modeling the electric field

According to Gauss' law, the charge Q_{BC} in the collector region is associated with the electric field E_{jc} at the BC junction (x=0 in Fig. 1) via the following relation,

$$Q_{BC}(V_{BC}, I_{Tf}) = \varepsilon E_{jc}(V_{BC}, I_{Tf}) \, . \quad (1)$$

Since it is more appropriate from a modeling and (circuit) application point of view, the (internal) BC terminal voltage V_{BC} and the quasi-static forward transfer current I_{Tf} have been selected as independent variables rather than V_{BC} and V_{BE}. For quasi-static operation, Q_{BC} can be obtained from a path independent integration over the independent variables. Thus, one can write for the incremental change

$$dQ_{BC}(V_{BC}, I_{Tf}) = C_{jCi}\, dV_{BC} + \tau_{BC}\, dI_{Tf} \quad (2)$$

where the variables

$$C_{jCi}(V_{BC}, I_{Tf}) = \left.\frac{\partial Q_{BC}}{\partial V_{BC}}\right|_{I_{Tf}} \text{ and } \tau_{BC}(V_{BC}, I_{Tf}) = \left.\frac{\partial Q_{BC}}{\partial I_{Tf}}\right|_{V_{BC}} \quad (3)$$

can be defined as bias dependent (internal) BC depletion capacitance and BC transit time, respectively.

According to (1), the above elements can be calculated as a function of bias, once the electric field is known. This also holds for other elements in a transistor, such as the base portion of the transit time and the avalanche current, which are strongly current dependent and have been difficult to describe sufficiently physics-based. In addition, “non-stationary” transport effects can be included in a compact model to first order. In practice, the difficulty though has been to find a sufficiently *simple* and *continuously differentiable* formulation for E_{jc} that at the same time ensures *accurate* derivatives.

Generally, the electric field in the collector can be obtained from solving Poisson’s equation. However, suitable compact analytical formulations require certain simplifying assumptions, mainly

- a spatially independent doping concentration N_{Ci} within the (lightly doped) collector region w_{Ci}, and
- an abrupt transition at w_{Ci} to the much larger buried layer doping concentration.

The consequence of these assumptions is that two mathematically different solutions are obtained, depending on the bias point (V_{BC}, I_{Tf}), which are repeated here for further discussion. For a partially depleted collector holds (note that E_{jc} is defined as positive here)

$$E_{jc} = E_{wc} + \sqrt{\frac{2qN_{Ci}}{\varepsilon}}\sqrt{\left(1-\frac{I_{Tf}}{I_{lim}}\right)(v_{ceff} - E_{wc}w_{Ci})} \qquad (4)$$

with the field at the buried layer side at $x = w_{Ci}$,

$$E_{wc} = \frac{I_{Tf}}{qA_E\mu_{nCi}(E_{wc})N_{Ci}} \quad . \qquad (5)$$

Here, μ_{nCi} is the field dependent mobility in the collector; $v_{ceff} = V_{DCi} - V_{BC}$ is the effective voltage across the collector region $[0,\ w_{Ci}]$ with V_{DCi} as built-in voltage; finally, $I_{lim} = qN_{Ci}v_{sn}$ with v_{sn} as electron saturation velocity.

In contrast, for a fully depleted collector holds

$$E_{jc} = \frac{v_{ceff} + V_{PT0}(1 - I_T/I_{lim})}{w_{Ci}} \qquad (6)$$

with the low-current punch-through voltage

$$V_{PT0} = \frac{qN_{Ci}}{2\varepsilon}w_{Ci}^2 \ . \qquad (7)$$

The above solutions were employed in, e.g., [2][3] for modeling the base transit time. However, there are several issues with the above formulations regarding their use in compact models. First of all, the square root dependence (4) on V_{BC} is not suitable for describing the BC depletion capacitance accurately enough for realistic applications. This will be discussed later on more detail. Second, the equations contain various numerical instabilities (i.e. conditions causing arithmetic errors) that have to be taken care of properly to arrive at a *reliable* compact model formulation. For non-zero current, there is neither a continuous first derivative of E_{jc} with respect to current or voltage nor a smooth transition from high to low voltages. Third, the equations are not valid in the high-current region (i.e. close to peak f_T and beyond). Some of these issues were addressed in [2], but a satisfying reliable formulation was not obtained; also, the square-root dependence was maintained, so that modeling of C_{jCi} could not be addressed.

Fig. 2 and Fig. 3 show the field and carrier distributions in the drift-type SiGe HBT investigated here. While the distributions in the collector are similar to that of the low-emitter concentration transistor in [2], they are quite different in the base region, where both the doping and Ge gradient cause a large field that influences E_{jc} at high current densities. Fig. 2 also contains a comparison between the electrostatic field, E_ψ, and the field E_φ that is defined by the gradient of the electron quasi-fermi potential. The latter is the actual driving force for the current in the BC region, and thus should be used in the model.

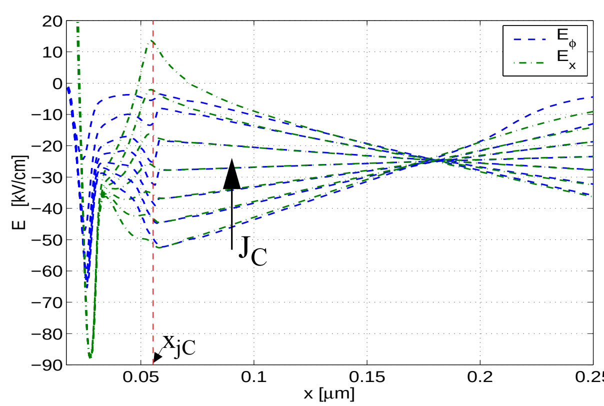

Fig. 2 Electric field distribution in base and collector region for selected bias points.
J_C/[mA/µm^2] = 0.074, 0.155, 0.225, 0.3, 0.37, 0.45, 0.51.

The consequences of the high electric field in the base are a dip in the electron density as shown in Fig. 3. Thus, the simple diffusion-type model applied in [2] to the transit time has to be generalized to make a model applicable to a wide range of process technologies.

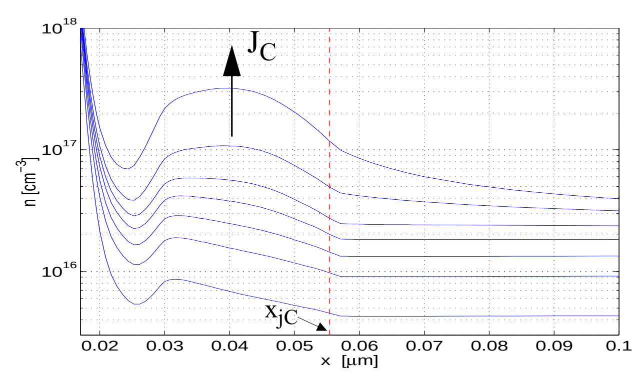

Fig. 3 Electron distribution in the base and BC region for selected bias points (cf. Fig. 2).

At this point, it is instructional to take a look at the current dependence of E_{jc}, which is shown in Fig. 4 for the transistor defined in Fig. 1. As expected, the field decreases with current. At low voltages, the square-root like dependence on current can be observed while in the punch-through region the dependence is quite linear as expected. At high current densities, the *electrostatic* field becomes negative in a SiGe (and any other) DHBT, while $|E_\varphi|$ reaches its minimum given by $E_\infty = 2V_T/w_C$ which can be derived from simple theory. Furthermore, it is interesting to note that the transition from the almost linear decrease of $|E_\varphi|$ to E_∞ can be well described as a function of voltage by the critical current I_{CK} in HICUM [4], which is indicated by arrows in Fig. 4.

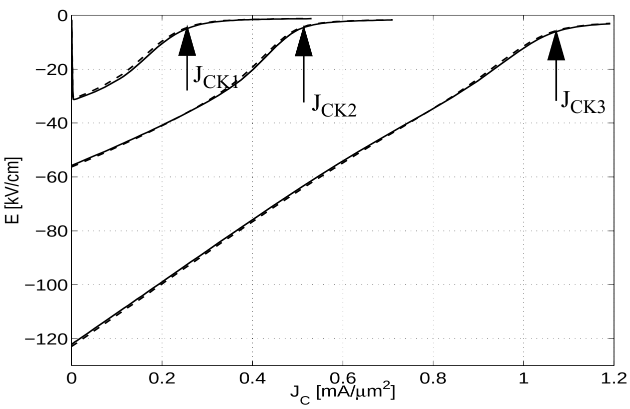

Fig. 4 Bias dependent electric field at the BC junction calculated from electron quasi-fermi potential (solid lines) and electrostatic potential (dashed lines) at V_{BC}/V = 0.5, 0, -2. The arrows indicate the critical current I_{CK}.

For a compact model, the analytical description of E_{jc} has to be kept as simple as possible mainly in order (i) to be able to guarantee the numerical stability of the overall formulation and (ii) to minimize the arithmetic operations. Thus, the bias dependent electric field is described here by

$$E_{jc} = E_\infty + f_e E_{lim} \tag{8}$$

with the field $E_{lim}=v_{sn}/\mu_{nCi}(E=0)=V_{lim}/w_{Ci}$. The smooth transition from medium to high current densities is accomplished by the function

$$f_e = \frac{e_j + \sqrt{e_j^2 + g_{jc}\frac{E_{CK}}{E_{lim}}}}{2} \tag{9}$$

with the (model) parameter g_{jc} and the argument [8]

$$e_j = \frac{(E_{jC0} - E_\infty) - (E_{jC0} - E_{CK})\frac{I_T}{I_{CK}}}{E_{lim}} \; . \tag{10}$$

It depends on the critical current I_{CK} and the bias dependent critical field

$$E_{CK} = E_{lim}\frac{v_{ceff}/V_{lim}}{\sqrt{1 + (v_{ceff}/V_{lim})^2}} \; , \tag{11}$$

which takes into account that at very low voltages the onset of high current effects occurs when the field curve becomes horizontal. Furthermore, E_{jc0} in (10) is the field at I_{Tf}=0 which, according to (1), can be described continuously differentiable as a function of v_{ceff} by using the depletion charge expression for Q_{jCi} already available in HICUM [4]. This allows to adjust the absolute value of the field based on measurements and eliminates the inaccurate square dependence on v_{ceff}. In order to capture the ohmic voltage drop, that occurs in the partial depletion case at the end of the collector, v_{BC} has been replaced by $v_{BC}+\Delta v_{pd}$ with

$$\Delta v_{pd} = V_{lim}\frac{I_{Tf}}{I_{lim}}\left(1 + \frac{I_{Tf}}{I_{lim}}\right) \; . \tag{12}$$

Above expression follows from (5) after converting the numerically unstable term $1/(1-I_{Tf}/I_{lim})$ to the numerically stable expression $(1+I_{Tf}/I_{lim})$.

Fig. 5 shows a comparison between the analytical field model and the results obtained from device simulation over a wide voltage range V_{BC}. The agreement is fairly good.

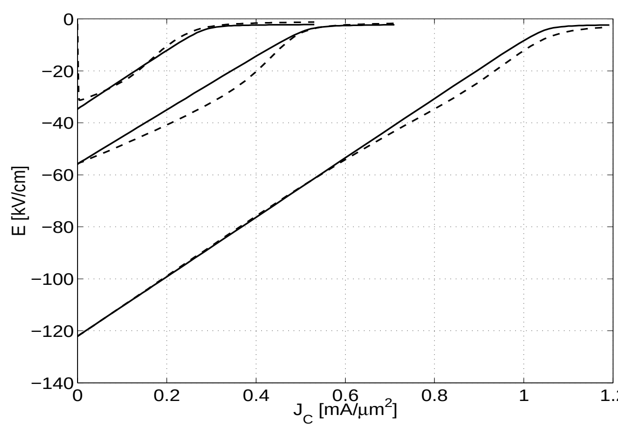

Fig. 5 Bias dependent electric field at the BC junction: comparison between analytical model (solid lines) and device simulation (dashed lines); V_{BC}/V = 0.5, 0, -2.

4 Base-collector depletion capacitance

The capacitance is calculated analytically from E_{jc} applying (3). In addition, the voltage dependent formulation for the critical current I_{CK} and its derivative have been included. Fig. 6 shows the depletion capacitance vs. current and voltage over a wide bias range. The peak in the current dependence at low voltages is caused by the ohmic voltage drop Δv_{pd} in the partial depletion case, which increases the forward biasing of the voltage across the junction. At higher voltages and medium current densities, the capacitance becomes flat when the punch-through case occurs. Once the electric field at the junction starts to collapse at I_{CK}, the capacitance decreases and then disappears at high current densities.

Overall, the analytical voltage *and* current dependent BC depletion capacitance agrees fairly well with the device simulation. The accuracy of the corresponding charge is equivalent to that of the electric field (cf. Fig. 5).

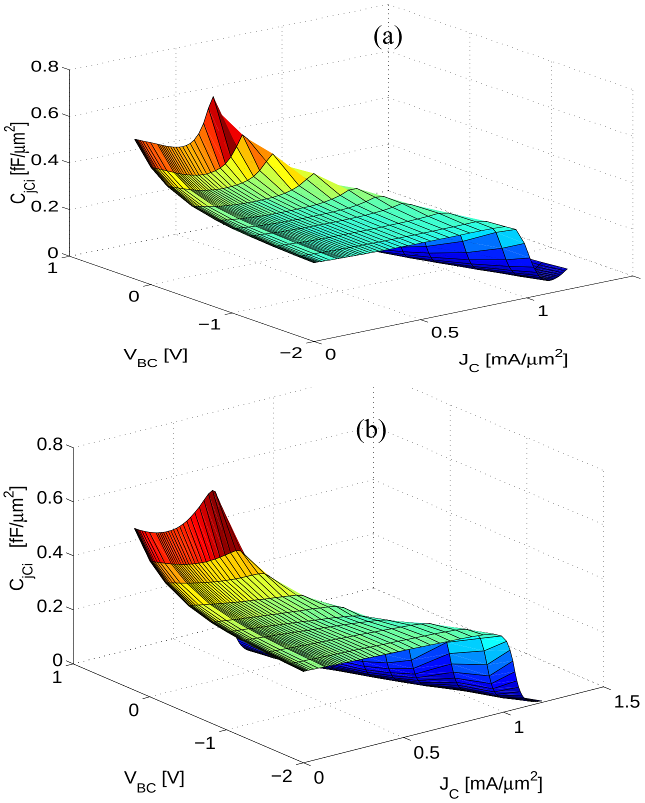

Fig. 6 Voltage and current dependence of the BC depletion capacitance: (a) device simulation, (b) analytical equation.

5 Transit time

The forward transit time τ_f represents the minority charge storage and determines the dynamic transistor behavior especially at medium and high current densities. τ_f can be partitioned into its components associated with the neutral and space-charge regions (SCRs),

$$\tau_f = \tau_{pE} + \tau_{BE} + \tau_{Bf} + \tau_{BC} + \tau_{pC}, \qquad (13)$$

which is shown in Fig. 7 for the selected transistor. Here, τ_{pE}, τ_{Bf}, τ_{pC} represent the charge storage in the neutral regions, and τ_{BE}, τ_{BC} are associated with the SCRs. Such a partitioning is very useful for compact modeling since it allows to separate the various effects, determine their relative importance, and derive adequate model expressions.

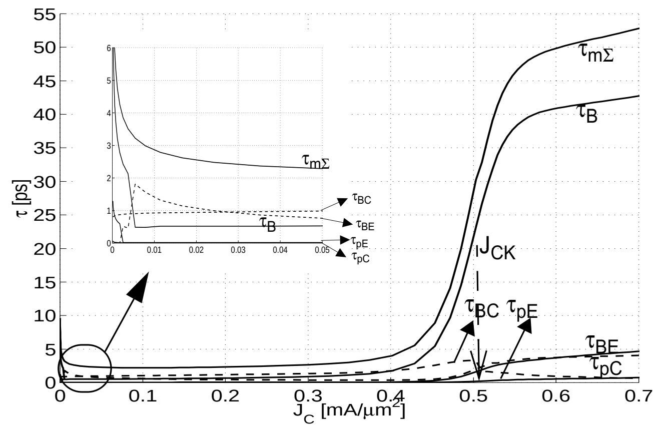

Fig. 7 Transit time components in a SiGe DHBT as a function of collector current density. V_{BC} = 0V.

Various partitioning methods have been proposed in literature. The classic regional approach (e.g. [5]) considers d.c. carrier concentrations and, thus, does generally not reproduce the small-signal transit time. The often referenced small-signal based method described in [6] only works for bipolar transistors with unrealistically high collector doping, but not for practical profiles and at high current densities. Furthermore, in advanced BJTs and HBTs the large doping gradients and intentional bandgap changes can produce "spikes" in the (small-signal) carrier densities, that turned out to make also other published partitioning methods (e.g. [7]) known to the authors unsuitable. As a consequence, the small-signal based carrier (and charge) partitioning definition described in [9] had to be extended in order to obtain clearly defined regions and a smooth bias dependence.

From Fig. 7, it can be observed that at low current densities (i.e. below I_{CK}) the contributions of neutral base and emitter as well as the BC SCR are very similar, while the other two components are negligible. At I_{CK} the base transit time increases rapidly due to the collapse of the electric field E_{jc} and the associated formation of the barrier at the location where the Ge decreases into the collector. In contrast to homo-junction transistors, the increase of the collector transit time is negligible and even the saturation value very small. The latter results from the relatively thin collector width, which gives τ_{pCs}=5.6ps according to theory [4][10], and can be considerably larger for power transistors with thicker epi. At very small current densities, an increase of the BE component, which is associated with the so-called neutral charge in the BE SCR, can be observed, but is irrelevant for circuit applications due to the very small absolute value of the charge in that bias region.

An analytical model for describing the transit time τ_f and its components was presented in [10] for Si BJTs. Although the formulation is flexible enough to allow any partitioning of the base and collector transit time at high current densities, it does not explicitly include the impact of the bias dependent electric field E_{jc} on the base component. Therefore, the existing theory for the base component has been extended to yield

$$\Delta\tau_{bfv} = \tau_{bfvl} f_u \left[1 - \frac{I_{Tf}}{E_{lim}} \frac{1}{u}\left(1 - \left(\frac{v_n}{v_{sn}}\right)^{\gamma_u}\right)\frac{dE_{jC}}{dI_{Tf}}\right]\exp\left(-b_{hc}\frac{u}{2}\right) \quad (14)$$

with [10] τ_{bfvl} as base transit time at negligible collector current, the normalized field variables

$$u = \frac{E_{jc}(V_{BC}, I_{Tf})}{E_{lim}} \quad , \quad u_0 = \frac{E_{jc}(V_{BC}, 0)}{E_{lim}} = \frac{E_{jc0}}{E_{lim}} \quad , \qquad (15)$$

$v_n(E_{jc})$ as field dependent electron velocity (γ_n =2) and the associated saturation velocity v_{sn}, and the function

$$f_u = \frac{(1 + u^{\gamma_u})^{1/\gamma_u} u_0}{(1 + u_0^{\gamma_u})^{1/\gamma_u} u} \underbrace{\frac{C_{jCi}(V_{BC}, I_{Tf})}{C_{jCi}(0, I_{Tf})}}_{1/c} . \qquad (16)$$

Furthermore, the last term $\exp(-b_{hc}u/2)$, which has already been given in [2], represents the barrier effect with the (new) model parameter b_{hc}.

Fig. 8 shows a comparison between the analytical model with the new description of the base transit time and the device simulation. Good agreement is obtained over a wide voltage and current density range.

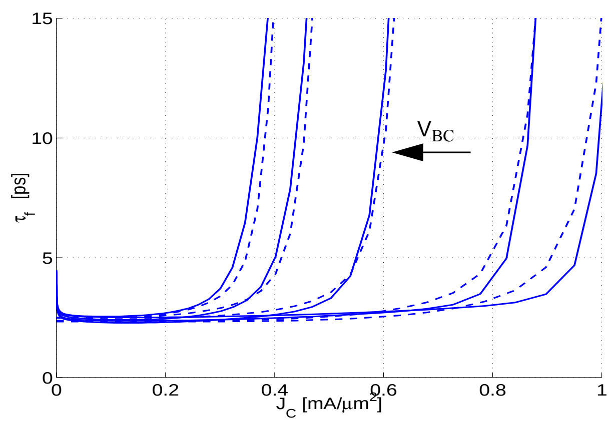

Fig. 8 Transit time vs. collector current density: comparison between device simulation (dashed lines) and new analytical equation (solid lines). V_{BC}/V= 0.2, 0, -0.5, -1.5, -2.

6 Modeling velocity overshoot

The analytical expression for the electric field as a function of bias can be used to model the effect of velocity overshoot observed in certain III-V HBTs. In this paper, the goal is only to evaluate the suitability of the new formulation for compact modeling of the transit time in AlGaAs HBTs. For this, the standard velocity-field expression for electrons in GaAs

$$v_n = v_{sn}\frac{(v_{max}/v_{sn})(E_{jc}/E_{lim}) + (E_{jc}/E_{lim})^4}{1 + (E_{jc}/E_{lim})^4} \qquad (17)$$

is inserted into the transit time formulation. The component most impacted by the different velocity-field relation is now the base-collector delay time τ_{BC}.

Fig. 9 contains a representation that is typically used to determine the transit time, and which can be used to display any velocity overshoot (e.g. [1]). Except for the transit time, all other parameters for calculating the transit frequency f_T were taken directly from device simulation, so that the difference is solely due to the different velocity-field expressions. The result in Fig. 9 clearly indicates for the GaAs expression a significant drop of $1/(2\pi f_T)$ before it starts to increase rapidly in the high-current region. Thus, it is expected that the new transit time formulation will be applicable to III-V HBTs, although the present model is based on an a-priori known bias dependence of the electric field; i.e. the influence of the now modified carrier distribution on the field is neglected. This would require an iterative solution which does not seem suitable for a compact model.

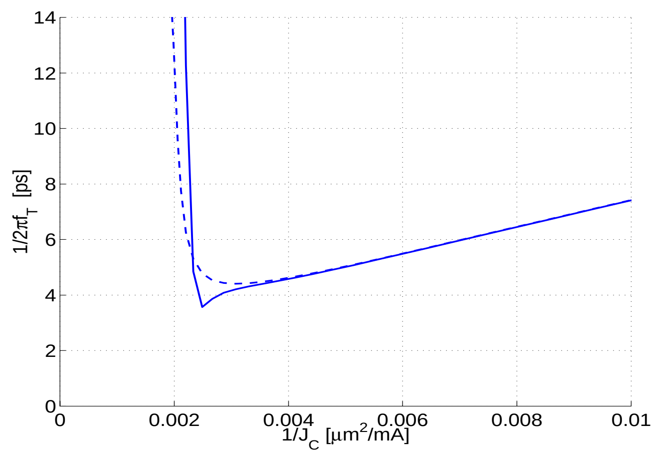

Fig. 9 Reciprocal of the transit frequency vs. reciprocal collector current with different velocity-field models: Si (dashed line) and GaAs (solid line).

7 Conclusion

A first version of a bias dependent description of the electric field in the collector of a bipolar transistor has been presented, which is suitable for compact modeling. Due to the complexity of the problem, caused especially by the various numerical instabilities in the fundamental (classical) solutions, the goal was to keep the new formulation as simple as possible.

The focus of the present work has been on transistors realized in advanced SiGe process technologies. However, as already indicated by Fig. 9, the formulation is also assumed to be applicable to III-V HBTs, since the availability of an analytical solution for the electric field in the collector allows to include velocity overshoot to first order. Further work will attempt to

- improve the accuracy in certain regions of operation,
- validate the formulations for a larger variety of device designs, including III-V HBTs and experimental data, and
- possibly simplify certain expressions further.

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References

[1] P. Zampardi et al., “III-V HBT modeling, scaling and parameter extraction using TRADICA and HICUM“, Power Amplifier Workshop, San Diego, Sept. 2003.

[2] S. Wilms and H.-M. Rein,”Analytical high-current model for the transit time of SiGe HBTs“, Proc. Bipolar/ BiCMOS Circuits and Technology Meeting, Minneapolis, pp. 199-202, 1998.

[3] M. Friedrich and H.-M. Rein, “A novel transistor model for simulating avalanche-breakdown effects in Si bipolar circuits “, IEEE J. Solid-State Circuits, Vol. 37, 2002, pp. 1184-1197.

[4] M. Schroter, “HICUM, a Scalable Physics-based Compact Bipolar Transistor Model”, http://www.iee.et.tu-dresden.de/iee/eb, December 2001.

[5] R. Schilling, “A Regional Approach for Computer-Aided Transistor Design”, IEEE Trans. Education, Vol. E-12, pp. 152-161, 1969.

[6] Van den Biesen, “A Simple Regional Analysis of Transit Times in Bipolar Transistors”, Solid-State Elec., Vol. 29, pp. 529-534 , 1986.

[7] Palestri et al. “A Drift-Diffusion/Monte Carlo Simulation Methodology for $Si_{1-x}Ge_x$ HBT Design”, IEEE. Trans. Electron Dev., Vol. 49, pp. 1242-1249, 2002.

[8] M. Schroter, SFB Project F2, Report for period 2000-2001.

[9] M. Schroter, "Physical models for fast silicon bipolar transistors - A review", Habilitation thesis (in German), Ruhr-University Bochum, Bochum, Germany, 1993.

[10] M. Schröter and T.-Y. Lee, „A physics-based minority charge and transit time model for bipolar transistors“, IEEE Trans. Electron Dev., vol. 46, pp. 288-300, 1999.

[11] M. Schroter, "A compact physical large-signal model for high-speed bipolar transistors with special regard to high current densities and two-dimensional effects", Ph. D. thesis (in German), Ruhr-University Bochum, Bochum, Germany, 1988.