Transfer of Atoms Between two Condensates by Raman Transitions

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ABSTRACT

Law and Bigelow have proposed a scheme for amplifying an atomic beam by using the interaction of the beam with a confined alkali-atom condensate. In this scheme the condensate acts as a source for increasing the intensity of the atomic beam. The amplification is achieved by the coupling of the atomic beam with the condensate through an optical laser, which induces Raman transitions. We have shown earlier that a reverse process, an opposite of the Law and Bigelow scheme, is also possible in which the atomic beam can be used as a source for amplifying the number of atoms in the condensate. By combining the two schemes, it is shown in this paper that number of atoms in the two physically separated condensates can be coupled. The coupling is used to exchange the atoms between the two condensates. Thus the number of atoms in one of the condensates can be depleted while the number in the other is increased. The parameters determining the exchange depend of the size of the condensates, the Raman interaction, the confining potentials and the intensity of the atomic beam. Numerical value for the exchange parameter is estimated.

Keywords: condensate, atomic-laser, Raman-transitions, Bose-Einstein condensation. Laser-cooling.

1 INTRODUCTION

There has been extensive research activity to study and manipulate the properties of condensates following the experimental observation of Bose-Einstein condensation in dilute alkali atoms. That the condensation would take place at sufficiently low temperatures was predicted a long time ago, but its observation was a major experimental achievement [1,2]. The low temperatures needed to observe the condensation was achieved by laser cooling. The interest in the subject arises because of the realization that a condensate is a coherent collection of atoms, often referred to as a new form of matter. Andrew, Townsend, Miesner, Durfee, Kurn and Ketterle [3] were the first to show that two freely expanding condensates when allowed to overlap give rise to high contrast interference fringes. The observation of the fringes confirmed the coherent nature of the condensate. It was also postulated that the condensate would form the basis for the construction of an atomic laser. It is anticipated that the fabrication of the atomic laser will not only advance our understanding of the basic concepts in Quantum Mechanics but also lead to technological developments based on the use of the coherent properties atomic lasers.

Developments in the field of condensates have been rapid over the last decade. The primary emphasis initially has been on the methods to form sufficiently large condensates. It is generally presumed that the fabrication of a large condensate would be necessary for the eventual production of an atomic laser. Holland, Burnet, Gardiner, Cirac and Zoller [4] have proposed a method for the fabrication of a condensate by using thermal atoms as the source. Using Raman transitions, May, Hope and Savage [5] on the other hand have shown how to produce a continuous beam of atoms by utilizing the condensate as the source.

Law and Bigelow [6] have recently proposed an ingenious method for the production of an atomic beam using the condensate as the lasing medium. The condensate is trapped in an optical cavity. Each atom of the condensate is in the same state denoted by $|\phi_a>$. Atoms in the external atomic beam have a different internal state denoted by $|\phi_b>$. We assume that the beam travels through the condensate without interacting with the trap potential. The condensate and the beam are Raman coupled through an input laser with frequency ω_L and an optical cavity field of frequency ω_C. The external laser field is described by a classical field expressed in terms of a plane wave with a constant amplitude. The condensate is considered to be sufficiently dilute so that the effects of the collision between the beam and the condensate are infrequent and these effects are ignored. The atoms of the condensate absorb a quantum of the laser field and concurrently emit a quantum of the cavity field. The interaction between the two optical fields is so adjusted that the atom from the condensate is transferred to the atomic beam. The scheme by Law and Bigelow [6] is intended to provide a method for amplifying an external atomic beam. A scheme which is converse of the Law and Bigelow [6] scheme is developed by Paranjape, Panat and Lawande [7]. In this scheme, the atoms from the beam absorb a laser quantum and emit an cavity photon such that the atoms from the beam are transferred to the condensate. In effect, in the alternate scheme the number of atoms in the beam are depleted while the number of atoms in the condensate are amplified. A new scheme is proposed in the present paper, which combines the original scheme of Law and Bigelow and the scheme which is the converse of the Law and Bigelow scheme. In the combined scheme an atomic beam is allowed to interact with two condensates. The interactions are arranged in such a way that they occur in a sequential manner. The interaction with the first condensate gives rise to the amplification of the beam and the depletion of the atoms in the condensate. The beam then interacts with the second condensate such that the atoms from the beam are transferred to the condensate resulting in increase in the number of atoms of the second condensate. The two schemes in combination has the effect of depleting the first condensate and amplifying the second.

2 METHOD

We denote the second quantized annihilation field operators $\phi_{a1}(\vec{x},t)$ for the atoms in the first Bose-Einstein condensate. $\phi_{a2}(\vec{x},t)$ stands for the annihilation operator for the atoms in the second condensate. $\phi_b(\vec{x},t)$ is the annihilation operator for the atoms in the beam. The Hamiltonian H for the system is given by

$$H = H_{BEC1} + H_{BEC2} + \hbar\omega_{c1}c_1^\dagger c_1 + \hbar\omega_{c2}c_2^\dagger c_2 + \int \phi_b^\dagger(\vec{x},t)\left[-\frac{\hbar^2\nabla^2}{2M}\right]\phi_b(\vec{x},t)d^3x + H_{\text{int}}, \quad (1)$$

where

$$H_{\text{int}} = \hbar g_1 e^{-i\omega_{L1}t}c_1^\dagger \int \phi_b^\dagger(\vec{x},t)\phi_{a1}(\vec{x},t)u_1(\vec{x})e^{i\vec{k}_{L1}.\vec{x}}d\vec{x} + \hbar g_2 e^{-i\omega_{L2}t}c_2^\dagger \int \phi_b(\vec{x},t)\phi_{a2}^\dagger(\vec{x},t)u_2(x)e^{i\vec{k}_{L2}.\vec{x}}d\vec{x} + H.C.$$

The first two terms in eq. (1) are respectively the Hamiltonians for the Bose-Eienstein condensates trapped in cavities one and two. c_1 and $c_1^\dagger$ are respectively the destruction and creation operators for the photons of the first cavity and similarly c_2 and $c_2^\dagger$ are the corresponding operators for the second cavity. ω_{c1} and ω_{c2} are the fundamental frequencies of the cavities number one and two respectively. The fifth term is the kinetic energy operator of the atoms in the beam and M is the atomic mass. ω_{L1} and ω_{L2} are the two laser frequencies directed at condensates one and two respectively. g_1 and g_2 are respectively the strengths of the coupling constants between the beam and the first and the second condensate.

The field operators satisfy the usual commutation relations:

$$[\phi_\mu(\vec{x},t), \phi_\nu^\dagger(\vec{x}',t)] = \delta_{\nu\mu}\delta(\vec{x}-\vec{x}'). \quad (2)$$

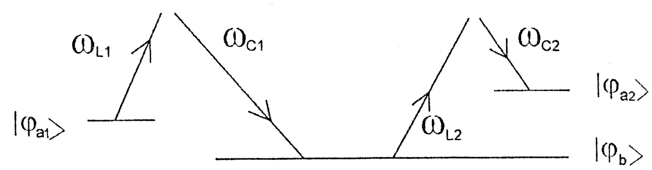

Figure 1. Schematic diagram showing the interaction term in the Hamiltonian.

The equation of motion for the field operators can be written by using the eqs. (1) and (2) and the Heisenberg's equation of motion for the field operator $\hat{O}$:

$$i\hbar\frac{\partial\hat{O}}{\partial t} = \left[\hat{O},\hat{H}\right]. \quad (3)$$

Thus:

$$i\hbar\frac{\partial\phi_b(\vec{x},t)}{\partial t} = -(\hbar^2/2M)\nabla^2\phi_b(\vec{x},t) + \hbar g_1 e^{-i\omega_{L1}t}c_1^\dagger\phi_{a1}(\vec{x},t)u_1(x)e^{i\vec{k}_{L1}.x} + \hbar g_2 e^{i\omega_{L2}t}c_2\phi_{a2}(\vec{x},t)u_2(x)e^{-i\vec{k}_{L2}.\vec{x}} \quad (4)$$

The equation of motion for c and $c^\dagger$ for the photon operators in cavities one and two can be written as

$$i\hbar\frac{\partial c_j}{\partial t} = \hbar(\omega_{cj} - i\kappa_j)c_j + \hbar g_j e^{-i\omega_{L1}t}\int \phi_b^\dagger(\vec{x},t)\phi_{a1}(\vec{x},t)u_1(x)e^{i\vec{k}_{L1}.\vec{x}}d\vec{x} \quad (5)$$

A similar equation can be written for the creation operator. In eq. (5) the suffix j stands for one and two representing the two cavities.

Assuming that the two condensates are sufficiently large in size so that the change in the number of the atoms due to Raman transitions is negligible compared to the total number of atoms in each of the condensates. In these situations it is possible to write

$$\phi_{a1} = \sqrt{N_1}\,\chi_1(\vec{x})e^{-i\mu_1 t} \quad (6)$$

and

$$\phi_{a2} = \sqrt{N_2}\,\chi_2(\vec{x})e^{-i\mu_2 t} \quad (7)$$

where N_1 and N_2 are respectively the number of atoms in the condensates one and two, $\chi(x)$ are the wave functions of the condensates determined by the mean value theory and $\hbar\mu$'s are the mean-field energies of the two condensates.

It is possible to rewrite the equations of motion from the field operators by removing a substantial part of its time dependence by redefining the field operators according as:

$$\phi_b(\vec{x},t) = \psi_b(\vec{x},t)e^{-i\varepsilon t} \quad (8)$$

$$c_j = \mathbb{C}_j e^{-i\omega_{cj}t} \quad (9)$$

Substituting the time dependence as defined above, it is possible to rewrite the field operators for the cavity photons according as:

$$i\hbar\frac{\partial \mathbb{C}_j}{\partial t} = -i\hbar\kappa_j\mathbb{C}_j + \hbar g_j\int\psi_b^{\dagger}(\vec{x},t)\sqrt{N_j}\,\chi_j(x)u_j(x)e^{i\vec{k}_{Lj}.\vec{x}}d\vec{x} \quad (10)$$

In eq. (10), the first term on the right hand side represents the leakage term and denotes the rate of loss of photons by the adjustable instrumental decay. In the present paper it is to our advantage to keep the decay term sufficiently large so that the number of photons in the cavity is close to zero. By keeping the cavity photons small we keep the processes in which the photons are emitted but the reverse processes are suppressed.

In the bad cavity limit and using the adiabatic approximation we get

$$\mathbb{C}_1 = -\frac{-ig_1\sqrt{N_1}}{\kappa_1}\int\psi_b^{\dagger}(\vec{x},t)\eta_1(x)d\vec{x}, \quad (11)$$

$$\mathbb{C}_2 = \frac{-ig_2\sqrt{N_2}}{\kappa_2}\int\psi_b(\vec{x},t)\eta_2(\vec{x})d\vec{x} \quad (12)$$

where

$$\eta_j(\vec{x}) = \chi_j(\vec{x})u_j(\vec{x})e^{i\vec{k}_{Lj}.\vec{x}} \quad (13)$$

In writing equation (10) we have used the energy conservation so that

$$\mu_1 + \omega_{L1} = \omega_{c1} + \varepsilon \quad (14)$$

and

$$\omega_{L2} + \varepsilon = \omega_{c2} + \mu_2 \quad (15)$$

We now combine the results obtained so far to express the time dependence of the field operator for the atomic beam according as

$$i\hbar\frac{\partial\psi_b(\vec{x},t)}{\partial t} = -\left[\frac{\hbar^2\nabla^2}{2M} + \hbar\varepsilon\right]\psi_b(\vec{x},t) + \frac{i\hbar|g_1|^2 N_1}{\kappa_1}\eta_1(\vec{x})\int\psi_b(\vec{x}',t)\eta_1^*(\vec{x}')d\vec{x}' - \frac{i\hbar|g_2|^2 N_2}{\kappa_2}\eta_2^*(\vec{x})\int\psi_b(\vec{x}',t)\eta_2(\vec{x}')d\vec{x}'. \quad (16)$$

The number of atoms in the beam can be written as

$$n_b(t) = \int d\vec{x} < \psi_b^{\dagger}(\vec{x},t)\psi_b(\vec{x},t) >. \quad (17)$$

Using the standard result of quantum mechanics, we express

$$\psi_b(\vec{x},t) = \int U(\vec{x},t;\vec{x}',0)\psi_b(\vec{x}',0)d\vec{x}' \quad (18)$$

and

$$\psi_b(\vec{x},0) = \hat{b}w(\vec{x},0) \quad (19)$$

In (18), $U(\vec{x},t;\vec{x}',0)$ is the standard propagator, $\hat{b}$ and w are respectively the creation operator for the atom in the beam and w is the wave packet of the beam-atoms at time t=0.

The rate of change of the atoms in the beam can now be expressed as

$$\frac{dn_b(t)}{dt} = \frac{2|g_1|^2 N_1}{\kappa_1}n(0)\left|\int\eta_1(\vec{x},t)w(x,t)d\vec{x}\right|^2$$

$$-\frac{2|g_2|^2 N_2}{\kappa_2} n(0)\left|\int \eta_2(\vec{x}) w(\vec{x},t) d\vec{x}\right|^2 \quad (20)$$

where

$$n_2(0) = <\hat{b}^\dagger \hat{b}>$$

is the number of the atoms in the beam at time t=0 i.e. before the beam interacts with the condensate.

The total change in the number of the atoms in the beam as it completes its passage through the condensate, is obtained by integration of eq.(20). We may express the number of atoms in the beam as

$$n_b(\infty) = n_b(0)[1 + G_1 - G_2], \quad (21)$$

where G_1 and G_2 are obtained by integration with respect to time the first and the second term in equation (20) respectatively.

3 NUMERICAL ESTIMATES

Clearly the first term in eq.(20) gives the rate of increase in the number of atoms in the wave packet as it interacts with the condensate in the first trap and the second term is the rate of decrease in the number of atoms in the wave packet as it interacts with the condensate in the second trap. Using the particle conservation, the first term gives the depletion rate of the atoms from the first condensate and the second term is the increase in the number of atoms in the second condensate.

We may assume the Gaussian shapes for the two condensates such that the first condensate is centered at the origin while the second condensate is centered at x_2. Thus η in x-direction is given by

$$\eta_1(x) = (\pi)^{-1/4} \sigma_1^{-1/2} e^{-x^2/2\sigma_1^2 + k_{L1}.x} \sin(k_{c1}x) \quad (22)$$

and

$$\eta_2(x) = (\pi)^{-1/4} \sigma_2^{-1/2} e^{-(x-x_2)^2/2\sigma_2^2} \sin(k_{c2}x) \quad (23)$$

For simplicity we assume that the two condensates have the same width of 10 μm and that the atomic wave packet is a Gaussian with width given by 10 μm. The wave packet initially is at a distance of 30 μm. We assume that the values of the coupling constants for the two condensates are $g_1 = 1.0\,kHz$ for the first condensate $g_2 = 1.0\,kHz$ for the second. The decay constant for the first and the second trap is assumed to be the same and given by $\kappa_1 = \kappa_2 = 100\,kHz$. Straightforward numerical integrations give the same value for G_1 and G_2 as 0.5. Thus the decrease in the number of atoms in the first condensate is $n_b(0) \times (0.5)$ while the corresponding increase of atoms in the second condensate is 0.5 $n(0)$.

ACKNOWLEDGEMENTS

(VVP) is grateful to NSERC of Canada for their partial support of this research. The authors wishes to thank the University of Pune for supporting the research collaboration.

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