

# Modelling and Simulation of Mechanical, Thermal and Electrical Behaviour of Si Cantilever with Implanted Strain Gauge

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## ABSTRACT

In the paper there is characterized physical model of implanted strain gauges; various girder topologies are designed and basic technological steps during realization of cantilever are described. Simulation using CoventorWare (MEMCAD) program is used for verification of mechanical properties and temperature distribution in cantilever structures. At realized structures of cantilever with strain gauges, there have been measured basic parameters, as dependence of electric parameters of strain gauges on mechanical deformation, temperature dependence at different mechanical load, temperature stability of output parameters, temperature dependence of pn junctions in the structure. From measured data there have been calculated piezoresistive coefficients, coefficients of deformation sensitivity, linearity, hysteresis, temperature coefficients of resistance, etc. Based on measured data, there has been designed connection of a simple electric equivalent model of the structure.

**Keywords:** piezoresistivity, cantilever, strain gauge, implantation, stress

## 1 DESIGN OF STRAIN GAUGE

When designing cantilever topology we start from the equation for relative change of resistance of strain gauges strained in transverse direction  $\sigma_x$  and longitudinal direction  $\sigma_y$ . For crystallographic plane (100) we can write the following equations [1]

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \pi_{11}' \cdot \sigma_x \quad (1)$$

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = \pi_{12}' \cdot \sigma_x \quad (2)$$

It can be written for crystallographic plane (100)

$$\Delta R_1 = -\Delta R_2 = \Delta R_3 = -\Delta R_4 = \Delta R \quad (3)$$

When designing system topology, we start from the required value of strain gauge resistance. It holds

$$R_i = \frac{\bar{\rho} \cdot l_i}{w_i \cdot x_j} \quad (4)$$

where  $\bar{\rho}$  is mean value of resistivity of diffusion layer,  $l_i$  is length of i-th resistor,  $w_i$  is width of i-th resistor and  $x_j$  is depth of pn junction. For practical calculation, the quantity resistance on square  $R_{\square}$  can be used. Then it holds

$$R_{\square} = \frac{\bar{\rho}}{x_j} \quad \text{and} \quad R_i = R_{\square} \cdot \frac{l_i}{w_i} \quad (5)$$

Example of a designed strain gauge geometry with long fibre [1] for connection to full bridge is shown in Figure 1.

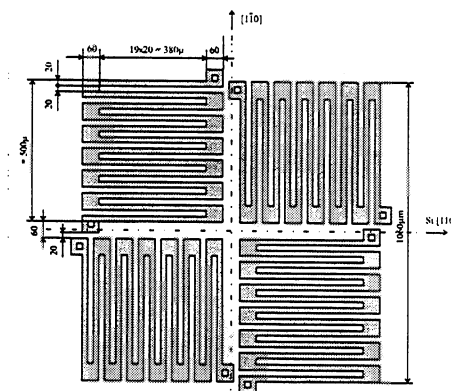


Figure 1: Geometry of strain gauge with long fibre for connection to full bridge.

Measured force is effective on the free end of the cantilever perpendicularly to its axis and causes its deformation. Strain gauges are located in the place where the maximum mechanical tension is. Due to piezoresistive phenomenon the maximum mechanical tension causes the greatest change of resistance.

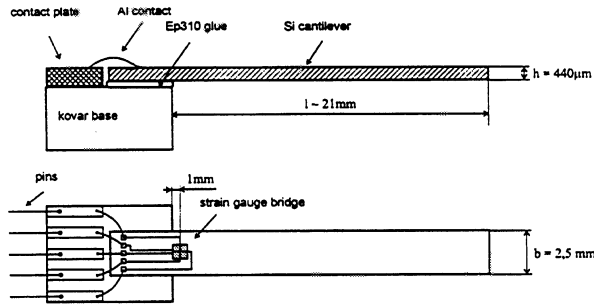


Figure 2: Geometry of Si cantilever with piezoresistors and fixation on kovar base.

Construction of cantilever fixed at one side has been chosen for maximum simplicity of production. Geometry of Si cantilever with piezoresistors and fixation on kovar base is shown in Figure 2. Length of the cantilever (to the point of fixation) is  $l=21\text{ mm}$ , width  $b=2.5\text{ mm}$  and thickness  $h=440\text{ }\mu\text{m}$ . Mechanical tension in place of implanted strain gauges is calculated from the equation [1]

$$\sigma_x = \frac{3E \cdot y_0 \cdot h \cdot x}{2l^3} = \frac{6F \cdot x}{b \cdot h^2} \quad (6)$$

where  $E$  is module of Si elasticity  $E=1.69 \cdot 10^{11} \text{ Nm}^{-2}$ ,  $y_0$  is cantilever bend,  $h$  is cantilever thickness,  $l$  is cantilever length and  $F$  is force effective on the free end of the cantilever. Maximum load and bend of the cantilever results from maximum allowed mechanical tension of Si  $\sigma_{DOV}$ . The value of  $\sigma_{DOV}$  has been found experimentally,  $\sigma_{DOV}=200\text{ MPa}$ . The equation (14) can be written as [1]

$$y_{\max} = \frac{2 \cdot \sigma_{DOV} \cdot l^2}{3 \cdot E \cdot h} \quad (7)$$

$$F_{\max} = \frac{\sigma_{DOV} \cdot b \cdot h^2}{6l} \quad (8)$$

$$m_{\max} = \frac{F_{\max}}{g} \quad (9)$$

where  $g$  is gravitational acceleration. After introducing numerical values into equations (7), (8) and (9) we get following numerical values for cantilever parameters:  $y_{\max}=0,713\text{ mm}$ ,  $F_{\max}=0,768\text{ N}$ ,  $m_{\max}=78\text{ g}$ .

Simple comparison of sensitivity of strain gauges with different concentrations of impurities can be done using coefficient of deformation sensitivity  $K_i$ . If the end of cantilever is loaded with weight  $m$ , the coefficient  $K$  can be written as

$$K_i = \frac{\Delta R_i}{R_i} \cdot \frac{E \cdot b \cdot h^2}{58,8 \cdot x} \cdot \frac{1}{m} \quad (10)$$

where  $R_i$  is resistance of strain gauge in undeformed state,  $\Delta R_i$  is change of resistance caused by deformation. At simplified design of Si cantilever we assume ideal fixation of Si cantilever to the base. We assume further that the

coefficients of thermal expansibility of Si and material of the base are identical [2]. However in reality this assumption does not hold, therefore we have to consider dilatation of the base to Si by value

$$\Delta l = l(\lambda_{\text{kovar}} - \lambda_{\text{Si}})\Delta T \quad (11)$$

where  $\lambda_{\text{kovar}}$  is thermal coefficient of longitudinal extension of kovar (material of the base),  $\lambda_{\text{Si}}$  is thermal coefficient of longitudinal extension of Si. The coefficient  $\lambda_{\text{kovar}}$  has been determined experimentally from measured values.

## 2 MODELLING AND SIMULATION OF STRUCTURES

The software package CoventorWare has been used for design of mechanical and thermal characteristics of the structure. The tools enable design, modelling and successive modification of designed MEMS structures. The program enables: drawing of 2D layout and its editing, simulation of production process, generation of 3D model from 2D masks, generation of network by the method of finite elements, solution of mechanical, electrostatic, thermal, piezoresistive, induction, optical, and further simulations.

The Coventor programme generates 3D model from 2D layout and from the file where production process is described in a simplified way. In our case, the layout consists of 3 main components, namely base, Si cantilever, and piezoresistive meanders. Generated model and detail of the piezoresistive system are shown in Figure 3.

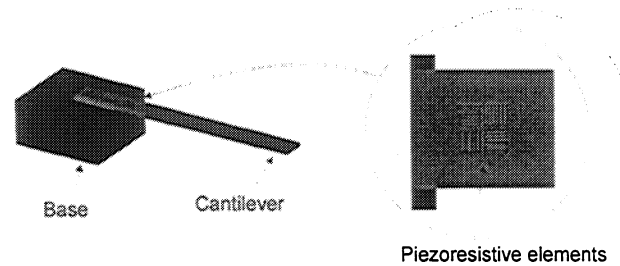


Figure 3: Model of Si cantilever with piezoresistive structure a) cantilever and base, b) detail of the piezoresistive structure.

### 2.1 Approach to simulation of structure

For successful simulation, it is necessary to input all material constants correctly. There have been realised following simulations.

**Mechanical simulation.** The simulation calculates bend of the cantilever and value of mechanical strain on the surface of the cantilever caused by this bend. The input values are coordinates and magnitude of the vector of effective force.

**Piezoresistive simulation.** The simulation calculates magnitude of voltage on meander at constant current in

dependence on cantilever deformation caused by effective force. The resistance of meander can be calculated from the voltage and known current. The resistance is proportional to surface tension. The surface tension has been acquired as a result of mechanical simulation. Inputs are piezoresistive coefficients and output files of mechanical simulation.

*Thermal simulation.* Results of this simulation are data on mechanical strain arising in connection of cantilever with base due to different thermal expansibility of material of base and Si at temperature changes. Input values are coefficients of thermal expansibility and environment temperature. For simulation it is assumed that the thickness of connecting layer is negligible in comparison with the thickness of Si cantilever.

## 2.2 Electric model

Electric model of piezoresistive cantilever is based on Wheatstone bridge. The model is designed in such a way that it covers temperature dependence of the resistor, change of resistance in dependence on load and influence of reverse voltage.

The influence of reverse voltage applies to model in the following way: The substrate diode that is spread under the total area of meanders is transformed to equivalent linear model of serial combination of voltage source and leaky resistor. Leaky resistor can be drawn in the bridge as resistance parallel to resistance of individual strain gauge. Each strain gauge in the bridge is characterised by the value of resistor  $R_0$  defined at zero load and room temperature, and further by dependence on deformation and temperature dependence according to relation

$$R_i = R_0 \left( 1 + \alpha_i \cdot \Delta T + K_i \cdot \frac{58,8 \cdot x \cdot m}{E \cdot b \cdot h^2} \right) \quad (12)$$

where the second term of addition in the parentheses expresses change of strain gauge resistance in dependence on temperature change and the third term of addition in the parentheses represents change of strain gauge resistance in dependence on deformation.

## 3 REACHED RESULTS

Si plates with (100) orientation, specific resistance and thickness have been used as basic material for realisation of cantilevers with implanted strain gauges. Standard technological steps have been used for sample production.

### 3.1 Simulation results

Results acquired from performed simulations show distribution of mechanical tension along x axis of the cantilever with acting force on the end of the cantilever as parameter – see Figure 4 (shown for one sample). The graph shows that the strain gauge bridge is located in the place of the greatest mechanical strain of the bridge.

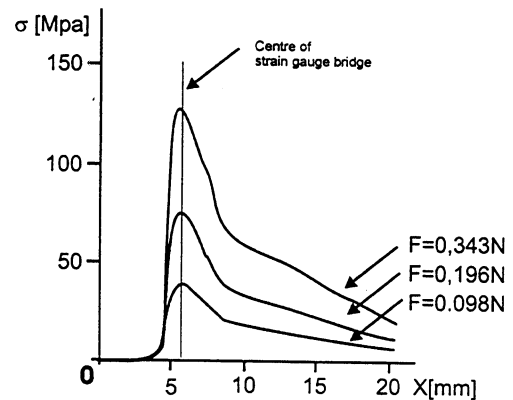


Figure 4: Simulation of distribution of mechanical tension in dependence on distance of fixation to base and acting force as parameter.

*Piezoresistive simulation* enables to calculate dependence of resistance for different concentrations of impurities.

For *thermal simulation* it is assumed that thickness of glue layer is negligible in comparison to thickness of Si cantilever. Further it is assumed that at room temperature ( $\sim 25^\circ\text{C}$ ) there is no mechanical strain in the fixation place. In

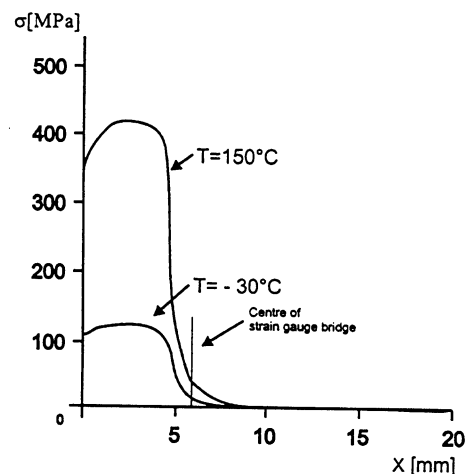


Figure 5: Simulation of distribution of mechanical tension in dependence on distance of fixation to base and temperature as parameter.

Figure 5 result of simulation of dependence of mechanical tension on distance in x axis with temperature as parameter ( $-30^\circ\text{C}$  and  $150^\circ\text{C}$ ) is shown. According to the measurement, the coefficient of temperature dilatation of kovar is  $\lambda_{\text{kovar}} = 17.8 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$  and that of silicon  $\lambda_{\text{Si}} = 2.5 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$ .

### 3.2 Measurement results

There have been measured a number of parameters of realised samples of cantilever structures with strain gauges in dependence on mechanical strain of cantilevers. Several temperature dependences have been measured as well.

From the measured values there have been calculated further parameters [3].

At mechanical strain, there have been measured values of strain gauge resistance and values of output voltage of bridge connection of strain gauges fed from the source of constant current.

Values of strain gauge resistance in dependence on temperature at zero and maximum load have been measured in temperature measurement. Temperature stability of bridge output voltage, temperature dependence of reverse voltage of pn junction and temperature dependence of capacity of pn junction have been measured as well.

From measured values there have been calculated values of piezoresistive coefficients, coefficients of deformation sensitivity, deviations from linearity, hysteresis and temperature coefficients of resistance. Reverse voltage and junction capacity of unfixed cantilevers have been measured as well.

Coefficients of deformation sensitivity were calculated according to the relation (10). Piezoresistive coefficients were calculated according to the relation (1).

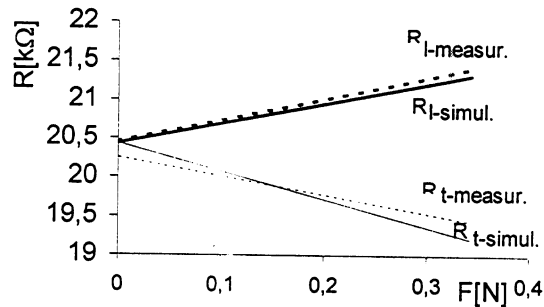


Figure 6: Comparison of measured and simulated values of strain gauge resistance for cantilever No.3 (20kΩ).

There is example of comparison of measured and simulated dependences of resistance of individual strain gauges on acting force for individual cantilevers in Figure 6.  $R_l$  denotes resistance in longitudinal direction of piezoresists and  $R_t$  denotes resistance in transverse direction.

Measurement and calculation of nonlinearity of cantilevers has followed from measured voltage on the bridge output in dependence on cantilever load.

Further, reverse voltage  $U_z$  on pn junction between implanted layers and Si substrate has been measured. Temperature has been chosen as parameter.

Temperature dependence of pn junction capacity for different cantilevers has been measured. Digital RCL meter has been used for measurement.

The last measurement has been measurement of temperature dilatation of kovar (material of base). The measurement has been done in temperature range  $80^\circ\text{F} \div 150^\circ\text{F}$ .

Cantilever	No. 1 (4kΩ)	No. 2 (4kΩ)	No. 3 (64kΩ)	No. 4 (20kΩ)
Sensitivity [mV/N]	311	314	952	262
Nonlinearity [%]	-0.454	-1.41	-1.04	-0.99
Hysteresis [%]	0.13	0,2	0,22	0,011
Temp. range [°C]	30÷140	30÷120	30÷100	30÷120

Table 1: Summary of basic parameters of some measured samples.

Table 1 shows summary of the most important measured and calculated parameters of some realised samples. Measured range is 0 to 0.35 N.

## 4 CONCLUSIONS

Structure of Si cantilever with piezoresistive strain gauges connected in bridge has been designed, simulated and realised. There have been performed mechanical and thermal simulations, measurement and calculation of basic parameters characterising properties of cantilever structure.

Measured and calculated characteristics of Si cantilevers with implanted strain gauges exhibit very good linearity, low hysteresis at load and very high sensitivity and moderate hysteresis at load. Strain gauges are determined for temperature range  $-30$  to  $+140$  °C. Simple equivalent model respects change of resistance in dependence on temperature and deformation, and undesirable influence of reverse voltage as well. CoventorWare simulator of COVENTOR Inc. can be used for design quality verification. Simulation results have corresponded well with measured values.

## 5 ACKNOWLEDGEMENT

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