A Miniature Laser Interferometer for Noninvasive Viscometry

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ABSTRACT

A miniature laser interferometer consisting entirely of an optical fiber held in close proximity to a fluid surface is described. The total path difference in this interferometer is less than a millimeter. The interferometer may be used to measure, for example, the amplitude of submicron capillary waves on fluids with a resolution of about 10 nm. In one application the interferometer is used to measure, noninvasively, the amplitude decay of capillary waves as a function of the distance from the wave generator. The amplitude decay data yields the viscosity of the fluid with unprecedented precision. As a test case, the viscosity of pure water as a function of temperature, measured by this system, will be presented.

Keywords: Interferometer, laser, capillary waves, viscosity

1 INTRODUCTION

Capillary waves are surface waves on fluids with wavelengths in the millimeter range. Surface tension and viscosity govern the propagation and attenuation of capillary waves while gravity plays a minor role. Thus in principle, data on dispersion and attenuation of capillary waves may be used to obtain the surface tension and viscosity of fluids [1-8].

More than a century ago, Stokes (1819-1903) pointed out that the attenuation of surface waves could be exploited to measure viscosity [9]. Since then, the determination of viscosity from the damping of surface waves has received much attention [10-14], particularly because the method presents the possibility of measuring viscosity noninvasively. But in practice no reliable technique has been available to measure the attenuation of capillary waves with the requisite precision to render the method useful for routine measurement of viscosity.

Here we describe a noncontact method for precision measurement of the wavelength and attenuation of capillary waves on fluids [15-17]. The technique utilizes a novel miniature laser interferometer [18] to map the wave profile with a resolution of about ten nanometers-- some fifty times better than the resolution of a typical optical microscope. We have used this interferometer to obtain the attenuation of capillary waves on pure water as a test case. To check the sensitivity of the method, the attenuation data was used to obtain the viscosity of pure water as a function of temperature.

2 THE INTERFEROMETER

The experimental technique has been described in more detail elsewhere [19]. Here we present a brief summary. Capillary waves are generated electronically by placing a metallic blade a few tenths of a millimeter above the fluid surface. A dc-biased sinusoidal voltage of a few hundred volts at a selected frequency is applied between the blade and the fluid. For polar fluids such as water or water-glycerin binary mixtures, the alternating electric field under the blade generates two capillary wave trains that recede from the blade on each side. Typically the amplitude of these waves is of the order of one micron.

To obtain the capillary wave amplitude we employ a fiber-optic detection system which functions like a miniature laser interferometer. The heart of the system consists of a single mode optical fiber, one end of which is positioned a short distance above the fluid surface. Laser light, traveling through the optical fiber is partially reflected from the cleaved tip of the fiber and again from the fluid surface. The two reflected beams travel back through the same fiber and generate an interference signal at the detector. As the fluid level changes due to the wave motion, the interference signal portrays an accurate record, in real time, of the variation of the gap. The interference signal is detected, amplified, and digitized for later analysis.

Since there is a one to one correspondence between the surface wave and the resulting interference pattern, the profile of the surface wave can be recovered by an analysis of the interference pattern. Indeed, as discussed below, the number of fringes in the interference pattern is directly proportional to the amplitude of the surface wave.

3 THEORY

To determine the wave amplitude of a traveling wave from the interference record, the vertical oscillation of the fluid surface under the probe may be represented by

$$y(t) = a \sin(\omega t + \beta), \quad (1)$$

where a is the wave amplitude, ω is the angular frequency, t is the time, and β is a phase which depends only on the position of the probe relative to the blade. If d_o is the air gap between the probe and the equilibrium surface of the fluid, then the path difference between the two reflected beams is,

$$\Delta = 2[d_o - a \sin(\omega t + \beta)]. \quad (2)$$

Thus, the ac component of the resulting interference pattern is given by,

$$Y(t) = A \cos[(2\pi \Delta / \lambda_l) + \pi]. \quad (3)$$

Here A is the amplitude of the interference signal, λ_l is the wavelength of the laser light, and π is added to the phase to account for the fact that the light beam reflecting from the fluid surface suffers a phase shift of π radians. When Eq.2 is substituted in Eq.3, the result is,

$$Y(t) = A \cos[b \sin(\omega t + \beta) - \varphi], \quad (4)$$

where $b = 4\pi a/\lambda_l$, gives the number of interference fringes and $\varphi = (\pi + 4\pi d_o / \lambda_l)$.

Thus the relation between the wave amplitude and the number of fringes in the interference signal is simply

$$a = b\lambda_l / 4\pi. \quad (5)$$

In light of Eq.5, to obtain the amplitude of the capillary wave from the associated interference data, it is only necessary to extract the parameter b by fitting the analytical expression in Eq.4 to the interference data. To accomplish this, the interference signal is digitized and used as input in a multi-variable fit routine, which adjusts the four parameters of Eq.4 until a good fit is achieved. Indeed, of the four parameters in Eq.4, A and β are readily available from the raw data, so the fit routine reduces to a search in the two-parameter space of b and φ. By this method we determine the attenuation of the wave amplitude as a function of the distance traveled from the source. But how is the wave attenuation related to viscosity?

The dissipation of wave energy due to viscosity manifests itself in the attenuation of the amplitude as the wave travels along the surface. Indeed, the wave amplitude as a function of distance x is given by,

$$a = a_o e^{-\alpha x}. \quad (6)$$

Here a_o is the reference amplitude at $x=0$, and α is the attenuation coefficient. As described elsewhere [21] the attenuation coefficient is related to the fluid viscosity η through the relation,

$$\alpha = (2k^2 \eta / \rho v_g). \quad (7)$$

Where $k=2\pi/\lambda$ is the wave number, λ is the capillary wavelength, and v_g is the group velocity of capillary waves. To a very good approximation, the group velocity for capillary waves is $3\omega/2k$. Therefore we obtain the viscosity in terms of four measurable quantities, namely, wave number $k=2\pi/\lambda$, surface tension σ, density ρ, and the attenuation coefficient α. Indeed, we have

$$\eta = 3\omega\alpha\rho / 4k^3. \quad (8)$$

The attenuation coefficient α is obtained from a plot of the wave amplitude vs. the distance from the blade. The wavelength λ is determined by the following method. First a standing wave is established on the fluid surface by use of two blades, separated by a few centimeters generating waves of the same phase, amplitude, and frequency. Since each blade sends a wave train toward the other, a standing capillary wave is established on the surface between the two blades. If the distance between the two blades is chosen to be a half odd-integer wavelength, the two wave trains interfere destructively on the outer sides of the blades. This judicious choice of the blades' separation produces a region of standing waves between the blades while the surface outside the blades remains calm.

The interferometer is used to locate the position of nodes. Measurement of the distance between several nodes yields the wavelength of the capillary wave for a given frequency. In our setup, the fiber optic probe is attached to a micropositioner, which in turn is equipped with a digital micrometer. This enables us to measure the wavelength of the standing capillary waves routinely to within a micron.

4 EXPERIMENTAL RESULTS

Using the method described above we have measured the viscosity of pure water as a function of temperature. Figure 1 gives the wave amplitude vs. distance from a reference point near the generating blade for pure water. The solid line is an exponential fit to the data. The fit yields an attenuation coefficient of 0.559/cm which when used in Eq.8 yields a value of 8.9×10^{-3} cm^2/s at 23° Celsius for the kinematic viscosity of water.

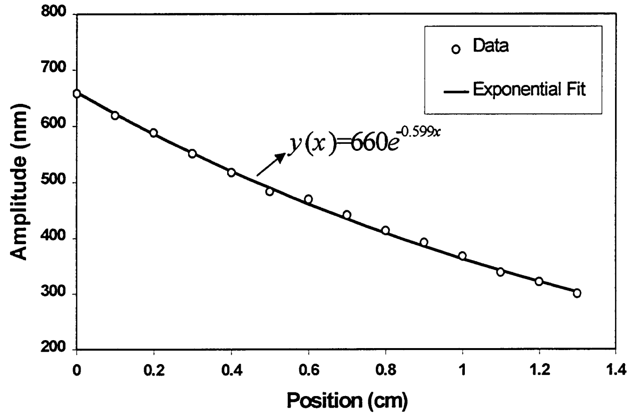

Figure 1: Wave amplitude vs. distance at 23° C

To obtain the attenuation data of Figure 1 two fiber interferometers are utilized, one stationary, the other movable. The amplitude data obtained by the movable probe is a function of position and is normalized by that obtained from the stationary probe to account for any changes in the equilibrium water level due to evaporation. While a small change in the water level has no effect on the detection system, it does affect the amplitude of the waves being generated under the blade. The normalization procedure outlined above eliminates this source of error. The solid graph in Figure 1 is an exponential fit to the data and gives the attenuation coefficient α for use in Eq.8

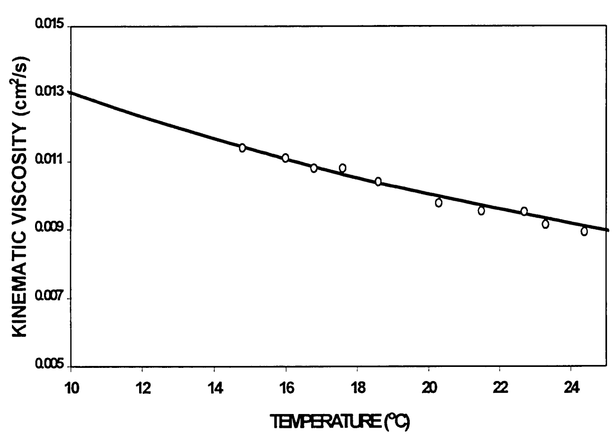

Figure 2: Kinematic viscosity vs. temperature for water

Figure 2 gives our measured values of the kinematic viscosity, η/ρ, vs. the temperature for pure water. The solid line is a second order polynomial fit to the published data (20) for pure water and is included for comparison. The water temperature was controlled by use of a thermoelectric unit in good thermal contact with the trough.

5 CONCLUSIONS

We present a new technique, based on laser interferometry, to obtain the wave amplitude of surface waves with a resolution of about ten nanometers-- some fifty times better than the resolution of a typical optical microscope. We use this technique to obtain the dispersion and attenuation of capillary waves on pure water as a test case. Furthermore, the attenuation data is used to obtain the viscosity of pure water as a function of temperature. The results are in excellent agreement with the most recent and reliable data obtained by traditional flow viscometry.

Since water has a very small viscosity to begin with, measuring the temperature variation of its viscosity constitutes a severe test of the sensitivity of our method. The excellent results presented in Figure 2 show that the noncontact method described here provides a sensitive new alternative to flow viscometry. Furthermore, the noncontact nature of the method provides another clear advantage by eliminating the possibility of contamination of the fluid under study.

In summary, we have described an elegant new technique that employs a miniature laser interferometer to measure the damping of capillary waves with unprecedented accuracy and without contact with the fluid, thus achieving what Stokes dreamed of more than a century ago. Furthermore, unlike the conventional flow viscometry, which is limited for use in Newtonian fluids, the new technique may be used to measure the viscosity of non-Newtonian fluids.

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