Semi-Analytic Boltzmann Equation Model for the Substrate Current of Short-Channel MOSFETs with Lightly Doped Drains

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ABSTRACT

This paper presents a new technique for obtaining practical formulae for the impact ionization (I.I.) generation rate (G_{ii}) in ultra-short-channel Lightly Doped Drain (LDD) MOSFETs. A new formulae for the substrate current (I$_{sub}$) is developed.

1 INTRODUCTION

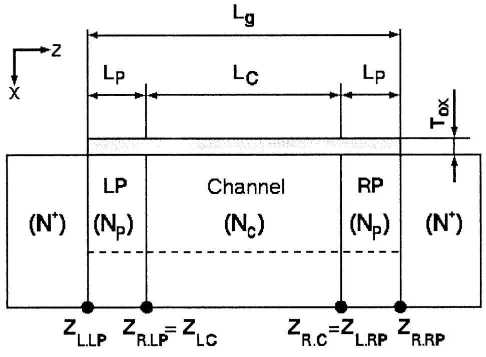

Figure 1: LDD MOSFET device model and its parameters.

Figure 1 illustrates our simplified conception of the device geometry. The regions LP and RP are the left and right lightly doped regions, respectively. We suppose that the doping level is constant in each of the three regions LP, Channel and RP.

The method developed by Yu [1] and expanded by Zanchetta [2] is used to obtain the following electric field $V_S(z)$ in each region $i = LP, Channel, RF$ of the channel

$$(V_{R,i}-V_{S,i})\frac{\sinh(\frac{z-z_{L,i}}{l_i})}{\sinh(\frac{L_i}{l_i})}+(V_{L,i}-V_{S,i})\frac{\sinh(\frac{z_{R,i}-z}{l_i})}{\sinh(\frac{L_i}{l_i})}+V_{S,i} \tag{1}$$

The quantity $V_{s,i} = V_{s,i}(V_G)$ is the long channel surface potential given by

$$V_{s,i}(V_G) = V_G - V_{FB} + \frac{1}{2}\gamma_i^2 - \gamma_i\sqrt{V_G - V_{FB} + \frac{1}{4}\gamma_i^2} \tag{2}$$

containing the body factors

$$\gamma_i = \sqrt{2q\epsilon_{si}N_i}/C_{ox} \tag{3}$$

for $i = LP, Channel$ and RP. Each interval is also characterized by a characteristic length parameter

$$l_i = \sqrt{\frac{\epsilon_{Si}T_{ox}}{\epsilon_{ox}}d_i} \tag{4}$$

where d_i is the average depletion layer thickness in region i. The constants $V_{R,i}$ and $V_{L,i}$ are found from the boundary conditions

$$V_{L,LP} = V_{bi} \tag{5}$$

$$V_{R,RP} = V_{bi} + V_{DS} \tag{6}$$

together with C^1 matching conditions at the end points of the central region

$$V_{L,Channel} = V_{R,LP} \tag{7}$$

$$V_{R,Channel} = V_{L,RP} \tag{8}$$

The satisfaction of these requirements leads to an expression for $V_{L,RP}$ as a function of V_{DS}, V_G and the device parameters such as the pocket and channel doping values. This gives the following mean field in the region RP

$$E_m = \frac{1}{L_{RP}}(V_{bi} + V_{DS} - V_{L,RP}) \tag{9}$$

The surface potential model (2) can easily be modified and should be regarded as an input to our model.

2 The BOLTZMANN EQUATION

The Boltzmann transport equation (BTE) is given by

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f + qE \cdot \nabla_k f = Q(f) \tag{10}$$

in which the collision operator $Q(f)$ has the form

$$\int (S(k',k)f(k')[1-f(k)] - S(k,k')f(k)[1-f(k')])dk' \tag{11}$$

To simplify the equation we include only phonon scattering terms and assume that $Q(f)$ has the linearized form

$$\int_R \left[S\left(k',k\right)f(t,k') - S(k,k')f(t,k)d^3k\right] \tag{12}$$

The acoustic and optical phonon scattering contributions to $S(k',k)$ are assumed to have the standard forms

$$K_{ac}(\varepsilon,\varepsilon')\delta(\varepsilon'-\varepsilon) \tag{13}$$

and

$$K_{op}(\varepsilon,\varepsilon')[(n_{op}+1)\delta(\varepsilon'-\varepsilon+\hbar\omega_{op})+n_{op}\delta(\varepsilon'-\varepsilon-\hbar\omega_{op})] \tag{14}$$

respectively. The quantity $\hbar\omega_{op}$ is the non-polar optical phonon energy and n_{op} is the equilibrium optical phonon number n_{op} defined by

$$n_{op} = \frac{1}{\exp(\frac{\hbar\omega_{op}}{k_B T_L})+1} \tag{15}$$

In each case the scattering kernels are assumed to be dependent only on initial and final enegies ε and ε' and we suppose that both $K_{ac}(\varepsilon,\varepsilon')$ and $K_{op}(\varepsilon,\varepsilon')$ are constant.

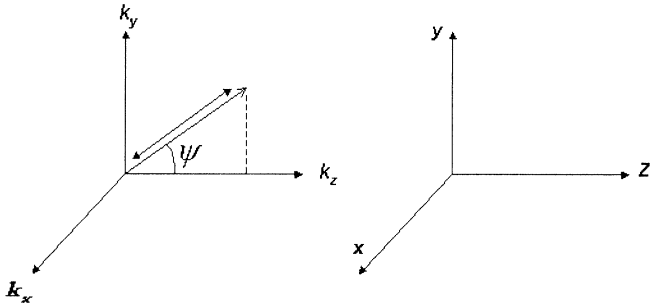

Figure 2: The k-space coordinate system.

We assume that $\vec{E} = E\hat{z}$ and seek a static solution for the carrier distribution function f in the form of the two term spherical harmonic expansion (SHE)

$$f = \alpha(\varepsilon) + \cos(\Psi)\beta(\varepsilon)|k|E. \tag{16}$$

The angle Ψ is the angle between the momentum vector $\vec{k}$ and the unit vector $\hat{z}$ along the channel. This illustrated in Figure 2. If we assume that $\vec{E} = E\hat{z}$, a solution can be found provided we can solve the transcendental equation

$$\frac{q^2\hbar^6}{48\pi^2 m^{*4}(K_{op}n_{op})^2(\frac{K_{ac}}{K_{op}n_{op}}+\frac{2n_{op}+1}{n_{op}})}\frac{E^2}{\theta^2}$$

$$+\frac{n_{op}+1}{n_{op}}e^{-\frac{\hbar\omega_{op}}{\theta}} + e^{\frac{\hbar\omega_{op}}{\theta}} - \frac{2n_{op}+1}{n_{op}} = 0 \tag{17}$$

for an electron energy variable θ. This equation includes both of the optical phonon (K_{op}) and acoustic phonon (K_{ac}) scattering kernel coefficients and the effective electron mass m^*. Dependence on the lattice temperature T_L is included through the equilibrium optical phonon number n_{op} and the parameter a defined by

$$a = \frac{n_{op}+1}{n_{op}} + \frac{K_{ac}}{n_{op}K_{op}} \tag{18}$$

It has been shown by Liotta [3] that for each $\theta = \theta(E)$ that solves (17), there exists a solution to the BTE of the form (16) with $\alpha(\varepsilon)$ and $\beta(\varepsilon)$ given by

$$\alpha(\varepsilon) = \alpha_0 e^{-\frac{\varepsilon}{\theta}},\ \beta(\varepsilon) = \frac{q\hbar}{\sqrt{2m^*}}\lambda_c\frac{1}{\sqrt{\varepsilon}}\frac{\partial\alpha}{\partial\varepsilon} \tag{19}$$

where λ_c is defined by

$$\lambda_c = \frac{\hbar^3}{4\pi n_{op}m^{*2}K_{op}(1+a)} \tag{20}$$

An approximate solution to (17) is given by the function

$$\theta = k_B T_e \tag{21}$$

with the electron temperature T_e given by

$$T_e = T_L(1+\kappa E^2) \tag{22}$$

where κ is given by

$$\kappa = \frac{\hbar^6 q^2 E^2 (K_{op}n_{op})^{-2}}{192M\pi^2(k_B T_L)^2(a+1)m^{*4}} \tag{23}$$

involving the parameter M defined by

$$M = \left(1 - \exp\left(\frac{\omega_{op}}{2k_B T_L}\right)\right)^2 \tag{24}$$

Equation (22) has exactly the theoretic form given in Lundstrom [4].

3 THE DRIFT VELOCITY

The electron drift velocity $\vec{v}_d$ is given by

$$\vec{v}_d = \frac{\hbar}{m^* n}\int\int\int_{R^3}\vec{k}fd^3k \tag{25}$$

with

$$n = \int\int\int_{R^3} fd^3k \tag{26}$$

We assume parabolic energy bands with the dispersion law $\varepsilon(k) = \frac{\hbar}{2m^*}k^2$ in which m^* is the value of the effective electron mass. Then, the drift velocity can be expressed as

$$v_d = \mu(E)E \tag{27}$$

where

$$\mu(E) = \frac{\mu_0}{\sqrt{1+\kappa E^2}} \tag{28}$$

and

$$\mu_0 = \frac{q\hbar^3}{3\pi\sqrt{2}(1+a)K_{op}m^{*\frac{5}{2}}} \tag{29}$$

4 THE IMPACT IONIZATION RATE

The generation rate due to impact ionization G_{ii} is calculated from

$$G_{ii} = \int \sigma(\varepsilon) f(\varepsilon) P_{ii}(\varepsilon) d\varepsilon \qquad (30)$$

where $\sigma(\varepsilon)$ is the density of states and $P_{ii}(\varepsilon)$ is the impact ionization probability which we approximate by the Keldysh formula [5]

$$P_{ii}(\varepsilon) = P_{ii0}(\varepsilon - \varepsilon_{th})^b \Theta(\varepsilon - \varepsilon_{th}) \qquad (31)$$

Our solution of the (SHE) equations is now normalized and used to compute the generation rate G_{ii}. This gives G_{ii} in the form

$$G_{ii} = \frac{2n}{\sqrt{\pi}} P_{ii0} \theta^b I(\frac{\varepsilon_{th}}{\theta}, b) \qquad (32)$$

where $\theta(E) = k_B T_L (1 + \kappa E^2)$ and

$$I(\varepsilon_{th}, b) = \int_{\varepsilon_{th}}^{\infty} \sqrt{\varepsilon}(\varepsilon - \varepsilon_{th})^b e^{-\varepsilon} d\varepsilon \qquad (33)$$

The factor 2 in (32) takes spin into account.

The integral (33) can be expressed in terms of Whittaker functions [6] but this is of little practical value. We regard both the threshold energy ε_{th} and b as fitting parameters. As a result it is necessary to evaluate the integral (33) in such a way that its dependence on ε_{th} and b is explicit.

5 AN INTEGRATION TECHNIQUE

In order to capture the parameter dependence of the integral (33) we cannot use regular quadrature technique such as an adaptive Simpson rule. Instead we use the Gauss-Laguerre formulae [7]. The general form of this integration technique is

$$\int_0^{\infty} e^{-x} \cdot f(x) dx = \sum_{k=1}^{n} \omega_k \cdot f(x_k) + \frac{(n!)^2}{(2n)!} f^{(2n)}(\zeta) \quad (34)$$

where $0 < \zeta < \infty$ and x_k's are the zeros (the nodes) of the Laguerre polynomials.

Using this integral approximation technique we can construct practical, closed-form, solutions that preserve the analytical dependence on ε_{th} and b. For example, the two-node approximation to G_{ii} is given by

$$G_{ii} = \frac{2n}{\sqrt{\pi}} P_{ii0} \theta^b I_2(\frac{\varepsilon_{th}}{\theta}, b) \qquad (35)$$

where

$$I_2(\xi, b) = \left[\sqrt{x_1 + \xi}\, x_1^b \omega_1 + \sqrt{x_2 + \xi}\, x_2^b \omega_2\right] e^{-\xi} \quad (36)$$

with

$$\xi = \frac{\varepsilon_{th}}{\theta} \qquad (37)$$

$$\theta = k_B T_L (1 + \kappa E^2) \qquad (38)$$

The parameters ω_1, ω_2, x_1 and x_2 are fixed numerical constants given by

$$\begin{aligned} x_1 &= 0.585786437626905 \\ x_2 &= 3.414213562373095 \\ \omega_1 &= 8.535533905932735e-01 \\ \omega_2 &= 1.464466094067262e-01 \end{aligned}$$

6 THE SUBSTRATE CURRENT

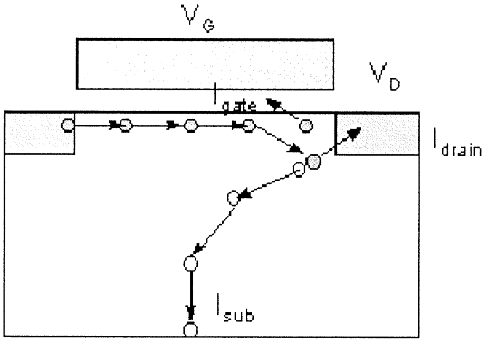

Figure 3: Illustrates Substrate Current I_{sub} resulting from I.I.

The probability that an ionizing collision occurs is given by the ionization coefficient

$$\alpha_n = \frac{1}{n|v_d|} G_{ii} \qquad (39)$$

The substrate current in an n-channel MOSFET is given by [8]

$$I_{sub} = I_{ds} \int_{RP} \alpha_n(z) dz \qquad (40)$$

where I_{ds} is the drain current at the end of the Channel region. If we now use the results (27) and (32) and approximate the integral in (40) by its mean value we obtain the final model formula

$$I_{sub} = I_{ds} \frac{2 L_{RP}}{E_m \mu(E_m) \sqrt{\pi}} P_{ii0} \theta(E_m)^b I_2 \left(\frac{\varepsilon_{th}}{\theta(E_m)}, b\right) \qquad (41)$$

where E_m is the mean electric field

$$E_m = \frac{1}{L_{RP}} \left[V_{bi} + V_{DS} - V_{L,RP}(V_{DS}, V_{GS})\right] \qquad (42)$$

μ is the electron mobility (28), V_{bi} and V_{DS} are the built-in potential and drain potential.

7 CONCLUSION

We have obtained analytical expressions to model impact ionization that capture the essential physics of the problem. We assumed the impact ionization occurs in the lightly doped pocket near to the drain. We also assumed an average electric field that accurately represents the LDD device geometry. We introduced a new integration technique to yield a practical approximation to the integral expression for the impact ionization rate obtained from the BTE. We presented an analytical formula for the substrate current that has both temperature and device geometry dependencies in an explicit form. This formula is suitable for use in SPICE type simulation packages.

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