Critical current oscillations versus pureness of superconductor-ferromagnet nanoscopic multilayers

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ABSTRACT

The influence of elastic electron scattering on nonmagnetic impurities in S/F multilayers to the oscillations of superconducting critical current J_c and critical temperature T_c versus the thickness of ferromagnetic layer d_F is studied. Using the quasiclassical Green function approach, we compute the electron excitation spectrum in the electrodes of S/F multilayer for an arbitrary pureness $\tau_i\Delta$ (except the limit $\tau_i\varepsilon_F \le \hbar$, ε_F being the Fermi energy; τ_i is the scattering time of electrons on nonmagnetic impurities). Using the same assumptions, we also derive expressions for J_c and T_c. The obtained results indicate that both the $J_c(d_F)$ and $T_c(d_F)$ dependencies are strongly affected by the electron elastic scattering in the electrodes.

Keywords: ferromagnet-superconductor multilayers, reentrant behavior.

1 INTRODUCTION

The artificial nanoscopic multilayered structures composed of superconducting (S) and ferromagnetic (F) spacers are potentially interesting for device applications and for study of basic interaction mechanisms between the superconductivity and magnetism. Changing the thickness and quality of superconducting (S) and ferromagnetic (F) layers (see, i.e., Fig. 1), one may substantially alter their electron transport properties in wide limits. In this way one may achieve controlled oscillating and reentrant behavior of critical parameters in such multilayers, which can be used to create the effective supercurrent valves [1,2]. The oscillations of critical supercurrent $J_c(d_F)$ (where d_F is the thickness of ferromagnetic layer) and critical temperature $T_c(d_F)$ are caused by the intrinsic exchange field h in F, which gives a large conducting band splitting[3]. The superfluid condensate leaking from S spacers to F due to the superconducting proximity effect has a coordinate dependent phase $\varphi(x)$ (where x is the coordinate in perpendicular to layers). The x-dependence originates from Cooper coupling between two electrons belonging to different spin subbands. Then the well-known Cooper coupling condition $\mathbf{p}+\mathbf{p}'=0$ of two electrons with momentums $\mathbf{p}$ and $\mathbf{p}'$ and opposite spins is modified. In the ferromagnetic material, the former condition is reformulated as $\mathbf{p}+\mathbf{p}'=\mathbf{Q}$ where $\mathbf{Q}=\mathbf{h}/v_F$.

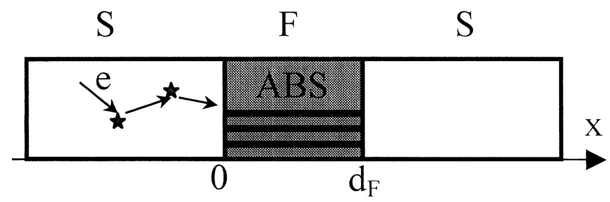

Fig. 1. The superconductor – ferromagnet - superconductor (SFS) junction with Andreev Bound States (ABS) in the middle F layer.

The finite magnitude of $\mathbf{Q}$ is actually responsible for the mentioned coordinate dependence of $\varphi(x)$ which causes many unconventional properties of S/F multilayers, including their reentrant behavior. Strictly speaking, the above interpretation of the proximity effect in S/F multilayers makes sense for perfectly "clean" systems only, where the electron momentum is a "good" quantum number. However, if the electron momentum $\mathbf{p}$ is not conserved (e.g., due to elastic scattering of electrons on nonmagnetic impurities), the condition $\mathbf{p}+\mathbf{p}'=\mathbf{Q}$ loses its sense already at $\hbar/\tau_i \ge h$. That means the above description of the proximity effect fails even for moderately "dirty" S/F-multilayers with $\tau_i\Delta \simeq \hbar$ (because in a typical experimental setup the exchange field magnitude is $h \sim \Delta$, Δ being the superconducting gap in S).

2 BASIC EQUATIONS

To study the influence of elastic electron-impurity scattering on the electron excitation spectrum and the basic transport properties of S/F multilayers we implement the quasiclassical Green function approach[3-6]. The Eilenberger equation is written in the form

$$iv_F \bullet \nabla \hat{g} + \left[i\omega_n \hat{\tau}_3 + h\hat{\sigma}_z \hat{1} - \hat{\Delta}, \hat{g} \right] = 0 \qquad (0.1)$$

where v_F is the Fermi velocity, $\hat{g}$ is the 4×4 matrix quasiclassical Green function having spin-dependent normal and anomal matrix elements, $\hat{\tau}_k, k=1\ldots 3$ are the Pauli matrices acting in the Nambu space, $\hat{\sigma}_m$, $m=\{x,y,z\}$ are the Pauli matrices acting in the electron spin-1/2 space

$$\hat{\Delta}=\begin{pmatrix} 0 & \Delta(i\hat{\tau}_2)\exp(i\varphi(i\hat{\sigma}_y)/2) \\ -\Delta(i\hat{\tau}_2)\exp(-i\varphi(i\hat{\sigma}_y)/2) & 0 \end{pmatrix} \quad (0.2)$$

The analytical solution of the above Eq. (0.1) is obtained for a piece-wise SFS geometry in assumption $\Delta = const$, $h=0$ inside the S electrodes and $\Delta=0$ and $h=const$ inside the ferromagnetic layer F. For the sake of simplicity we assume that there is no sharp interface barrier at the S/F- and F/S-interfaces. This means we apply continuous boundary conditions at the interfaces when solving Eq. (0.1), and take for $\hat{g}(x)$ its steady state value in the bulk of S electrodes. Implementing the mentioned boundary conditions, one obtains the Green function inside the middle F layer. The energy integrated quasiclassical electron Green function $g_\omega^{\pm}(x)$ is obtained in the form

$$g_\omega^{\pm}(x)=\frac{1}{2i}\frac{\omega\cosh\chi^{(\pm)}+\Omega\sinh\chi^{(\pm)}}{\Omega\cosh\chi^{(\pm)}+\omega\sinh\chi^{(\pm)}} \quad (0.3)$$

where $g_\omega^{+}(x)=\hat{g}_\omega^{11}(x)$, $g_\omega^{-}(x)=\hat{g}_\omega^{22}(x)$. In Eq. (0.3), the total phase shift $\chi^{(\pm)}$ is determined as

$$\chi^{(\pm)}=\frac{\omega d}{v_F z}\eta_\omega \mp i\frac{ehd}{v_F z}\pm i\frac{\varphi}{2};\quad \Omega=\sqrt{\omega^2+\Delta^2} \quad (0.4)$$

where d is the F layer thickness, $z=\cos\vartheta$, ϑ is the electron incidence polar angle, $\eta_\omega = 1+1/(2\tau_i\Omega)$ is the electron-impurity scattering factor.

2.1 The electron density of states in the SFS junction

The local electron excitation spectrum in the SFS junction electrodes is computed using retarded Green function $g_E^{\pm R}(x)$ which is obtained by analytical continuation of (0.3). The angle-resolved DOS, e.g., for electrons with spin "↑" is given by

$$N_i(x,E)=\mathrm{Re}\left[\hat{g}_{ii}^R(x,i\omega_n\to E+i\delta)\right] \quad (0.5)$$

where $\hat{g}_{ii}^R$ is the *ii*-th matrix element of retarded Green function, ω_n is the Matsubara frequency, and $\delta\to 0$. In Eq. (0.5) index $i=1$ corresponds to the DOS of electron with spin "↑"; $i=2$ for electrons with spin "↓"; $i=3$ corresponds to the DOS of holes with spin "↑"; and $i=4$ for holes with spin "↓". The above Eqs. (0.3)-(0.5) are used to study how the electron-impurity scattering affects the partial electron density of states. In Fig. 2 we plot $N_1(E)$ for the middle F electrode. The curves A, B, and C correspond to different thickness of the middle F layer. Namely, curve A corresponds to the thickness of the middle F layer d_F=0.5 (in units of the BCS coherence length), curve B to d_F=2, and curve C to d_F=7. For all the three curves we used the exchange field h=0.3 (in units of Δ), $\tau_i=0.3$ (in units $\hbar/\Delta$), z=1 (for a normal electron incidence).

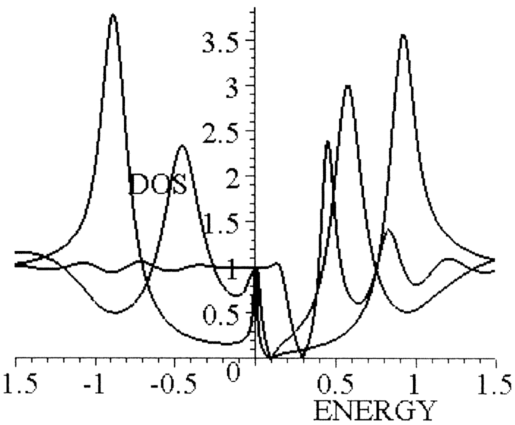

Fig. 2. The partial density of states for electrons with spin "↑". Curve A corresponds to the thickness of the middle F layer d_F=0.5 (in units of the BCS coherence length), curve B to d_F=2, and curve C to d_F=7.

From the above Fig. 2 one may see that $N_1(E)$ for d_F=0.5 has sharp peaks at $E\simeq\Delta$ associated with so-called Andreev bound state energy levels[4-7]. A qualitatively new feature as compared to the regular SNS junctions (where the middle layer N is an ordinary metal with h=0), is the zero-energy peak anomaly (see curves A, B at E=0). This anomaly is smoothed and shifted toward higher energies for relatively thick F layers (see, i.e., curve C, for which $d_F=7\gg\xi_{BCS}$).

The influence of electron-impurity scattering of different strength is shown in the next Fig. 3 where we compare the electron density of states calculated for different τ_i.

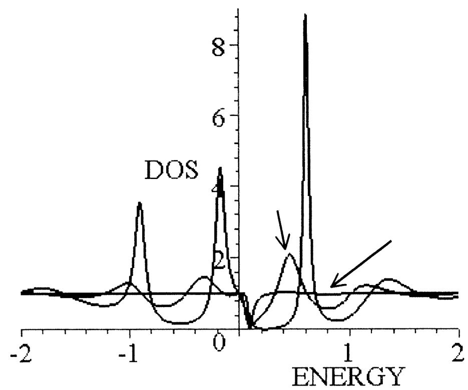

Fig. 3. The DOS curves A, B, and C related to different τ_i. The F layer thickness d_F=3 is the same for all the curves. Curve A corresponds to the "clean" limit with $\tau_i = 20$ (in units of $\hbar/\Delta$), curve B to $\tau_i = 2.5$, and curve C to $\tau_i = 0.6$.

From Fig. 3 one may infer that the elastic electron scattering drastically affects the electron excitation spectrum of the SFS multilayer. In the "clean" limit (i.e., curve A for which $\tau_i = 20$), there are sharp peaks correspondent to the ABS energy levels. Their position is not symmetric in respect to E=0. As the electron-impurity scattering intensifies (which corresponds to shorter τ_i), the former sharp peaks are gradually smoothed. For small $\tau_i << \hbar/\Delta$, there is only one sharp dip in the electron DOS (see, e.g., curve C plotted for $\tau_i = 0.6$ where the mentioned sharp dip is positioned at $E \simeq h$).

2.2 Critical current oscillations in SFS junctions

Formerly, the nonmonotonic dependence of $J_c(d_F)$ was studied theoretically either for perfectly "clean" (i.e., when $\tau_i\Delta >> \hbar$, τ_i is the electron impurity scattering time, Δ is the superconducting order parameter) or for "dirty" ($\tau_i\Delta << \hbar$) samples. However, such simplifying assumptions are not always fulfilled in real experiments. The sample purity in most cases practical interest is rather in between the two mentioned limiting cases (i.e., $\tau_i\Delta \sim \hbar$).

In this paper we study how the oscillation of $J_c(d_F)$ depends on the elastic scattering of electrons on nonmagnetic atomic impurities in the SFS junction. The expression for the electric supercurrent was obtained from solutions of quasiclassical Eilenberger equations using the continuous boundary conditions. The electric current is

$$j = ev_F N(0) T \sum_{\omega} \int_0^{2\pi} d\phi \int_0^1 z dz \left(g_{\omega}^{+}(x) + g_{\omega}^{-}(x) \right) \qquad (0.6)$$

where v_F is the Fermi velocity, $N(0)$ is the electron density of states at the Fermi surface, the summation in (0.6) is conducted over the Matsubara frequency ω, ϕ is the azimuthal angle of electron incidence to the interfaces.

The above equations (0.3)-(0.6) are used in this work to study the influence of elastic electron impurity scattering to the non-monotonic behavior of the critical parameters. The obtained results indicate that the dependence of critical current across the SFS junction $J_c(d_F)$ is strongly affected by elastic scattering in the electrodes. Since the Josephson current mechanism is closely related to existence of the quantized states, the elastic electron scattering may drastically affect the critical parameters, because its relation to the quantization conditions. In this paper we pay attention mostly to the intermediate case $\tau_i\Delta \sim \hbar$, which is less studied as compared to the marginal "dirty" and "clean" (when either $\tau_i\Delta << \hbar$ or $\tau_i\Delta >> \hbar$) cases [1,2]).

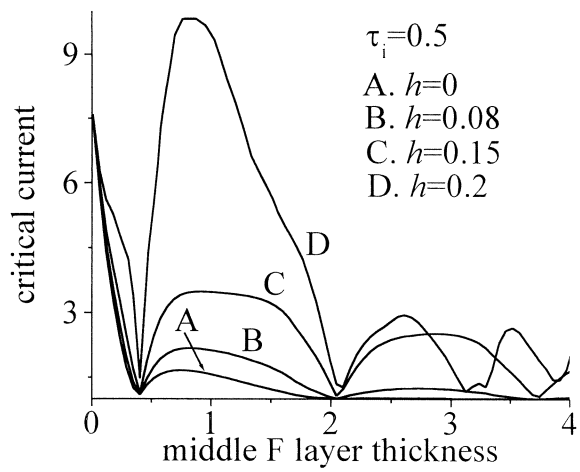

Fig. 4. The critical current oscillations versus the middle F layer thickness d_F in the SFS junctions

The obtained dependence of critical current versus the thickness of the middle F layer is shown in Fig. 4 for $\tau_i = 0.5$ where curve A corresponds to exchange field energy $h = 0$, curve B to $h = 0.08$, curve C to $h = 0.15$, and curve D to $h = 0.2$. One can see that the critical current oscillations versus d_F are taking place even at $h = 0$ (curve A), which corresponds to Andreev bound state levels (each lobe contain a certain number of ABS levels). Because the exchange field splits the levels, their total number is increased at finite $h \neq 0$ as compared to the case $h = 0$. Since ABS levels carry the supercurrent across the junction, its magnitude may increase at finite $h \neq 0$ when the number of ABS levels is larger.

3 REENTRANT BEHAVIOR OF THE SFS JUNCTION

The temperature dependence of the critical current of SFS junctions $J_c(T)$ shows a reentrant behavior. In Fig. 5 we show the dependence of critical current on temperature computed for $\tau_i = 0.5$. From Fig. 5 one can see that when $h = 0$ (curve A), the $J_c(T)$ dependence coincides with the curve for an ordinary SNS junction. When the exchange field inside the middle F layer is finite, $J_c(T)$ exhibits local maximums at some temperatures.

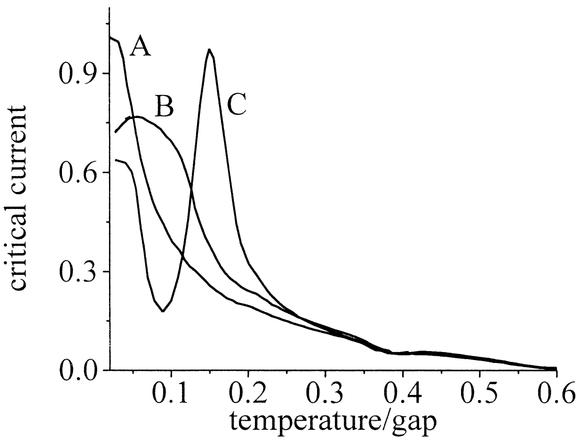

Fig. 6. The temperature dependence of critical current for three SFS junctions with different magnitudes of exchange field inside the middle F layer, $h = 0$ (curve A), $h = 0.1$ (curve B), and $h = 0.2$.

Such a maximum in $J_c(T)$ is visible in curve B (which corresponds to $h = 0.1$) at $T \simeq 0.07$, while in curve C computed for $h = 0.2$ a sharper maximum is visible at a higher temperature $T \simeq 0.15$.

4 CONCLUSIONS

We conclude that that the elastic electron-impurity scattering can change the properties of SFS multilayers drastically. This influence is manifested in the oscillatory dependence of critical current on the thickness of the middle F layer $J_c(d_F)$ and in reentrant dependence $J_c(T)$. The physical reason of oscillating behavior $J_c(d_F)$ is caused by Andreev bound state energy levels which are shifted and split by the intrinsic exchange field. In sufficiently "clean" junctions (where $\tau_i\Delta \geq \hbar$), the levels are visible as sharp peaks in the density of electron states. The peaks are non-symmetric in respect to the zero of energy, while their width is strongly affected by the electron-impurity scattering. The electron-impurity scattering tends to smooth them out, therefore ABS levels are not well pronounced at $\tau_i\Delta << \hbar$.

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