Conductance of a disordered double quantum wire in a magnetic field: boundary roughness scattering

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ABSTRACT

We investigate the effect of the boundary roughness scattering on the conductance of a disordered tunnel-coupled double quantum wires in the presence of a perpendicular magnetic field. We show that the average distance between the neighboring discontinuities of the boundary profile plays an important role in the transport properties of the system and the manifestation of the localization-delocalization effect. We have found a new effect consisting of conductance decreasing with the increasing of the disorder correlation length.

Keywords: quantum wires, localization-delocalization effect

1 INTRODUCTION

A tunnel-coupled double quantum wire system (DQW) is expected to exhibit interesting transport properties when an external magnetic field is applied perpendicularly to both the axis of the wires and the direction of tunneling [1], [2]. In the case of narrow quantum wires the electronic properties of the system are mostly determined by the peculiarities of the lowest subbands. In a sufficiently strong magnetic field the lowest subband become strongly nonparabolic acquiring the camel-back shape (Fig.1). As a result one obtains two energy regions containing two propagating modes, separated by a partial energy gap where only one mode remains propagating. At energies in the partial energy gap the magnetic field localizes the two electronic states of the only propagating mode in the opposite wires and the elastic backscattering between these states is effectively suppressed [1]. Thus a localization-delocalization transition may occur in a disordered DQW whenever the Fermi energy falls into the partial energy gap [2] and can be achieved by either the variation of the carriers concentration or the strength of the applied magnetic field.

The presence of the disorder is crucial for the effect of localization-delocalization transition to manifest itself. In one's turn the disorder is determined by the experimental realization of the DQW, being produced during the growing of the multilayered structure and defining the 1D channel. The properties of the elastic electron scattering as well as the electronic transport

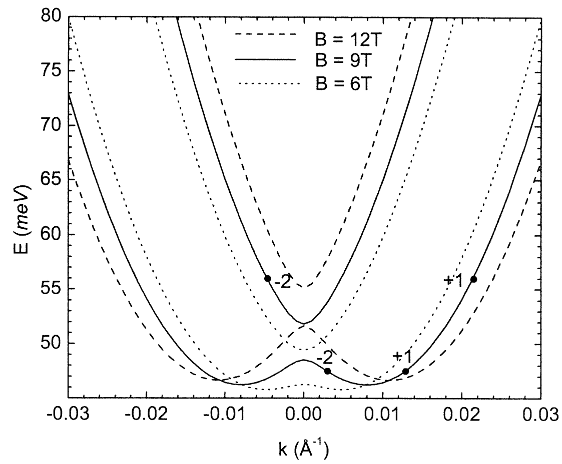

Figure 1: Energy dispersion for the lowest subbands of an ideal DQW at different magnetic fields.Points on dispersion curves shows the right-moving (+1) and left-moving (+2) propagating mode pertaining to the same shell.

properties of the DQW in the presence of the disorder due to random impurities and boundary roughness at the interfaces between different semiconductor layers have been investigated in a number of recent studies [1]–[4]. Such kinds of disorder do not significantly change the electronic states of the system and can be treated via the perturbation theory [1], [3] or, as in the case of short-range scatterers, by exactly solving the Lippman-Schwinger equation [2], [4]–[6]. At the same time the boundary roughness produced at the lateral boundaries of sufficiently narrow 1D channels during the etching or cleaving processes can considerably affect the electronic properties of a DQW. In a DQW of relevant lateral width of the wires the small fluctuations of the wire width within a few layers lead to an effective shift of the position of the partial energy gap which is comparable or may exceed the typical width of the partial gap. In a sense, the partial energy gap becomes effectively smeared by disorder, putting a question mark on the possibility to observe the localization-delocalization effect.

In the present paper we investigate the problem of scattering from disorder in a DQW due to boundary roughness produced in the result of an imperfect lateral confinement. We consider the disordered DQW consisting of an array of ideal DQW segments of randomly varying width and length. The conductance of the disordered DQW in a perpendicular magnetic field is computed using the scattering-matrix (S-matrix) method [7]. We show that the conductance enhancement occurs in a certain range of the values of the disorder correlation length defined as the average distance between the boundary discontinuities. We also found that in a wide range of the system parameters the conductance decreases with the correlation length, i. e. it decreases with the decreasing of the number of effective scatterers. To the best of our knowledge, such an effect has not been reported yet and can be regarded as a new effect of the boundary roughness on conductance.

2 AN IDEAL DQW IN MAGNETIC FIELD

We assume the DQW system to be formed on the basis of an ideal semiconductor double quantum well structure, grown along the z-axis, with the coupled 1D channels defined by means of a lateral confinement $V_L(x)$, applied along the x-axis. The double quantum well structure is assumed to be made of $GaAs$ well layers with electron effective mass $0.067m_0$ sandwiched between $Al_{0.3}Ga_{0.7}As$ barrier layers with electron effective mass $0.073m_0$and barrier height $V = 280meV$. We further assume the 1D channels to be defined by either etching process or cleaving followed by overgrowing the wire boundary planes, that is, the tunnel-coupled $GaAs$ wires are embedded into $Al_{0.3}Ga_{0.7}As$ material.

In the case of an ideal DQW system with sufficiently narrow wires the eigenstates of the Hamiltonian can be characterized by the wave vector k of the longitudinal motion along the y-axis and discrete quantum number n of the quantization along the z-axis

$$\Psi_{n,k}(\mathbf{r}) \simeq \phi_0(x)\psi_{n,k}(z)e^{iky}, \quad (1)$$

where $\phi_0(x)$ is the ground level eigenstate of the transverse motion along the x-axis determined by the following equation

$$\left[-\frac{\hbar^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{m(x)}\frac{\partial}{\partial x}\right) + V_L(x) - E_0^x\right]\phi_0(x) = 0, \quad (2)$$

and $\psi_{n,k}(z)$ is the solution of the equation

$$\left[-\frac{\hbar^2}{2}\frac{\partial}{\partial z}\left(\frac{1}{m(z)}\frac{\partial}{\partial z}\right) + \frac{\hbar^2}{2m(z)}\left(k - \frac{z}{l_B^2}\right)^2\right]\psi_{n,k}(z) = \left[E_{n,k} - E_0^x - V(z)\right]\psi_{n,k}(z). \quad (3)$$

where $l_B = \sqrt{\hbar c/eB}$ is the magnetic length. In the equation above the Zeeman splitting has been neglected and Landau gauge $\mathbf{A}(\mathbf{r})$=(0,$-Bz$,0) has been used.

Since $m(z)$ and $V(z)$ are piecewise constants the eigenvalues $E_{n,k}$ and corresponding eigenstates $\psi_{n,k}(z)$ can be found by solving Eq.(3) in each semiconductor layer, and then matching the solutions at the interfaces, requiring the wave function and the probability current density to be continuous across the interface. Making the substitution $\zeta = \sqrt{2}\left(l_B^{-1}z - l_B k\right)$, we obtain that the function $\psi_{n,k}(z)$ satisfies in the j-th semiconductor layer the equation for parabolic cylinder functions [8]

$$\left[\frac{\partial^2}{\partial\zeta^2} + \left(\nu_j + \frac{1}{2} - \frac{1}{4}\zeta^2\right)\right]\psi_j(\zeta) = 0 \quad (4)$$

and can be written as

$$\psi_j(\zeta) = A_j D_{\nu_j}(\zeta) + B_j D_{\nu_j}(-\zeta), \quad (5)$$

where $\nu_j = (E - V_j)/\hbar\omega_j - \frac{1}{2}$ and $\omega_j = eB/m_j c$.

The spectrum of a symmetric DQW system consists of pairs of 1D subbands splitted as a result of electron tunneling through the inter-wire barrier. Denoting one half of the symmetric-antisymmetric energy splitting by δ and the energy of the center of the gap by Δ, we can write the ideal DQW Hamiltonian in the truncated basis of the lowest symmetric and antisymmetric states $\{\psi_{s,0}(z), \psi_{a,0}(z)\}$ in the following matrix form

$$H = \begin{pmatrix} u + \frac{q^2}{2} - 1 & q_0 q \\ q_0 q & u + \frac{q^2}{2} + 1 \end{pmatrix}, \quad (6)$$

where $u = \Delta/\delta$, $q = \hbar k/\sqrt{\mu\delta}$, and $q_0 = -\hbar k_0/\sqrt{\mu\delta}$ are the new dimensionless quantities and δ is used as a unit of energy. Parameter $k_0 = \langle s\,|z|\,a\rangle/l_B^2$ determines the strength of the magnetic field and μ is the carrier effective mass in the well region. The roots of the equation $\|H - \varepsilon\| = 0$ determine dispersion of the lower (l) and upper subbands (u)

$$\varepsilon_{l,u}(q) = u + q^2/2 \mp \sqrt{q_0^2 q^2 + 1}. \quad (7)$$

The dispersion is nonparabolic at finite q_0 and small q due to the term $\pm\sqrt{q_0^2 q^2 + 1}$. At sufficiently large magnetic fields it produces a local maximum at the center ($q = 0$) of the lower subband, which for $q_0 > 1$ acquires the camel-back shape with two local minima at the points $q = \pm\sqrt{q_0^2 - 1/q_0^2}$ (Fig.1). The partial energy gap is formed between the local maximum of the lower subband at $\varepsilon_l(0) = u - 1$ and the local minimum of the upper subband at $\varepsilon_u(0) = u + 1$.

3 DISORDERED DQW WITH BOUNDARY ROUGHNESS

As a disordered DQW system we consider a structure which consists of three regions. The left (L) and right

(R) regions are semi-infinitely long ideal DQW with the same lateral width d. The middle one represents an array of ideal DQW segments of average length Λ and width fluctuating about d. Considering small fluctuations $|\Delta d|/d << 1$ of the wire width we neglect the variations of the ground state wave function $\phi_0(x)$ in the different segments. This allows us to solve the problem of the electronic transport through a disordered DQW system using the model Hamiltonian (6), with a piecewise constant effective potential $u(y)$, which describes the shifts of the middle-gap position due to wire width fluctuations. The transmission coefficients are computed by matching the scattering states in the left and right ideal DQW regions to the appropriate eigenstates of the Hamiltonian (6) in the disordered DQW region.

There are four scattering (propagating) states for a carrier energy outside the partial gap and two scattering states within the gap. The states of an energy shell ε are described by the solutions of the equation $\|H-\varepsilon\|=0$ in q variable with ε playing the role of a parameter

$$q=\pm\sqrt{2\left(q_0^2+\varepsilon-u\pm\sqrt{q_0^4+2q_0^2(\varepsilon-u)+1}\right)}. \quad (8)$$

Four different values of $q(\varepsilon)$ corresponds to the right- and left-moving states ±1 and ±2, where the sign coincides with the sign of the velocity $\partial\varepsilon/\partial q$ [4]. In the case of the evanescent states, we will associate the "right"- and "left"-moving states to the solutions with $\mathrm{Im}q>0$ and $\mathrm{Im}q<0$, respectively. In the j-th segment of the disordered region the wave function can be expressed as follows

$$\mathbf{\Psi}_j(y)=\sum_{i=1,2;\sigma=\pm}A_{ij}\mathbf{F}_{ij}^{\sigma}\exp\left[\sigma i q_{ij}(y-y_j)\right], \quad (9)$$

where q_1 and q_2 correspond to the right-moving channel 1 and channel 2 states, respectively, and

$$\mathbf{F}_{1j}^{\pm} = \frac{1}{N_1}\begin{bmatrix}\frac{q_{1j}^2}{2}+u_j+1-\varepsilon\\ \mp q_0q_{1j}\end{bmatrix}, \quad (10)$$

$$\mathbf{F}_{2j}^{\pm} = \frac{1}{N_2}\begin{bmatrix}\pm q_0q_{2j}\\ \varepsilon-\frac{q_{2j}^2}{2}-u_j+1\end{bmatrix}, \quad (11)$$

are the normalized eigenvectors of the Hamiltonian (6) for the given segment.

The transmission coefficients T_{nm} from the n-th propagating state in region L into the m-th propagating state of the region R are determined using a method based on the scattering matrix formalism [7]. The scattering matrix $\mathbf{S}(L,R)$ relates explicitly the outgoing states $\mathbf{A}_R$, $\mathbf{B}_L$ to the incoming states $\mathbf{A}_L$, $\mathbf{B}_R$ of the system

$$\begin{bmatrix}\mathbf{A}_R\\ \mathbf{B}_L\end{bmatrix}=\mathbf{S}(L,R)\begin{bmatrix}\mathbf{A}_L\\ \mathbf{B}_R\end{bmatrix} \quad (12)$$

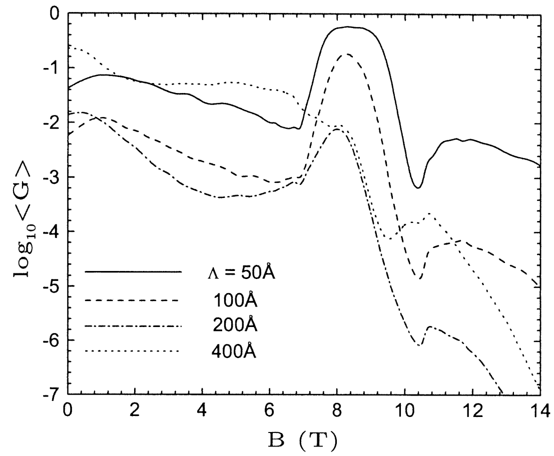

Figure 2: Dependence of the $\log_{10}\langle G\rangle$ on magnetic field for different values of the disorder correlation length Λ.

and is computed by applying the matching conditions at the interfaces between ideal wire regions and using the S-matrix composition law. The linear conductance at low temperature can be written as [9]:

$$G=\frac{2e^2}{h}\sum_{n,m}\left|\frac{v_m}{v_n}\right|\left|T_{nm}(\varepsilon_F)\right|^2, \quad (13)$$

where $v=\partial\varepsilon/\partial q$ is the group velocity and the summation is performed over the propagating states at Fermi level ε_F.

We will be considering a DQW sample with the inter-well barrier and quantum well layer widths of 40Å and 80Å, respectively. Taking into account the discrete nature of the width fluctuation, we assume the length l_j and the width d_j of an ideal DQW segment to be multiples of quantity $\lambda=2.83$Å, corresponding to $GaAs$ monolayer width. The disordered region is modeled by assigning random values to ideal DQW segment width $d_j=d+n_j\lambda$ and length $l_j=m_j\lambda$, where n_j and m_j are random integers. In the following calculations we assume that the amplitude of the width fluctuation does not exceed three $GaAs$ layers, i.e. $-6\leq n_j\leq 6$, and m_j is assumed to be distributed according to the discrete Poisson distribution. The length of the disordered DQW is $2\mu m$ and the energy Fermi of the system is fixed at $50meV$. The conductance is averaged logarithmically [10] over $1000\div3000$ realizations of the disordered DQW region and normalized to $2e^2/h$.

In Fig.2 $\log_{10}\langle G\rangle$ versus magnetic field is shown for DQW with average lateral width of 150Åand different values of correlation length Λ. In the case of a narrow DQW structure the fluctuations of the energy gap position due to wire width fluctuations are comparable with the width of the gap and the conductance is strongly affected by disorder within the whole range of magnetic fields. First, the dependence of the conductance versus magnetic fields shows a well pronounced peak in the range of magnetic fields at which Fermi level lies inside the partial energy gap. Second, one can see a remarkable dependence of the conductance on the correlation length of the disorder Λ. It turns out that the conductance decreases with the decreasing of the number of scatterers. This somewhat counterintuitive dependence of conductance on the average distance between the discontinuities of the boundary profile is a consequence of the fact that the motion of the electrons through a disordered DQW is similar to a passing through an array of randomly distributed potential barriers and well. The segments where the width of the wires is smaller than the average play the role of the barriers for a propagating electron while the wider segments are effectively the potential wells. It should be noticed, however, that the mechanism of scattering here is more complicated then in the usual one-dimensional case thanks to the complex energy spectrum of the DQW in a magnetic field. For example, if an electron with the energy below the gap enters the segment where this energy lies in the gap, the nature of the scattering through the propagating channel will be similar to the above-the-barrier reflection while the scattering through the evanescent states will be similar to the process of the under-barrier tunneling. For high magnetic field an electron can even fall into the region where both modes are evanescent, in which case the situation will be exactly the same as in the case of pure tunneling. The two-terminal conductance for a single such barrier as a function of magnetic field, is shown in Fig.3. As is seen, the conductance decreases rapidly with the longitudinal dimension of the "barrier" segment. There are also the backscattering resonances inside the partial energy gap close to the local maximum of the lower subband (repulsive scatterer) [5].

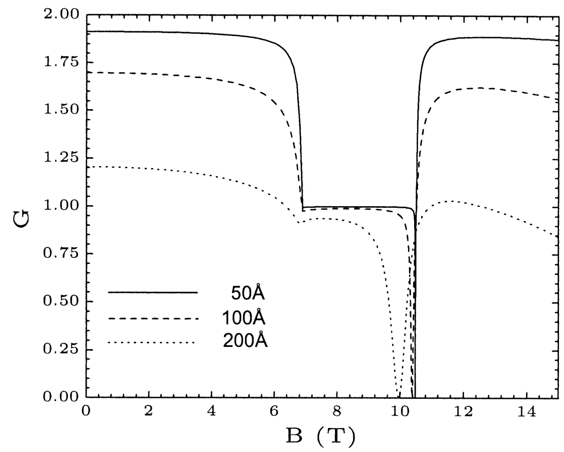

Figure 3: Dependence of the normalized conductance versus magnetic field for different disorder region lengths, when the disordered region consists of only one "barrier" DQW segment.

4 CONCLUSION

In conclusion, we have studied the electronic transport properties of a disordered DQW with rough boundaries in the presence of a perpendicular magnetic field focusing on the conditions when the localization-delocalization effect [2] and the corresponding enhancement of the conductance of the DQW [1] take place. The conductance is found to depend strongly on the amplitude of the fluctuation of the effective disorder potential relative to the width of the partial gap. In the regime of strong disorder the dependence of the conductance versus magnetic field shows a well pronounced characteristic peak when the energy Fermi falls into the energy gap. A new remarkable dependence of the conductance on the longitudinal correlation length of the disorder is observed: the conductance decreases with the average length between the discontinuities of the boundary profile.

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