Simulation of Micromachined Inertial Sensors with Higher-Order Single Loop Sigma-Delta Modulators

Yufeng Dong, and Michael Kraft

University of Southampton, School of electronics and computer science,
Highfield, Southampton, SO17 1BJ, UK, yd02r@ecs.soton.ac.uk

ABSTRACT

Micromachined capacitive inertial sensors incorporated in sigma-delta force-feedback loops have been proven to improve linearity, dynamic range and bandwidth, and also provide a direct digital output. Previous work mainly focused on using only the sensing element to form a 2nd-order single loop sigma-delta modulator (ΣΔM). Therefore, the advantages of higher-order (4th-order and 5th-order) single loop electro-mechanical ΣΔM have not been explored, especially for inertial sensors that require higher Signal to Quantization Noise Ratio (SQNR), wide-band signal and low power dissipation. This paper presents architecture for higher-order single loop electro-mechanical ΣΔM with optimal stable coefficients that lead to better SQNR. Simulations show the maximum SQNR of 3rd-order, 4th-order and 5th-order ΣΔM is 88dB, 105dB and 122dB, respectively, using an Oversampling Ratio (OSR) of 256.

Keywords: higher-order, micromachined inertial sensors, noise transfer function, sigma-delta modulation.

1 INTRODUCTION

There is increasing demand for high performance (micro-g accuracy) micromachined inertial sensors that comes from inertial navigation/guidance systems, space micro-gravity, unmanned aerial vehicles, and seismometery for oil exploration and earthquake prediction. High performance inertial sensors usually exploit the advantages of closed loop control strategy to increase the dynamic range, linearity and bandwidth of the sensors. To avoid the electrostatic pull-in problems in purely analogue force feedback closed loop scheme, a ΣΔM closed loop force feedback control scheme has become very attractive for capacitive inertial sensors. Its output is digital in the format of a pulse density modulated bitstream and can directly interface to a digital signal processor.

Previous work mainly focused on using a sensing element to form a 2nd-order, single loop, electro-mechanical ΣΔM [1]-[2]. A sensing element can be approximated by a second order mass-damping-spring system and can be regarded as analogous to the two-cascaded electronic integrators commonly used in 2nd order electronic ΣΔM A/D converters. However, the equivalent d.c. gain of the mechanical integrator functions is considerably lower than their electronic counterparts. This leads to a much lower SQNR (for example, most below 66dB at OSR=256) for electro-mechanical ΣΔM compared with a purely 2nd order electronic implementation. The most obvious method to increase the SQNR is to increase the sampling frequency of the system, but increasing the sampling frequency will also increase the electronic thermal noise and power dissipation. Recently, some researchers used an additional electronic integrator to form a 3rd-order ΣΔM [3]-[4]. Kajita et al [3] demonstrated the higher noise shaping at low frequencies by adding an electronic integrator to a 2nd-order single loop electro-mechanical ΣΔM, but the SQNR is around 90dB at OSR=2500, which may be still low for high performance applications. Furthermore, Kraft et al [4] presented a 3rd-order multi-stage noise shaping (MASH) electro-mechanical ΣΔM, but it is known that MASH loop is more sensitive to the coefficient variations than single loop [5]. The microfabrication usually exhibits large manufacturing variations, the characteristics of a micromachined inertial sensor are not precisely fixed without calibration, therefore, the cascaded MASH structures, which rely on accurate quantization noise cancellation, may be problematic in implementation. Therefore, single loop electro-mechanical ΣΔM architecture seems the most practical configuration for high performance inertial sensors. However, the advantages of higher-order (4th-order and 5th-order) single loop electro-mechanical ΣΔM have not been explored, especially for inertial sensors that require higher SQNR, wide-band signal and low power dissipation. This work is to design a higher-order single loop electro-mechanical ΣΔM with optimal stable coefficients that lead to better SQNR. To achieve the same performance as 2nd-order ΣΔM, a higher-order ΣΔM can use lower OSR and this leads to lower electronic thermal noise and power dissipation.

2 DESIGN METHODOLOGY

2.1 Topology of a Higher Order Single Loop Electro-Mechanical ΣΔM

Although interpolative topologies with multiple feedback and forward paths are very successful approaches to implement single loop high-order delta-sigma A/D converters, these topologies can not be directly applied to an electromechanical ΣΔM because the sensing nodes can not be connected to feedback & feedforward paths. Here, we propose a practical topology to maximize the SQNR using the fewest feedback paths that give the simplest implementation and thus the lowest circuit complexity. Such a topology in a 5th order loop is shown in Figure 1. K_1 to K_3 are coefficients of the electronic integrators, K_4 is the variable gain of the quantizer, K_{dv} is the gain of displacement to voltage of the sensor, K_{po} is the pick-off gain of capacitance position sensing circuitry, and K_{fb} the gain through the voltage to force conversion in the feedback path. K_m=$K_{po}K_{fb}$ is an equivalent coefficient. In fact, K_{po} and K_1 are effective one coefficient, splitting them is meaningful only on convenient comparisons among different order electro- mechanical ΣΔM.

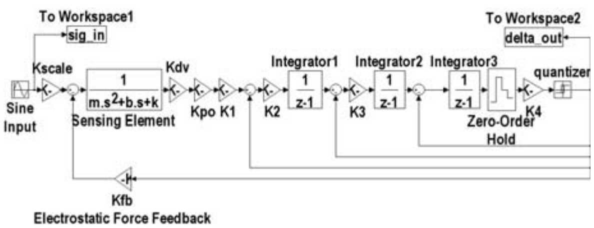

Fig. 1: Topology of a 5th order electromechanical ΣΔM.

2.2 Noise Transfer Function of a n^{th} Order Electro-Mechanical ΣΔM

The linearized quantizer, modeled by a white noise source and variable gain, is still good approximation for a higher order ΣΔM with great OSR to determine its properties [5]. The noise transfer function (NTF) H_e(z) of n^{th}-order electromechanical ΣΔM in a topology of Figure 1 is given by:

$$H_e(z) = \frac{1}{1 + \prod_{i=1}^{n-1} K_i K_m M(z)[H(z)]^{n-1} + \sum_{i=2}^{n-1}\prod_{j=i}^{n-1} K_j [H(z)]^{n-i}} \quad (1)$$

where M(z) is the mechanical transfer function of the sensing element by transforming to discrete-time z-domain:

$$M(z) = K_f \cdot \frac{(z-a_f)}{(z-b_f)\cdot(z-c_f)} \quad (2)$$

where K_f, a_f, b_f and c_f are gain, zero and poles of a sensing element, which are a function of the sampling frequency. According to the z-transformation $z = e^{j\omega}$ and assuming $f \le f_b << f_s$:

$$z^{-1} = e^{-j2\pi f/f_s} = \cos(\frac{2\pi f}{f_s}) - j\sin(\frac{2\pi f}{f_s}) \approx 1 - j\frac{2\pi f}{f_s} \quad (3)$$

$$\left|1-z^{-1}\right| \approx \left|j\frac{2\pi f}{f_s}\right| = \left|j\pi\frac{f}{f_b}\frac{2f_b}{f_s}\right| = \left|j\pi\frac{f}{f_b}\right|\cdot\left|\frac{1}{OSR}\right| << 1 \quad (4)$$

where f_b is the signal bandwidth, f_s the sampling frequency, and OSR=f_b / f_s /2 the oversampling ratio.

After omitting the items containing $|1\text{-}z^{-1}|$ in the denominator of (1), the NTF H_e(z) can thus be approximated at low frequencies:

$$\left|H_e(z)\right| = \frac{\left|1-z^{-1}\right|^n + \left|2-b_f-c_f\right|\left|1-z^{-1}\right|^{n-1} + \left|(1-c_f)(1-b_f)\right|\left|1-z^{-1}\right|^{n-2}}{K_m K_f(1-a_f)\prod_{i=1}^{n-1}K_i + \left|(1-c_f)(1-b_f)\right|\prod_{i=2}^{n-1}K_i} \quad (5)$$

It can be seen from (5) that the ability of n^{th} order noise shaping will degrade by the denominator of (5) in an n^{th} order electromechanical ΣΔM. It is also worth to be noted from (5) that for a given sensing element, the in-band noise power degrades with increasing the product of the loop coefficients $\prod_{i=1}^{n-1} K_i$, but there is an upper limit on the product due to the stability of the ΣΔM.

2.3 Stability and Optimal Coefficients

Similar to the design of higher-order sigma-delta A/D converters [6], there is no precise analytic approach to ascertain the stability of a modulator without resorting to simulation due to a highly nonlinear element, the quantizer. For the time being, the most reliable method for verifying stability of high-order loop is simulation [7]. Any gain introduced before the quantizer does not affect the output, the coefficient of the last integrator is also irrelevant for simulation purpose. In modern VLSI fabrication, the mismatches in switch-capacitor amplifiers may be between 0.1% and 1%, however, the tolerance in fabrication of MEMS will be large (3%) depending on technology, device size and circuit topology. In order to guarantee the optimal SQNR with robust stability, an empirical approach is suggested to find an optimal product of loop coefficients after exhaustive simulations. We use a normalized sinusoidal input with amplitude of -10dB to avoid overloading, and the SQNR and gain of NTF are calculated by Hann window 128*1024 bin FFT. The constraints set for simulations are: ①SQNR loss less than 3dB due to 3% variation of the coefficients, ②maximization of SQNR, ③ stability criteria (|NTF|'s gain <1.5) and ④ coefficients should be between 0.001 and 1 according to normalized input-output signal amplitude of 1 volt, which are practical in circuit implementation. The approach to find stable and optimal coefficients can be illustrated in three steps:

1. Simulations start from a 2nd order electro-mechanical ΣΔM to identify the stability of the loop. If the loop is unstable, as for an underdamped sensing element, add a lead filter to stabilize it.

2. Next is to find the stable and optimal coefficient $K_{po}*K_1$ in a 3rd-order electro-mechanical ΣΔM: sweeping the coefficient across a range of values 0.001 to 1 in steps of 0.05, meanwhile the data in simulation process are saved to a file according to the $K_{po}*K_1$, SQNR and gain of NTF.

3. Use a similar strategy for 4th-order and 5th-order loops to get $K_{po}*K_1$, K_2 and K_3. After applying the constraints to these files, post-processing can find a relatively maximum of the SQNR.

To illustrate this procedure, Figure 2 shows the SQNR distribution of a 4th order electro-mechanical ΣΔM as function of the coefficients K_1, K_2. It was found that when $K_1 \approx 0.2$, $K_2 \approx 0.5$, the SQNR=82dB is optimized using an OSR=64.

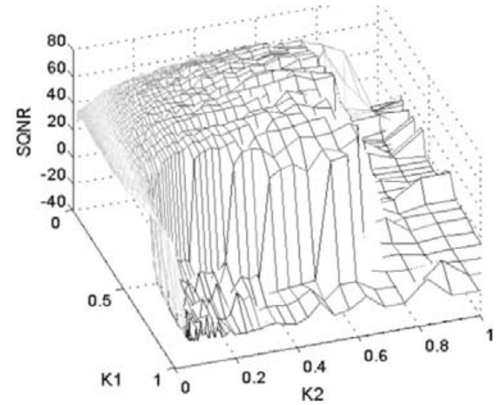

Fig. 2: SQNR distribution of a 4th order electro-mech. ΣΔM.

A stable and optimal 3rd-order electro-mechanical ΣΔM is fundamental for designing a higher-order loop. The constraint that fabrication uncertainty 3% leading to SQNR variation less than 3dB is the most important stability criterion in practical implementation. Based on extensive simulations of 3rd-order to 5th-order loops and careful post processing, it was found that the product of the electronic pick-off gain and integrator coefficients is found to be nearly a constant according to order level. The SQNR of the high-order electro-mechanical ΣΔM is maximized with stable constraints when the integrator coefficients are approximately:

$$K_{po} \cdot \prod_{i=1}^{n-2} K_i \approx K_{po} \cdot (\frac{1}{2}, \ \frac{1}{10}, \ \frac{1}{50}), \ (n = 3,4,5) \tag{7}$$

The coefficients of the electronic integrators are chosen by borrowing the expertise from a mature high-order ΣΔM A/D converter [6]:

$$\begin{aligned} &K_1 \approx 0.5, && (n = 3) \\ &K_1 \approx 0.2, \ K_2 \approx 0.5, && (n = 4) \\ &K_1 \approx 0.2, K_2 \approx 0.2, K_3 \approx 0.5, && (n = 5) \end{aligned} \tag{8}$$

3 SIMULATION RESULTS

The sensor used to demonstrate the approach is a bulk micromachined overdamped accelerometer (mass m = 1.5x 10^{-6} kg, damping coefficient b=0.0867 Nm/s, and spring stiffness k=98.1 N/m) provided by QinetiQ. The input is sinusoidal and signal bandwidth 1kHz.

3.1 3rd Order Electro-Mechanical ΣΔM

The simulation model of a 3rd-order electro-mechanical ΣΔM is shown in Figure 3. The maximum SQNR obtained from this architecture is 88dB at OSR=256. Figure 4 shows the power spectrum density (PSD) of noise shaping in this 3rd order electro-mechanical ΣΔM.

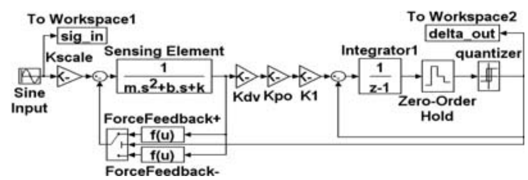

Fig. 3: Model of a 3rd-order electro-mechanical ΣΔM.

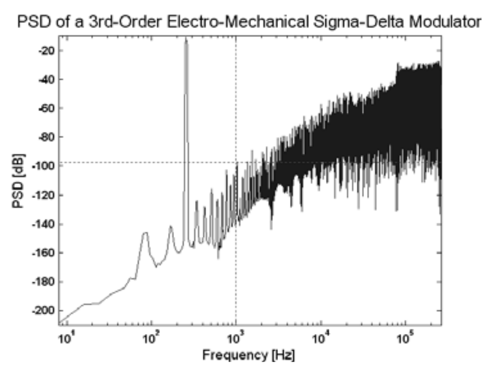

Fig. 4: Noise shaping: a 3rd-order electro-mechanical ΣΔM.

3.2 4th Order Electro-Mechanical ΣΔM

The simulation model of a 4th-order electro-mechanical ΣΔM is shown in Figure 5. The maximum SQNR obtained from this architecture is 105dB at OSR=256. Figure 6 shows the PSD of noise shaping in this 4th-order electro-mechanical ΣΔM.

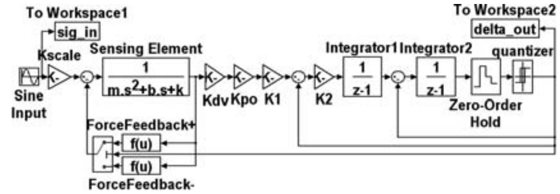

Fig. 5: Model of a 4th-order electro-mechanical ΣΔM.

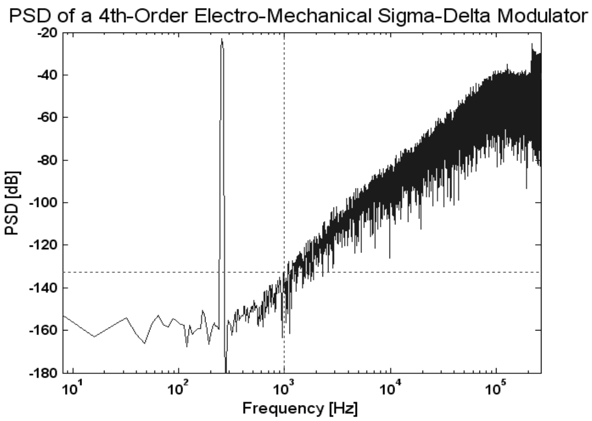

Fig. 6: Noise shaping: a 4th-order electro-mechanical ΣΔM.

3.3 5th Order Electro-Mechanical ΣΔM

The simulation model of a 5th-order electro-mechanical ΣΔM is shown in Figure 7. The maximum SQNR obtained from this architecture is 122dB at OSR=256. Figure 8 is the comparison of the PSD of noise shaping between a 5th-order and a 2nd-order electro-mechanical ΣΔM at OSR=256. The SQNR difference between the two loops is about 60dB.

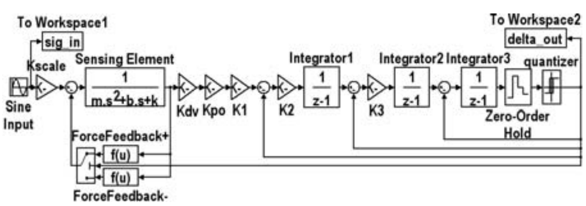

Fig. 7: Model of a 5th-order electro-mechanical ΣΔM.

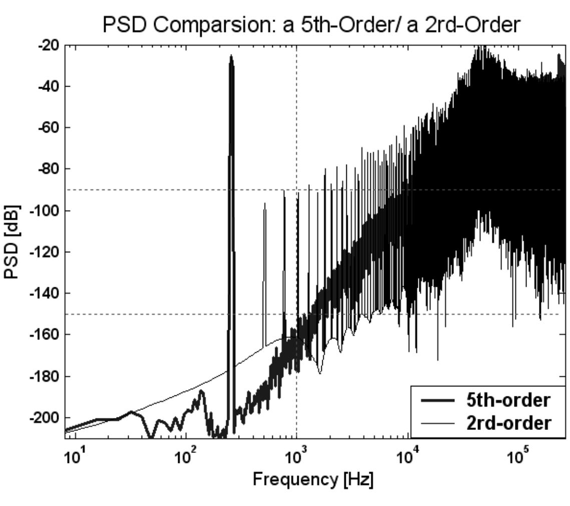

Fig. 8: Noise shaping comparison: a 5th-order /a 2nd-order electro-mechanical ΣΔM.

The coefficients and performance for higher-order electro-mechanical ΣΔM are summarized in Table 1. DR (dynamic range) refers to the maximum SQNR achievable for sinusoidal input signal, and OL the maximum input signal amplitude for which the SNR degrades less than 6dB from the maximum SQNR.

Order	**3**		**4**		**5**	
Integrator Coefficients	K1=0.5		K1=0.2 K2=0.5		K1=0.2 K2=0.2 K3=0.5	
OSR	***DR***	***OL***	***DR***	***OL***	***DR***	***OL***
64	69dB	0.95	79dB	0.85	88dB	0.75
128	79dB	0.9	91dB	0.80	105dB	0.65
256	88dB	0.85	105dB	0.75	122dB	0.55

Table 1: Coefficients and performance in a higher-order electro-mechanical ΣΔM.

4 CONCLUSIONS

Using the same OSR, a 5th-order electro-mechanical ΣΔM can improve the SQNR by 60dB compared with a 2nd-order electro-mechanical ΣΔM at OSR=256. To achieve the same performance as 2nd-order electro-mechanical ΣΔM, higher-order ΣΔM can use lower oversampling frequency to reduce electronic thermal noise and power dissipation. The present work involves simulation of the optimal higher-order single loop electro-mechanical ΣΔM. To validate these architecture, design and fabrication of a high performance inertial sensor and associated interface ASIC are currently underway. Other important issues in a higher-order single loop electro-mechanical ΣΔM such as force feedback linearization, various noises analysis and fabrication mismatch on performance degradation by Monte-Caro analysis will be published in the near future.

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