# Switching Time of a Double Barrier Josephson Junction Based Qubit Gate

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## ABSTRACT

The switching time of a qubit gate consisting of a three-terminal double-barrier SINIS junction is examined. The quantum states $|0\rangle$ and $|1\rangle$ are associated with conventional and unconventional Josephson currents. The switching time, $\tau_{sw}$, and decoherence (dephasing) time, $\tau_{\varphi}$, of such a device are computed. The mentioned switching characteristics are closely related to the junction geometry and to the recombination time of quasiparticles from the narrow Andreev bound state level into the superfluid condensate.

***Keywords***: Andreev bound state, double-barrier junction, Josephson current, qubit.

## 1 INTRODUCTION

It is now widely recognized that various configurations involving Josephson junctions (JJ) are potentially capable of performing quantum logic operations (so-called qubits) involving a superposition of two macroscopic quantum states, $|0\rangle$ and $|1\rangle$ (see, e.g., Ref. [1]). JJ-based qubit circuits have a number of advantages over other systems under study. Most important are: (i) the JJ-based approach allows significant freedom in the design of both the elementary qubit gate and the overall circuit [2]; (ii) in a low-temperature superconductor, almost all of the internal degrees of freedom are "frozen out", eliminating most of the dissipative mechanisms, and thereby minimizing decoherence (as compared with other solid-state designs) [3]; and (iii) full use can be made of well-developed JJ fabrication and measurement techniques. Since the two basic JJ qubit schemes were proposed, exploiting the conjugate quantum variables phase and charge [1], a significant theoretical and experimental effort has been made to study the dissipation mechanisms, switching, and manipulation of various JJ qubit gates [1]). However, realization of a quantum computer on the basis of existing qubit layouts is still very far from reality, and it is not clear whether either approach will be up to the task. (One difficulty involves integration of the qubits into large circuits.) Therefore, at this early stage of the development, it seems obvious that other proposals for qubit realization should be considered.

A different approach to realizing a qubit gate was suggested in Ref. [4]. This approach is based on the two quantum states present in appropriately engineered SINIS junctions (here S, I, and N denote a superconductor, an insulator, and a normal metal, respectively) under conditions to be discussed. These two energy levels can be associated with the two quantum states, $|0\rangle$ and $|1\rangle$.

## 2 A SINIS BASED QUBIT GATE

The basic element of the suggested qubit gate is the double-barrier SINIS junction. In such a device, the position of the energy levels can be controlled by applying appropriate bias voltages across the individual barriers, and by passing a transport current, $I_{tr}$, through a separate strip positioned on top of the SINIS junction (the geometry may be designed so that the transport current can pass directly along the top or bottom S electrode).

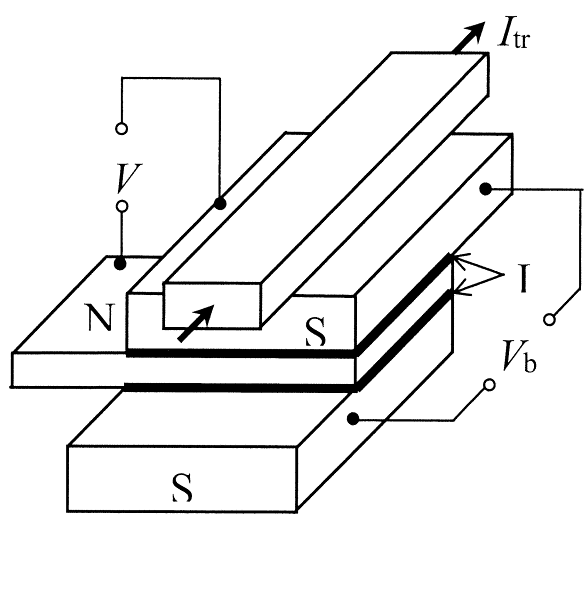


Fig. 1. Schematic configuration of the SINIS qubit gate

We start by recalling, qualitatively, some properties of SINIS junctions. Inside the normal layer, N, the Cooper pairing potential vanishes; this creates a quantum well for electron and hole quasiparticles, which experience conventional as well as Andreev reflections [5] at the SIN and NIS interfaces. For the latter, the interference between the incident electrons and reflected holes results in the appearance of the localized states, so-called Andreev bound states (ABS). In the absence of the insulating barriers, the ABS band structure (in particular, the energy spacing between the levels) is governed by the thickness of the N layer and typically there are a few bound states. Introducing the barriers dramatically changes the ABS structure; if the thickness of the N layer, $d_N$, is of order the cohererence length in the S layers, $\xi_S$, while the strength of the interface barriers, $Z$, is finite, then it is easy to achieve a situation in which there are only two levels in the subgap region [6].

The transport current, $I_{tr}$, and the voltage, $V$, which are applied to the junction electrodes as shown in Fig. 1, play the role of the "magnetic fields" $B_x^m$ and $B_z^m$, respectively, in the qubit Hamiltonian (1). In addition, a bias voltage, $V_b$, is applied between the top and bottom S electrodes of the device (see Fig. 1). In an SINIS junction, the superconducting current between the external S electrodes can flow via two "channels", as shown schematically in Fig. 2. Channel 1 corresponds to a conventional Josephson process, where the Cooper pairs, upon leaving the left electrode and traversing the total INI barrier, join the superfluid condensate in the right S electrode within a distance of the order of the coherence length.

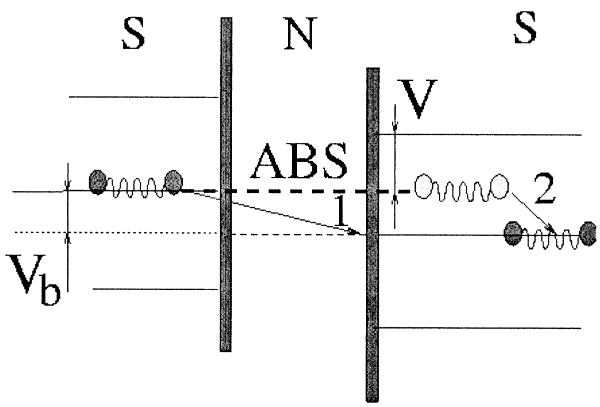


Fig. 2. The elementary processes responsible for the conventional ac Josephson current (channel 1) and unconventional dc Josephson current (channel 2).

The energy difference between the initial and final state of the Cooper pairs, $\delta E = 2eV_b$, is emitted as a photon during this process. However, if the Cooper pair recombination time, $\tau_r$, exceeds the electron transit time between the two S electrodes, $\tau_{tr} \sim d_N/v_F$, a fraction of the superconducting electrons can be transferred by the second channel (channel 2).

Electrons in an ABS level, $E_n \neq 0$, are trapped for a much longer time, and (as detailed calculations show) the distance over which these electrons rejoin the superfluid condensate on the right-hand side considerably exceeds the coherence length (in realistic structures, such a nonequilibrium Cooper pair population can be maintained throughout the whole thickness of the right-hand S electrode). In the simplest case, where $V = 0$, the ABS-level energy, $E_n$, is approximately equal to $\Delta$ (where $\Delta$ is the superconducting energy gap in S) [7]. This means that, besides the ordinary dc Josephson supercurrent $I_s^{(0)}$ at $V_b = 0$, an additional dc supercurrent appears at a finite bias voltage $V_b = E_n/e \sim \Delta/e$. The critical magnitude of the unconventional supercurrent at finite bias voltage is determined by the inelastic recombination time of quasiparticles. The dwell time during which two phase-correlated electrons reside in the upper ABS level (shown as channel 2 in Fig. 2) will be computed in Section 3. This property of the SINIS junctions differs noticeably from the ordinary SIS junctions that can support only the first component of the Josephson current, i.e., the dc Josephson current at $V_b = 0$. This unconventional dc supercurrent at finite bias voltage was recently observed as a current step in the current-voltage characteristics (CVC) of the double-barrier Nb/Al/AlO$_x$/Al/AlO$_x$/Al/Nb junctions [7]. From the slope of the current step at $V_b = E_n/e$ in the experimental CVC [7], the dissipation $\Gamma_n$ of the Cooper pair energy in the finite bias Josephson supercurrent state is estimated to be less than $0.05\Delta$ per single electron. This means that inside the S electrodes of an SINIS junction $\Gamma_n$ is remarkably low. Physically, the magnitude of $\Gamma_n$ is determined by the ABS level width that is reasonably small if the junction is not too "dirty" (i.e., when $\tau_i\Delta \geq 1$, $\tau_i$ being the electron-impurity scattering time). Because the ABS are delocalized over relatively long distances $l_{ABS} >> d_N \approx \xi_S$, the nonequilibrium Cooper pairs injected into the middle N layer from the left S electrode at $V_b = E_n/e \sim \Delta/e$ may travel across the rest of SINIS junction with almost with no energy loss. Indeed, such pairs eventually lose their energy after they escape to the external environment (i.e., to the external normal electrodes).

## 2.1 Quantum logic operations and rotations in the spin space

The elementary quantum logic operations in qubits involve controlled manipulations of two states denoted as $|0\rangle$ and $|1\rangle$.

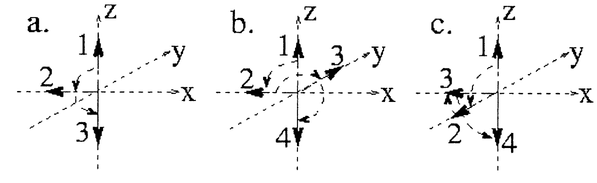


Fig. 3. Quantum logic operations as rotations of a spin state.

In particular we must be able to prepare *intermediate* states, e.g., $\psi = (|0\rangle \pm |1\rangle)/\sqrt{2}$, in addition to the *pure* states $|0\rangle$ and $|1\rangle$.[1]

For a theoretical description of the system, it is convenient to use the notation for spin-1/2 states, and to map the two states on to the "↑" and "↓" spin states. Then the quantum logic operations may be described as rotations in the spin space. The transition between $|0\rangle$ and $|1\rangle$ states may proceed in different ways, passing various intermediate states, as shown in Fig. 3.
The spin rotations may be generated by appropriate "magnetic fields" $B_x^n$ and $B_z^n$ applied individually to each spin. The physical equivalent of these fields, which is relevant to our SINIS system, will be described below. To perform quantum logic, one must also *couple* two *different* qubits, 1 and 2, so as to create mixed states of the kind

$$\psi_a^{12} = \left(|\uparrow_1\downarrow_2\rangle \pm |\downarrow_1\uparrow_2\rangle\right)/\sqrt{2}$$

or

$$\psi_b^{12} = \left(|\uparrow_1\downarrow_2\rangle \pm |\downarrow_1\uparrow_2\rangle\right)/\sqrt{2}$$

(where the indices 1 and 2 denote two different qubits).

The Hamiltonian for a two-spin system may be written as

$$\boldsymbol{H} = -\frac{1}{2}B_z^m\hat{\tau}_z^m - \frac{1}{2}B_x^m\hat{\tau}_x^m + J_{mn}\hat{\tau}_z^m\hat{\tau}_z^n \qquad (1)$$

where the indices $m$, $n$ are to be summed. The fields $B_x^n$ and $B_z^n$, which must be independently controllable, can be introduced in several different ways, depending on the particular setup; these fields, as well as the coupling constants $J_{mn}$, are determined by various Josephson interaction parameters.

# 3 SWITCHING TIME OF THE SINIS BASED QUBIT

The switching dynamics of three-terminal double-barrier SINIS junction (cf. Fig. 1) working as a qubit gate [1] based on two quantum states is analysed using the spin-boson model [8,9]. The quantum states $|0\rangle$ and $|1\rangle$ are associated with conventional and unconventional Josephson current components observed in the SINIS junctions [4,7].

If the coupling to the environment is weak compared to the qubit energy, $E_Q$, the decoherence of the qubit quantum states proceeds in two stages [8,9]: (*i*) dephasing and (*ii*) recombination (or mixing). Dephasing means that the phase difference between the two quantum states $|0\rangle$ and $|1\rangle$ randomizes on the time scale of a dephasing time $\tau_\varphi$ evaluated in the spin-boson model [8,9]

$$\tau_\varphi^{-1} = \frac{1}{2}\tau_r^{-1} + \pi\alpha\cos^2\eta\frac{2k_BT}{\hbar} \qquad (2)$$

where $\alpha$ is a dimesionless dissipation strength [8,9] and $\eta = B_x/B_z$ is a mixing angle, which determines the degree of entanglement between the two quantum states $|0\rangle$ and $|1\rangle$. The dissipation strength $\alpha$ is related to the energy loss in the system when it resides in the state $|1\rangle$ and is calculated from a microscopic model for our SINIS setup. The main source of dissipation is related to the energy recombination of the phase-correlated quasiparticles from the upper ABS level into the superfluid condensate. The relaxation time $\tau_r$ determines the dissipation dynamics during the next stage when the system makes a transition from the state $|1\rangle$ with energy $E_n$ to $|0\rangle$ with zero energy:

$$\tau_r^{-1} = \pi\alpha\sin^2\eta\frac{E_n}{\hbar}\coth\frac{E_n}{2k_BT} \qquad (3)$$

From the above expressions one can conclude that the effects of longitudinal ($\propto \cos\eta$) and transverse ($\propto \sin\eta$) terms in the qubit Hamiltonian contribute independently.

## 3.1 The recombination time of quasiparticles in an ABS level

The dimensionless dissipation strength $\alpha$ is essentially energy-dependent and is related to the recombination time $\tau_E$ of quasiparticles from the upper ABS level $E_n$ into the superfluid condensate as

$$\alpha_E = \frac{\hbar}{E_n\tau_E} \qquad (4)$$

Because the width of the ABS level is small, $\tau_E$ is much longer as compared to the traversal time $\tau_{tr} = d_N/v_F$ ($d_N$ is the thickness of the middle N spacer). For this reason, the dwell time of two phase-correlated electrons, which penetrate from adjacent S layers to the middle N spacer is sufficiently long to form a dc supercurrent when a finite bias voltage $V \cong \Delta/e$ is applied across the junction. Using a standard nonequilibrium function approach [10], one obtains $\alpha_E$ due to inelastic electron-phonon collisions in the form:

$$\alpha_E = \frac{\pi g E_n}{4(sp_F)} \int dE'(E+E')^2$$
$$(u_E u_{E'} + v_E v_{E'} - 1) \quad (5)$$
$$[(1-n_{E'})(N_{E-E'}+1) - n_{E'} N_{E-E'}]$$

where $g$ is the dimensionless electron-phonon coupling constant, $s$ is the sound velocity, and $p_F$ is the Fermi momentum. Functions $u_E$ and $v_E$ correspond to the local electron density of states and to the local Cooper pair amplitude. They are obtained from a solution of the Eilenberger equation [9] in the junction's electrodes. Detailed calculations show that in a "clean" limit, $\tau_i \Delta / \hbar >> 1$, the electron excitation spectrum inside N consists of quantized levels, while in the opposite "dirty" limit, $\tau_i \Delta / \hbar << 1$, the spectrum of N is a rather smooth function of energy, $E$. Such a difference affects $\alpha(\varepsilon)$ dramatically. Due to the reasons mentioned above, the dissipation strength $\alpha_E$ is strongly dependent on the electron excitation spectrum inside N, which, in turn, is very sensitive to the presence of nonmagnetic impurities (i.e., to the magnitude of electron impurity scattering time $\tau_i$).

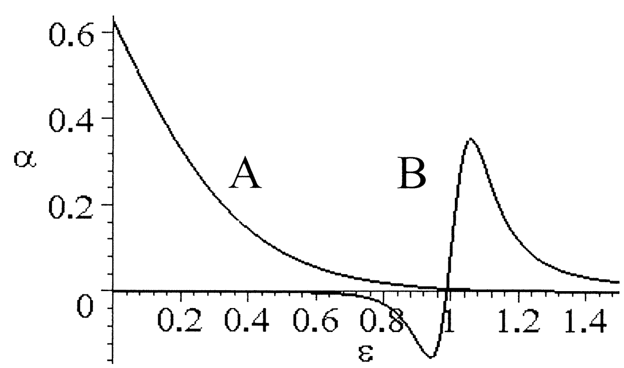


Fig. 4. The energy dependence of dissipation strength $\alpha(\varepsilon)$ (where $\varepsilon = E/\Delta$, $E$ is the energy variable, and $\Delta$ is the energy gap in S).

In Fig. 4 we plot the energy dependence of $\alpha(\varepsilon)$ inside the middle N layer of an SINIS gate for two different cases: $\tau_i = 0.1\hbar/\Delta$ (curve A), and $\tau_i = 5\hbar/\Delta$ (curve B). One can see that $\alpha(\varepsilon)$ vanishes at $E \cong \Delta$ (curve B for the "clean" case) while it still remains finite for a "dirty" junction (curve A). For such reasons, the dynamics of SINIS qubit gates are quite different in the two limits mentioned above.

# 4 CONCLUSIONS

We conclude that the switching dynamics of the SINIS-based qubit gate is determined mainly by the width and intensity of the ABS energy level positioned at $E \cong \Delta$. In sufficiently "clean" SINIS junctions, the energy-dependent dissipation strength, $\alpha_E$, vanishes at the position of the upper ABS level (i.e., at $E \cong \Delta$). Then the dephasing and recombination times of the qubit gate become very long. The parameters mentioned depend mostly on the temperature, and on the thickness and purity of the middle N layer.

# 5 ACKNOWLEDGMENT


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