Stability of Charge-Controlled Electrostatic Actuators:

A general theorem and a novel charge Pull-In extraction numerical scheme

Ofir Bochobza-Degani*, David Elata** and Yael Nemirovsly*

*Electrical Engineering Department, Technion – Israel Institute of Technology, Haifa, 32000, Israel, PHN: +972-4-8292763, FAX: +972-4-8322185, Email: ofird@tx.technion.ac.il

**Mechanical Engineering Department, Technion – Israel Institute of Technology, Haifa, 32000, Israel, PHN: +972-4-8293184, FAX: +972-4-8322185, Email: elata@tx.technion.ac.il

ABSTRACT

This work deals with the stability of charge-controlled electrostatic actuators. First, a general proof is presented showing that for any electrostatic actuator, the stability range under charge-control is always larger than the stability range under voltage-control. Second, a novel numerical scheme for analyzing the stability and extracting the charge Pull-In parameters of charge-controlled actuators is proposed. Third, a strategy for further extending the travel-range of charge-controlled actuators is suggested.

Keywords: Charge Control, Pull-In, Electrostatic actuator, numerical scheme.

1 INTRODUCTION

Electrostatic actuation is the most prevalent means of manipulating deformable elements in MEMS devices. The coupled electro-mechanical response of such devices is often characterized by the Pull-In instability [1-5]. The instability of the electro-mechanical coupled field is due to the non-linear dependency of the electrostatic forces on the deformation of flexible elements. Due to the limitations the Pull-In posses on the stable range of electrostatic actuator, it is important to characterize and understand this phenomenon.

By applying a voltage difference across the electrodes of an electrostatic actuator (Fig.1), the electrodes are charged and an electrostatic force is generated that tends to reduce the gap between the electrodes. For a sufficiently low voltage, the electrostatic forces are balanced by the elastic restoring forces of the deformable structure, and this balance is a stable equilibrium. At a critical value of the applied voltage (Pull-In voltage) the equilibrium state becomes critically stable and any virtual increase in the applied voltage or virtual decrease in the gap results in a dynamic collapse. This dynamical collapse is due to an uncontrolled rapid increase of the charge. This in turn results in an increase of the electrostatic forces that rapidly decrease the gap between the electrodes [2,3].

It may seem that by controlling the applied charge rather than the applied voltage, the Pull-In instability can be prevented. Indeed, it was shown that a charge-controlled parallel plate actuator does not suffer from Pull-In instability [2]. However, it has been shown that other charge-controlled electrostatic actuators may exhibit the Pull-In instability (e.g., in the case of torsion actuator [3] or circular plate [4]). Although the total charge is controlled under charge excitation, the charge distribution in the electrodes is determined by the deformation.

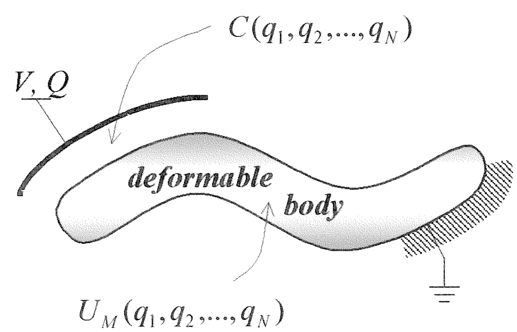


Figure 1: General configuration of an electrostatic actuator with *N-DOF*. The electrode may be subjected to either voltage or charge excitation.

In general deformable elements, increasing the total applied charge above a critical value (Pull-In charge) also results in a dynamic collapse. In this process, a re-distribution of the charge rapidly increases the electrostatic forces even though the total charge is fixed. Since the gap in the parallel-plate actuator is and remains uniform, such a re-distribution cannot occur and therefore the parallel-plate actuator is always stable under (total) charge excitation.

In the torsion actuator the charge Pull-In is due to charge re-distribution and concentration away from the axis of rotation. Furthermore, for this actuator it was shown that the range of stability associated with charge excitation is larger than that of voltage excitation (i.e., in this sense the charge Pull-In occurs after the voltage Pull-In) [3]. Figure 2 presents typical equilibrium curves for an electrostatic actuator under voltage and charge control.

So far, no counter example for which the voltage Pull-In occurs after the (total) charge Pull-In was presented. The question whether such a counter example can be found or whether voltage Pull-In always precedes charge Pull-In (as in the torsion actuator), is still open.

In this work a general proof that charge Pull-In cannot occur before voltage Pull-In for a general actuator with an arbitrary number N of degrees-of-freedom (DOF), is presented.

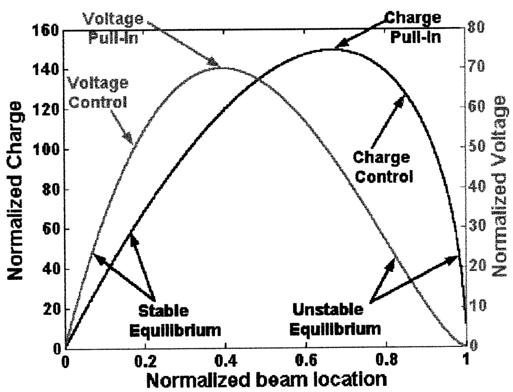


Figure 2: Typical equilibrium curve of an electrostatic actuator under charge or voltage control. The charge-control travel range is larger than the voltage-control travel range in accordance with the general proof.

Recently, charge-control actuation scheme was demonstrated [4]. This makes charge-control a practical means of driving electrostatic actuators. Therefore, there is need for modeling tools that can predict the charge Pull-In parameters. Much effort has been invested in developing numerical schemes for simulating voltage pull-in but such schemes for charge Pull-In are yet to be developed.

In this work a novel scheme for extracting the Pull-In parameters of general charge-controlled electrostatic actuators is proposed. The new charge Pull-In numerical scheme is based on the recently presented *DIPIE* scheme [5], and can be implemented in existing MEMS-CAD tools.

2 THE GENERAL PROOF

The essence of the proof is showing that any equilibrium state that is stable under voltage control is necessarily a stable equilibrium state under charge control. This means that the range of stability of the charge-controlled actuator is always equal or larger than that of the voltage-controlled configuration of the same actuator.

To this end, consider an electrostatic actuator with *N-DOF*, $(q_1, q_2, \ldots, q_N)$, where N is an arbitrary number (Fig.1), formed by two suspended conducting bodies. An energy source applies either a voltage difference or a fixed charge to the free space capacitor formed between the two conducting bodies (Fig.1). The free space capacitor, C, is assumed to be a linear electrical element, for which the charge Q is proportional to the voltage V, but the capacitance may be a non-linear function of the degrees of freedom. The mechanical strain energy stored in the suspending elements is denoted by U_M.

The total energy, U^T, of the charge-controlled actuator and the total potential energy [2,3], Ψ^T, of the voltage-controlled actuator are given by,

$$U^T(q_1, q_2, \ldots, q_N, Q) = U^M(q_1, q_2, \ldots, q_N) + \frac{Q^2}{2C(q_1, q_2, \ldots, q_N)} \qquad (1a)$$

$$\Psi^T(q_1, q_2, \ldots, q_N, V) = U^M(q_1, q_2, \ldots, q_N) - \frac{1}{2}C(q_1, q_2, \ldots, q_N)V^2 \qquad (1b)$$

The equilibrium states of the charge-controlled actuator and the voltage-controlled actuator are those for which the total energy or the total potential have an extremum, respectively. Hence, the derivative of the total energy/total potential with respect to all the degrees of freedom vanish at the equilibrium state $(q_{10}, q_{20}, \ldots, q_{N0})$,

$$\left(\frac{\partial U^T}{\partial q_i}\right)_Q = \frac{\partial U^M}{\partial q_i} - \frac{Q^2}{2C^2}\frac{\partial C}{\partial q_i} = 0 \qquad i = 1..N \qquad (2a)$$

$$\left(\frac{\partial \Psi^T}{\partial q_i}\right)_V = \frac{\partial U^M}{\partial q_i} - \frac{V^2}{2}\frac{\partial C}{\partial q_i} = 0 \qquad i = 1..N \qquad (2b)$$

Since C is a linear electrical element, both configurations will exhibit an equilibrium state at $(q_{10}, q_{20}, \ldots, q_{N0})$. This state can be reached by applying the required voltage across the plates or by applying the equivalent charge $Q = V \times C(q_{10}, q_{20}, \ldots, q_{N0})$.

The stability of the equilibrium state $(q_{10}, q_{20}, \ldots, q_{N0})$ of the *N-DOF* actuator is determined by the eigenvalues of the stability matrix, **A**, which is given by

$$A_{ij}^Q = \left(\frac{\partial^2 U^T}{\partial q_i \partial q_j}\right)_Q = \frac{\partial^2 U^M}{\partial q_i \partial q_j} - \frac{Q^2}{2C^2}\frac{\partial^2 C}{\partial q_i \partial q_j} + \frac{Q^2}{C^3}\frac{\partial C}{\partial q_i}\frac{\partial C}{\partial q_j} = \frac{\partial^2 U^M}{\partial q_i \partial q_j} - \frac{\partial U^M / \partial q_i}{\partial C / \partial q_i}\frac{\partial^2 C}{\partial q_i \partial q_j} + \frac{Q^2}{C^3}\frac{\partial C}{\partial q_i}\frac{\partial C}{\partial q_j} \qquad (3a)$$

$$A_{ij}^V = \left(\frac{\partial^2 \Psi^T}{\partial q_i \partial q_j}\right)_V = \frac{\partial^2 U^M}{\partial q_i \partial q_j} - \frac{V^2}{2}\frac{\partial^2 C}{\partial q_i \partial q_j} = \frac{\partial^2 U^M}{\partial q_i \partial q_j} - \frac{\partial U^M / \partial q_i}{\partial C / \partial q_i}\frac{\partial^2 C}{\partial q_i \partial q_j} \qquad (3b)$$

Eqs. (2a) and (2b) were used in the derivations of Eqs. (3a) and (3b), respectively.

The relation between the stability matrices of the charge control actuator, $\mathbf{A}^{Q}$, and the voltage control actuator, $\mathbf{A}^{V}$, at the equilibrium state $(q_{10}, q_{20}, \ldots, q_{N0})$ can be derived from Eqs. (3),

$$\mathrm{A}_{ij}^{Q} = \mathrm{A}_{ij}^{V} + \frac{Q^2}{C^3}\frac{\partial C}{\partial q_i}\frac{\partial C}{\partial q_j} \tag{4}$$

The equilibrium state $(q_{10}, q_{20}, \ldots, q_{N0})$ is a stable one when the stability matrix is positively defined, i.e. the extremum is a local minimum point. Thus, for any vector $\beta \in \Re^N$, the value of $\beta \mathbf{A} \beta$ is positive. Using the Einstein summation convention on repeated indices, Eq. (4) can be rewritten in the form

$$\begin{aligned} \beta_i \mathrm{A}_{ij}^{Q} \beta_j &= \beta_i \mathrm{A}_{ij}^{V} \beta_j + \frac{Q^2}{C^3}\beta_i\left[\frac{\partial C}{\partial q_i}\frac{\partial C}{\partial q_j}\right]\beta_j \\ &= \beta_i \mathrm{A}_{ij}^{V} \beta_j + \frac{Q^2}{C^3}(\beta \cdot \nabla C)^2 \end{aligned} \tag{5}$$

where $\beta \cdot \nabla C$ is the projection of the capacitance gradient on the vector β. The second term on the RHS of Eq. (5) is always positive (C>0).

Therefore, it is concluded that if the equilibrium state $(q_{10}, q_{20}, \ldots, q_{N0})$ is a stable state of the voltage-controlled actuator (i.e., $\beta_i \mathrm{A}_{ij}^{V} \beta_j > 0$), it is necessarily a stable equilibrium state of the charge-controlled actuator (i.e., $\beta_i \mathrm{A}_{ij}^{Q} \beta_j > 0$).

3 THE NUMERICAL SCHEME

Much effort was invested in developing numerical schemes that simulate the electromechanical response and extract the Pull-In parameters of voltage-controlled actuators. In what follows a numerical scheme for simulating and extracting the Pull-In parameter of charge-controlled actuators is proposed.

Recently, a DIPIE scheme for extracting the voltage Pull-In parameters was presented [5]. The DIPIE scheme equally converges to stable or unstable voltage-controlled equilibrium states. To calculate an equilibrium state, the displacement of a pre-chosen node is postulated. The deformation of the actuator is calculated by solving the 'voltage-free' electromechanical equations, while the pre-chosen node is fixed. Finally the applied voltage associated with the pre-chosen node postulated displacement is calculated [5].

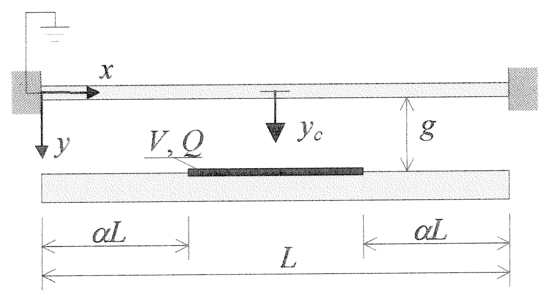


Figure 3: Schematic view of a clamped-clamped beam actuator with either charge or voltage excitation.

By a slight modification, the new scheme calculates the total charge associated with each of the voltage-controlled equilibrium states, computed by the DIPIE scheme. Then, the new scheme seeks the state associated with the maximum applied charge (i.e., charge Pull-In).

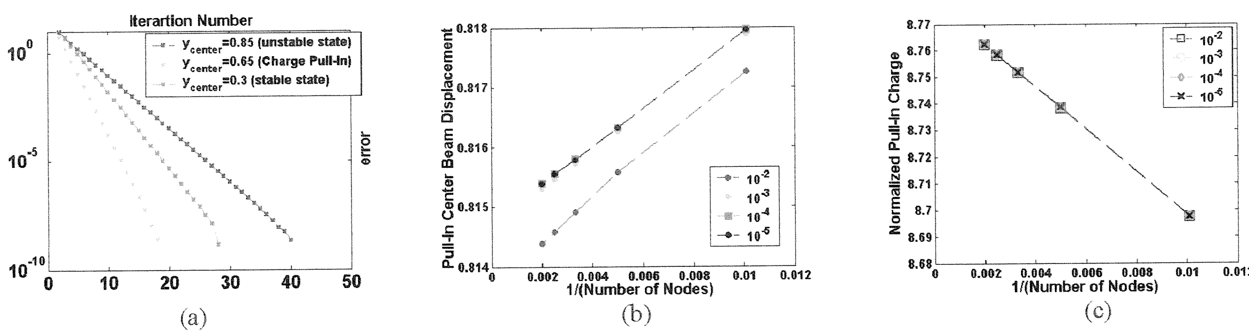


Figure 4: Convergence characteristics of the proposed numerical scheme for charge Pull-In extraction, (a) convergence to various equilibrium states, (b)-(c) convergence of the Pull-In parameters with increasing accuracy and mesh refinement.

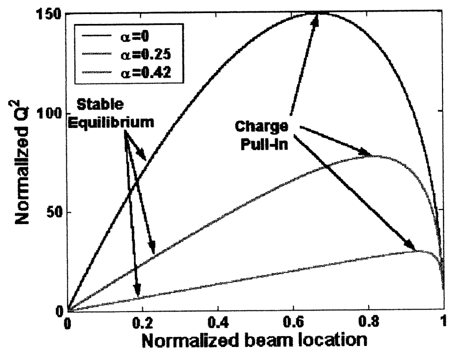
To illustrate the performance of the numerical scheme the charge-controlled clamped-clamped beam actuator (Fig.3) was analyzed. Fig.2 presents the equilibrium curve of this actuator with full bottom plate, under voltage and charge excitation.

The convergence properties of the suggested scheme are presented in Fig.4 showing regular and consistent behavior. Fig.4a presents the convergence to equilibrium states. It is clearly observed that the scheme converge to stable and unstable equilibrium states with almost the same numerical effort.

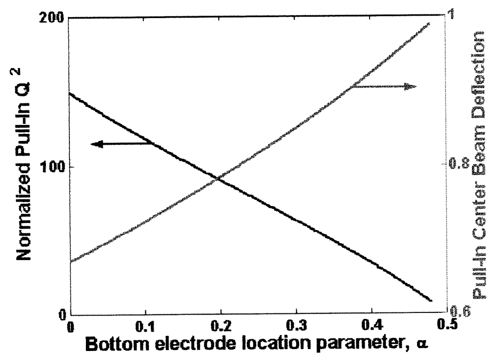
Fig.4b and fig.4c exhibit the convergence of the Pull-In parameters with mesh refinement and increased accuracy. The well-behaved linear convergence suggests that the exact solution can be easily estimated using a few coarse discretizations.

To further extend the travel range under charge-control, the bottom electrode can be confined to the center region of the beam, as shown in Fig.5. This differs from leverage-bending proposed for voltage-controlled actuators [1], where the electrode is confined to the beam edges.

As the electrode is centralized the actuator becomes more like a parallel plate actuator, where there is no charge Pull-In. This is also observed in the electromechanical response that becomes more linear as the expected for the parallel-plates actuator (Fig.5a).



(a)



(b)

Figure 5: Extending the stable travel range of a charge controlled clamped-clamped beam actuator by reducing the bottom electrode extension, α; (a) the equilibrium curves for increasing α exhibiting the more linear response in stable domain, (b) the increased travel-range as function of α and the related Pull-In applied charge.

4 SUMMARY

To conclude, it was shown that the extended travel range under charge control is indeed a fundamental rule for electrostatic actuators with arbitrary geometry. This ensures that the use of charge control for driving MEMS devices (rather than the prevalent voltage control) will extend the travel range.

A simple extension of the DIPIE scheme was used to facilitate a charge Pull-In scheme. The well-behaved properties of the charge Pull-In scheme were demonstrated on the clamped-clamped beam actuator.

Moreover, a scheme that allows further extending the travel range of charge controlled electrostatic actuators beyond the charge Pull-In point was proposed and analyzed. This scheme also allows linearizing the electromechanical response of the actuator.

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