Quantum-Classical Transition Induced by Electrical Measurement

D. Mozyrsky and I. Martin

Theoretical Division, Los Alamos National Laboratory
Los Alamos, NM 87545, USA, mozyrsky@cnls.lanl.gov, ivar@viking.lanl.gov

ABSTRACT

A model of an electrical tunnel junction coupled to a mechanical system (oscillator) is studied to simulate the dephasing effect of measurement on a quantum system. The problem is solved at zero temperature under conditions of strong non-equilibrium in the measurement apparatus. For linear coupling between the oscillator and tunneling electrons, it is found that the oscillator dynamics becomes damped, with the effective temperature determined by the voltage drop across the junction. It is demonstrated that both the quantum heating and the quantum damping of the oscillator manifest themselves in the current-voltage characteristic of the point contact.

Keywords: Quantum-Classical Transition, Quantum Dissipative Systems, Non-Equilibrium Dynamics.

1 INTRODUCTION

There is a dramatic difference in the observed behaviors of microscopic particles and of macroscopic objects. The everyday-scale objects obey the rules of classical Newtonian mechanics, while microscopic particles command the use of quantum physics for their description. The effects of quantum coherence are almost never observed at the macroscale. The only known exceptions are realized when the macroscopic quantum state is particularly robust against external perturbations, as is the case for superconductors and quantum Hall liquids. Hence, it appears natural to assume that it is the coupling to the external world, or *environment*, that leads to *decoherence* and consequently to a transition from quantum to classical behavior. This process was explored in detail in numerous works. It has been shown that within a phenomenological model of environment, at sufficiently high temperatures, a quantum mechanical system becomes effectively classical [1], [2]. The environment provides both the decoherence and the dissipation needed for the quantum-classical transition.

Another important distinction between the classical and quantum systems is in their response to measurement. Measurement of a classical system in principle can have no effect on the state of the system; on the other hand, in the quantum regime, the measurement itself is a source of decoherence that inevitably changes the state of the system [3]. The main difference between the measurement process and the environment induced dephasing is that measurement is an intrinsically non-equilibrium process. In this work we demonstrate that despite the apparent differences, the measurement can also induce a quantum-classical transition. Recently, Gurvitz *et al.* [4] and Korotkov *et al.* [5] have shown that electrical measurement leads to dephasing of a finite state system that is being measured. The systems that they have studied did not have however a classical analogue. Here, we extend their approach to the problem of the measurement of a mechanical system [6]. Specifically, we consider a *quantum* oscillator coupled to an electrical point contact in the tunneling regime. We solve this combined problem in the non-equilibrium limit of large voltage across the point contact. From the general solution, we separately extract the dynamics of the oscillator and of the current through the contact. For the oscillator, our main findings are that (1) coupling to the tunneling electrons leads to dissipation with damping coefficient independent of the applied voltage, (2) current shot noise generates fluctuations and decoherence, and (3) behavior of the mechanical system becomes effectively classical with the temperature equal to one half of the voltage drop across point contact. For the point contact, we determine the stationary non-linear current-voltage (I-V) characteristic, which implicitly measures the effects of the quantum heating and quantum dissipation in the oscillator. Thereby we explicitly demonstrate how measurement can induce a quantum-classical transition.

2 MODEL

The setup we consider is presented schematically in Fig. 1. Our system is described by the Hamiltonian

$$\hat{H} = \sum_l \epsilon_l a_l^\dagger a_l \;+\; \sum_r \epsilon_r a_r^\dagger a_r + \hat{H}_0(x) + \sum_{l,r} \hat{\Omega}(x)\left(a_l^\dagger a_r + a_r^\dagger a_l\right). \quad (1)$$

Here $a_l^\dagger$ ($a_r^\dagger$) creates an electron at the energy level ϵ_l (ϵ_r) in the left (right) reservoir. $\hat{H}_0(x) = -(1/2m)\nabla_x^2 + U(x)$

is the unperturbed Hamiltonian of an oscillator of mass m in classical confining potential $U(x)$, where x is the oscillator coordinate. In what follows we set both electron charge e and Planck constant $\hbar$ to unity unless stated otherwise. Generalization to multi-dimensional case when x is a vector is straightforward. The last term in Eq. (1) describes tunneling of electrons between reservoirs, modulated by the oscillator position x. For simplicity we assume that the transition matrix element $\hat{\Omega}(x)$ is independent of single electron states l and r in the reservoirs. The latter dependence can be included without qualitative modification of our principal results. Moreover, $\hat{\Omega}(x)$ can be a general hermitian operator of the particle's coordinates, i.e. it can be a function of momenta. Finally, there is an electrical bias, V, applied across the tunnel junction, so that the chemical potentials in the reservoirs are related as $\mu_L - \mu_R = eV > 0$.

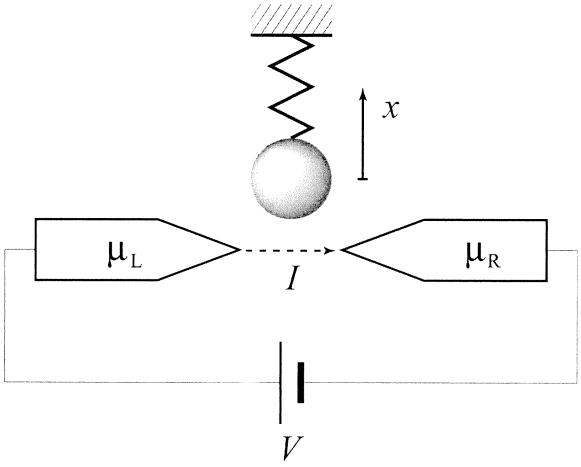

Figure 1: Model setup. An oscillator with a confining potential $U(x)$ (shown as spring) is coupled to an electrical point contact. The motion of the oscillator modulates the electron tunneling between the reservoirs.

3 DERIVATION OF MASTER EQUATIONS

To determine the time evolution of the system described by the above Hamiltonian, we use the many-body Shrödinger equation approach developed in Refs. [4] for solution of highly-nonequilibrium quantum transport problems. The wavefunction of the full system is

$$|\mathbf{\Psi}(t)\rangle = \Big\{ b_0(x,t) + \sum_{l,r} b_{lr}(x,t) a_r^\dagger a_l + \sum_{l,l'r,r'} b_{ll'rr'}(x,t) a_r^\dagger a_{r'}^\dagger a_l a_{l'} + \ \ldots \ \Big\} |\mathbf{0}\rangle \,, \qquad (2)$$

where the initial state $|\mathbf{0}\rangle$ corresponds to fully occupied Fermi seas in the reservoirs, with no electrons above the respective chemical potentials. This wavefunction is a superposition of all possible electron-hole combinations that can be generated by the Hamiltonian H; note that H conserves the total number of electrons in the reservoirs. The first term in $\mathbf{\Psi}$ is the time-dependent part of the wavefunction that corresponds to unchanged occupancy of Fermi seas; the second term describes a state in which a hole is created in the left reservoir and an extra electron occupies the right reservoir, etc. In our representation, the amplitudes $b(x,t)$ explicitly depend on the coordinate x of the oscillator. Substituting wavefunction (2) into the Scrödinger equation with Hamiltonian (1), $i|\dot{\mathbf{\Psi}}\rangle = H|\mathbf{\Psi}\rangle$, and performing imaginary Laplace transform on $\mathbf{\Psi}$, $\tilde{b}(\omega) = \int_0^\infty dt e^{i\omega t} b(t)$, we obtain an infinite set of equations for the amplitudes $\tilde{b}$:

$$\Big(\omega - \hat{H}_0(x)\Big)\, \tilde{b}_0(x,\omega) - \sum_{l,r} \hat{\Omega}(x) \tilde{b}_{lr}(x,\omega) = i\psi_0(x) \,, \quad (3)$$

$$\Big(\omega - \hat{H}_0(x) + \epsilon_l - \epsilon_r\Big)\, \tilde{b}_{lr}(x,\omega) - \hat{\Omega}(x)\tilde{b}_0(x,\omega) - \sum_{l',r'} \hat{\Omega}(x) \tilde{b}_{ll'rr'}(x,\omega) = 0 \,, \quad (4)$$

$$\ldots \ .$$

Here, $\psi_0(x)$ is the initial state of the particle. In order to proceed, we solve the n^{th} equation for the n^{th} amplitude $\tilde{b}(\omega, x)$ in terms of the $(n-1)^{th}$ and $(n+1)^{th}$ amplitudes and substitute it into the $(n-1)^{th}$ equation. Then, as it was shown in Ref. [4], the contribution of the $(n+1)^{th}$ amplitude to the $(n-1)^{th}$ equation is negligible. Not only it is of higher order in the coupling constant $\hat{\Omega}(x)$, but it also vanishes in the limit of large bias. As an example, let us solve Eq. (4) for $\tilde{b}_{lr}(\omega, x)$ and substitute the resulting expression into Eq. (3). The sum in Eq. (3) becomes proportional to $\tilde{b}_0$,

$$\sum_{l,r,n} \int dx' \frac{\hat{\Omega}(x)\hat{\Omega}(x')\phi_n(x)\phi_n^*(x')}{\omega - E_n + \epsilon_l - \epsilon_r} \tilde{b}_0(\omega, x') \ , \qquad (5)$$

where $\phi_n(x)$ and E_n are eigenfunctions and eigenvalues of $\hat{H}_0(x)$, $\hat{H}_0\phi_n(x) = E_n\phi_n(x)$. Replacing summations over l and r by integrals over the corresponding densities of states, ρ_L and ρ_R (here assumed constant), $\sum_{l,r}(\omega - E_n + \epsilon_l - \epsilon_r)^{-1} = -i\pi\rho_L\rho_R(V + \omega - E_n)\Theta(V + \omega - E_n)$. The real part of the sum only weakly renormalizes the self energy of the oscillator. The step-function Θ can be dropped provided the typical energy spacing for $\hat{H}_0$ is much smaller than V and the particle's dynamics involves only relatively low-lying energy levels, such that condition $V > E_n$ is satisfied. The latter assumption is consistent with the final results of our calculation and thus $\Theta(V + \omega - E_n)$ is omitted. Then, using the completeness relation for $\phi_n(x)$, the above sum yields $-i\pi\rho_L\rho_R[\hat{\Omega}^2(x)(V + \omega) - \hat{\Omega}(x)\hat{H}_0(x)\hat{\Omega}(x)]\tilde{b}_0(\omega, x)$. Applying this procedure to all equations in the chain (3,4) and neglecting terms of order $O(\hat{\Omega}^4)$ and higher, we ob-

tain a set of simplified equations,

$$\left(\omega - \hat{H}_0(x)\right)\tilde{b}_0(x)$$
$$-i\frac{\eta}{2}\left\{\hat{\Omega}\left[\hat{H}_0,\hat{\Omega}\right](x) - V\hat{\Omega}^2(x)\right\}\tilde{b}_0(x) = \Delta(x)\,, \qquad (6)$$
$$\left(\omega - \hat{H}_0(x) + \epsilon_l - \epsilon_r\right)\tilde{b}_{lr}(x) - \hat{\Omega}(x)\tilde{b}_0(x)$$
$$-i\frac{\eta}{2}\left\{\hat{\Omega}\left[\hat{H}_0,\hat{\Omega}\right](x) - V\hat{\Omega}^2(x)\right\}\tilde{b}_{lr}(x) = 0\,, \qquad (7)$$
$$\ldots\,,$$

where square brackets denote commutator, $\hat{\Omega}[\hat{H}_0,\hat{\Omega}](x) \equiv \hat{\Omega}(x)[\hat{H}_0(x),\hat{\Omega}(x)]$, $\eta = 2\pi\rho_L\rho_R$, and $\Delta(x) = i\psi_0(x)(1 - i\pi\rho_L\rho_R\hat{\Omega}^2(x))$. Eqs. (6,7) allow us to derive the quantum rate equations for our system. We introduce the density matrix of the system defined as

$$\sigma^0(x,x',t) = b_0(x,t)b_0^*(x',t)\,,$$
$$\sigma^1(x,x',t) = \sum_{l,r} b_{lr}(x,t)b_{lr}^*(x',t)\,,$$
$$\ldots\,.$$

Each of these objects is the density matrix of the oscillator conditioned by the number of electrons (the superscript in σ's) that have arrived in the right reservoir. By following steps of Refs. [4], [6], we obtain a set of equations for density matrices σ^n:

$$\dot{\sigma}^n = -i\left[\hat{H}_0,\sigma^n\right] + \frac{\eta}{2}\Big\{\hat{\Omega}\left[\hat{H}_0,\hat{\Omega}\right](x) + \hat{\Omega}\left[\hat{H}_0,\hat{\Omega}\right](x')$$
$$-V\left(\hat{\Omega}^2(x) + \hat{\Omega}^2(x')\right)\Big\}\sigma^n - \frac{\eta}{2}\Big\{\hat{\Omega}(x)\left[\hat{H}_0,\hat{\Omega}\right](x')$$
$$+\hat{\Omega}(x')\left[\hat{H}_0,\hat{\Omega}\right](x) - 2V\hat{\Omega}(x)\hat{\Omega}(x')\Big\}\sigma^{n-1}\,.$$

From the above equations of motion for the full density matrix, one can obtain a closed form equation for the density matrix of the oscillator alone, $\sigma(x,x',t)$. Summing the above equations over n we find that the equation for σ can be written in an invariant form,

$$\dot{\sigma} = -i\left[\hat{H}_0,\sigma\right] + \frac{\eta}{2}\left[\hat{\Omega},\left\{\hat{\Lambda},\sigma\right\}\right] - \frac{V\eta}{2}\left[\hat{\Omega},\left[\hat{\Omega},\sigma\right]\right]\,, \quad (8)$$

where $\hat{\Lambda} = [\hat{H}_0,\hat{\Omega}]$ and curly brackets denote anticommutator. Similarly, one can determine the time dependence of the average current in terms of the oscillator density matrix. The current is defined as $\langle I(t)\rangle = \langle\dot{N}(t)\rangle$, where $\langle N(t)\rangle = \sum_n n\int dx\sigma^n(x,x,t)$ is the expectation value of the number of electrons that have tunnelled into the right contact by time t. Using the above equation for $\dot{\sigma}^n$, we find the current

$$\langle I(t)\rangle = V\eta\mathrm{Tr}\left(\hat{\Omega}^2\sigma(t)\right) - \frac{\eta}{2}\mathrm{Tr}\left(\left[\hat{\Omega},\hat{\Lambda}\right]\sigma(t)\right)\,, \qquad (9)$$

in terms of the oscillator density matrix. Eqs. (8) and (9) constitute the principal results of this work. They describe the evolution of a system modified by its interaction with non-equilibrium environment, as well as influence of the oscillator on the current between reservoirs.

4 RESULTS AND DISCUSSION

It is easy to see that for linear coupling between the two subsystems, $\Omega(x) = \Omega_0 + Cx$, we recover the Caldeira and Leggett equation (CL) [2] for the density matrix of the oscillator. Indeed, for the linear coupling, the second term in the RHS of Eq. (8) produces the dissipative term $(\eta C^2/2m)(x-x')(\partial_{x'}-\partial_x)\sigma$, while the last term becomes $(\eta VC^2/2)(x-x')^2\sigma$. It is responsible for fluctuation/decoherence induced by the tunnel current.

Both the CL equation and our Eq. (8), describe high temperature dynamics of a mechanical system interacting with a heat bath. There is a substantial difference, however. Unlike the CL work where the heat reservoir is in equilibrium at high temperature T, our calculation is at T = 0. The effective temperature that arises in Eq. (8) is not the temperature of the reservoirs, but rather is a result of the *non-equilibrium* fluctuations that arise in the course of the evolution of the full system described by Hamiltonian (1). In this "classical" limit, the oscillator dynamics specified by the master equation (8), is given by the Langevin equation, $m\ddot{x} + \gamma m\dot{x} + \nabla_x U(x) = f(t)$, where $f(t)$ is white noise satisfying $\langle f(t)f(t')\rangle = 2m\gamma T_{\rm eff}\delta(t-t')$. Comparing Eq. (8) for $\hat{\Omega} = \Omega_0 + Cx$ with Caldeira and Leggett equation [2], we deduce the damping coefficient and the effective temperature of the oscillator:

$$\gamma = \eta\hbar C^2/2m\,, \quad T_{\rm eff} = eV/2.$$

The fluctuations that give rise to the effective temperature can be traced back to the shot noise of the tunnel current.

Due to the self-consistent coupling between the oscillator and the tunnel current, the measurement-induced effective oscillator temperature can be directly extracted from the non-linear part of the I-V characteristic. The current dynamics is given by Eq. (9). The first term on the RHS of Eq. (9) is simply the Landauer formula with the transmission coefficient $\eta\Omega^2(x)$ modulated by the position of the oscillator. This is what one would expect from the Hamiltonian (1) assuming elastic electron tunneling. The second term, however, is non-trivial and accounts for the possible inelasticity of electron tunneling due to the coupling to the oscillator. For linear coupling $\hat{\Omega}(x) = \Omega_0 + Cx$, this term yields $(-\gamma)$. Hence, the expression for the time-dependent current is

$$\langle I(t)\rangle = \frac{e^2}{\hbar}\eta V(\Omega_0^2 + C^2\langle x^2(t)\rangle) - e\gamma. \qquad (10)$$

For a linear oscillator with frequency ω_0, such that $U(x) = (m\omega_0^2/2)x^2$, the stationary I-V characteristic can be determined by solving Eq. (8) for the stationary state. We obtain that in the stationary regime $\langle x^2\rangle = T_{\rm eff}/(m\omega_0^2)$. This generates a quadratic in V term in the I-V characteristic, which is a signature of

the "quantum heating" of the oscillator. The constant negative offset term $(-e\gamma)$ is more subtle and has to be properly interpreted. Indeed, for low bias this term could dominate, which would seem to imply a current flowing in the direction opposite to the applied voltage. This is however only an artifact which signifies the break down of our approach at low biases. To trace the origin of this term we calculate the tunnel current in the low voltage limit at zero temperature from the linear response theory. Assuming that the oscillator is in the ground state, we obtain

$$\langle I\rangle_{\rm L.R.} = \frac{e^2}{\hbar}\eta\Omega_0^2 V + \frac{\eta e C^2}{\hbar}\langle x^2\rangle_0 (eV-\hbar\omega_0)\Theta(eV-\hbar\omega_0),$$

where $\langle x^2\rangle_0$ is the average square of the zero point motion. The Θ-function indicates that at zero temperature, for small bias neither the oscillator is excited by the current, nor the current is affected by the oscillator's zero point motion. For a bias exceeding the oscillator excitation energy, an additional channel in the tunneling opens up. This new channel however requires the excitation of the oscillator, which introduces inelasticity and effectively reduces the applied voltage by $\hbar\omega_0$. Since the negative offset term in the linear response equation is identical to the $(-e\gamma)$ term that appears in Eq. (10), we conclude that both can be attributed to the dissipative effect of the oscillator on the tunnel current.

Given the correspondence between the large-voltage expression and the linear response result, we expect that the expression for current, Eq. (10), remains approximately valid for arbitrary voltage down to $eV = \hbar\omega_0$. Below this threshold voltage, oscillator and point contact become decoupled and the traditional expression for the tunnel current in a point contact applies, assuming that the oscillator is initially in a ground state. With this modification included, the current-voltage equation (10) could be used to fit experimental data in order to extract the measurement-induced dissipation coefficient γ.

Based on the above discussion, it would seem impossible to measure zero point fluctuations of the oscillator. At low voltages, $eV < \hbar\omega_0$, while the oscillator is in the ground state, it is completely decoupled from current. On the other hand, at high voltages the oscillator is no longer in the ground state. We will now demonstrate that it is indeed possible to measure the effect of zero point fluctuations using *time-dependent* measurement of the I-V characteristic. Assuming that at $t = 0$ oscillator was in its ground state and solving the time-dependent density matrix equation (8) for a linear oscillator, we find that the time-dependent oscillator dispersion is

$$\langle x^2(t)\rangle = \langle x^2\rangle_0 Q(t) + \langle x_{cl}^2(t)\rangle,$$

where $\langle x^2\rangle_0$ is the oscillator ground state dispersion and $Q(t) = e^{-\gamma t}(\sin^2(\omega_0 t+\phi_0) + \sin^2\omega_0 t)$, with $\cos\phi_0 = \gamma/2\omega_0$. The term $\langle x_{cl}^2(t)\rangle$ arises due to the "quantum heating." It coincides with the solution of the classical Langevin equation with the damping coefficient η and temperature $T_{\rm eff}$ for initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$, $\langle x_{cl}^2(t)\rangle = (\gamma eV/m\omega_0^2)\int_0^t d\tau e^{-\gamma\tau}\sin^2\omega_0\tau$ for $\omega_0 \gg \gamma$. In the stationary state for $t = \infty$, we obtain results discussed above. For small times, $t \ll \gamma^{-1}$, the $\langle x_{cl}^2(t)\rangle$ term can be neglected and thus from Eq. (10) we obtain

$$\langle I(t \ll \gamma^{-1})\rangle = \frac{e^2}{\hbar}\eta V(\Omega_0^2 + C^2\langle x^2\rangle_0) - e\gamma. \qquad (11)$$

Hence we find that the zero-point motion can be extracted as the change in the linear part of the I-V characteristic from short to long time dynamics.

A setup similar to the one considered here, was recently proposed in Ref. [7], where the authors argued that single electron transistor (SET) can be used as a displacement detectors for a charged mechanical cantilever. It was suggested that such setup could provide the resolution sufficient to detect the cantilever zero point motion. We expect that despite the differences in the proposed systems, our framework and results will be relevant for the SET configuration as well. Another system that may prove to be suitable for observation of the effects discussed here is a molecule trapped near a point contact [8].

REFERENCES

[1] R. P. Feynman and F. L. Vernon, Jr., Ann. Phys. (N.Y.) **24**, 118 (1963); K. Mohring and U. Smilansky, Nucl. Phys. A **338**, 227 (1980).

[2] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983); A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1984).

[3] M. Namiki, S. Pascazio, and H. Nakazato, *Decoherence and Quantum Measurements*, (World Scientific, Singapore, 1997).

[4] S. A. Gurvitz, Phys. Rev. B **56**, 15215 (1997); S. A. Gurvitz and Ya. S. Prager Phys. Rev. B **53**, 15932 (1996).

[5] A. N. Korotkov, Phys. Rev. B **60**, 5737 (1999); A. N. Korotkov and D. V. Averin, Phys. Rev. B **64**, 165310 (2001).

[6] D. Mozyrsky and I. Martin, Phys. Rev. Lett. **89**, 018301 (2002); M. B. Hastings, I. Martin and D. Mozyrsky, cond-mat/0207005.

[7] M. P. Blencowe, M. N. Wybourne, Appl. Phys. Lett. **77**, 3845 (2000).

[8] H. Park, J. Park, A. K. L. Lim, E. H. Anderson, A. P. Alivisatos, and P. L. McEuen, Nature (London) **407**, 57 (2000).