Extensions of spring model approach for continuum-based topology optimization of compliant mechanisms.

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ABSTRACT

Spring Model approach for topology optimization of compliant mechanisms has been extended to include simultaneous multiple output formulation, taking into account the workpiece interaction at different output ports. Functional specifications including Mechanical Advantage, Geometric Advantage and Mechanical Efficiency are used as objective functions while material and behavior constraints are imposed. A new constraint is formulated to restrict the motion at the output port in a direction perpendicular to the desired motion direction. SIMP material interpolation scheme is used. A topology optimization software tool has been developed with ANSYS as the linear FE solver and globally convergent MMA as the optimization solver. Several optimal mechanism designs have been obtained, analyzed in ANSYS and some of them have been manufactured in microscale and tested.

Keywords: Topology Optimization, Compliant Mechanism, Spring Model, Optimal Design, Multiple Outputs

1 INTRODUCTION

Topology Optimization is an automated systematic design technique wherein best arrangement of limited amount of material in a specified domain is obtained for optimal mechanical performance of the device/structure. Structural Topology Optimization method solves the material distribution problem in a design domain under load and support conditions subject to resource and behavior constraints [2]. Originally intended for maximum stiffness design of elastic structures, Ananthasuresh et al. [1] extended this technique for systematic design of compliant mechanisms.

Compliant mechanism is a device that transfers or transforms motion, force or energy through the deflection of its flexible members instead of movable rigid links. The main advantage of compliant mechanisms is that part count is reduced due to elimination of joints and links which in turn leads to reduction in wear and tear, and need for lubrication. There is little or no assembly required and thus these are easier to manufacture. They show great potential in MicroElectroMechanical Systems (MEMS) applications which cannot have rigid links due to sub-millimeter scale and are manufactured via layered fabrication process.

In topology optimization of compliant mechanisms, two conflicting objectives, stiffness and flexibility, are optimized simultaneously. Thus, success of systematic design problem is heavily dependent on the problem formulation. Three models namely spring model, force-deflection model and multi-criteria model, have been proposed [1] to describe the design problem. Spring model allows for functional specifications to be used as objective functions [6, 8] and captures the interaction between compliant mechanism and the workpiece held at the output port.

Larsen et al. [5] designed topologically complicated mechanisms for multiple input and output ports assuming that only one of the output ports is loaded at a time. Frecker et al. [4] used multi-criteria optimization approach, which limits the objective functions to a combination of Strain energy and Mutual Potential energy, for multiple output requirements. However, to our knowledge, there is no literature available on simultaneous multiple outputs formulation using the spring model approach that takes into account the interaction between the workpieces at different output ports.

This work extends spring model approach to include multiple output formulation at arbitrary angles for a given actuation at input port. This formulation is required for the design of microsystems for parallel assembly. A new constraint is formulated to restrict the motion at the output port in the direction perpendicular to the desired motion direction to a user-specified tolerance.

This paper is divided into seven sections: section 2 describes the design problem, spring model approach for multiple outputs is detailed in section 3, section 4 explains calculation of objective functions and constraint values, sensitivities are derived in section 5, section 6 describes numerical implementation and section 7 shows results obtained using the formulations described.

2 DESIGN PROBLEM

A general formulation for topology optimization problem can be stated as follows:

$$\min f(\rho)$$

$$\text{s.t. } g_j(\rho) - g_j^* \leq 0 \qquad j = 1,\ldots\ldots,m$$

$$0.01 \leq \rho_i \leq 1 \qquad i = 1,\ldots\ldots,N \tag{1}$$

where f represents objective function, g_j and g_j^* represent j^{th} global constraint and its upper bound, ρ_i denotes i^{th} element's relative density, m is the number of constraints and N is the number of elements in finite element mesh.

3 SPRING MODEL APPROACH

The mechanical analysis for compliant mechanism with single output force using spring model approach is well documented in Sigmund [8]. For multiple output ports, the equations get modified (Fig. 1) as follows:

$$\alpha_i - \sum_{j=1}^{P} c_j \beta_{ij} = \Delta^i_{out} = \Delta^i_{gap} + \frac{c_i}{K_i} \tag{2}$$

$$\Delta_{11} - \sum_{i=1}^{P} c_i \gamma_i = \Delta_{in} \tag{3}$$

where α_i is displacement at i^{th} output port due to input force, β_{ij} is displacement at i^{th} output port due to force at output port j, Δ_{11} is displacement at input port due to input force, γ_i is displacement at input port due to force at i^{th} output port, c_i is scaling factor at i^{th} output port (unit dummy load technique), Δ^i_{gap} is gap between i^{th} output port and workpiece at that port, K_i is the workpiece stiffness, P is the number of output ports, Δ^i_{out} is the displacement at output port i and Δ_{in} is displacement at input port. All displacement components are along the desired motion direction at respective ports. The reader should note that linear elasticity has been assumed and hence the superposition of displacement vectors for different load cases resulting in Δ^i_{out} and Δ_{in}. Equation (1) is a linear set of equations that is solved simultaneously to give the values for c_i. This is done to account for the mutual interaction between the loadings at the output ports that are active simultaneously.

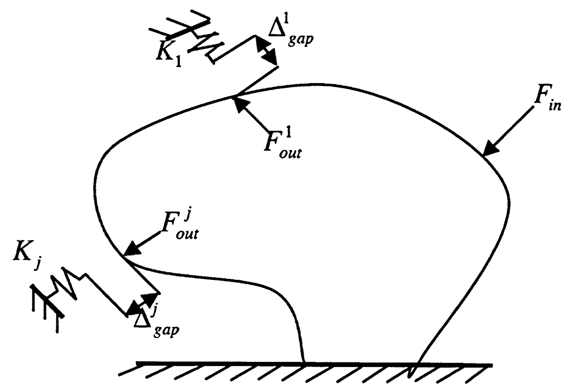

Figure 1: Multiple Output Ports

4 OBJECTIVE FUNCTIONS AND CONSTRAINT VALUES

The objective functions implemented in the software tool include Mechanical Advantage (MA), Geometric Advantage (GA), linear combination of MA and GA, and Mechanical Efficiency [6, 8]. The constraints are imposed on displacement at input port, Δ_{in} (3) and on displacement in direction perpendicular to the desired motion direction at the output port. The latter constraint is formulated as follows:

$$\alpha_{i,perp} - \sum_{j=1}^{P} c_j \beta_{ij,perp} = \Delta^i_{out,perp} \tag{4}$$

where labels $(*)_{,perp}$ have the same meanings as $(*)$ explained in Section 3 except that these are now along a direction perpendicular to the desired motion direction at their respective ports.

5 SENSITIVITY ANALYSIS

In this section, sensitivity of perpendicular direction constraint will be derived. For sensitivities of objective functions and other constraints, reader is referred to Sigmund [3].

Adjoint method is used to formulate sensitivity of $\Delta^i_{out,perp}$. For simplicity, sensitivity is derived for a single output load case. The same equations can be extended to include multiple output ports. From equation (4) (substituting $P = 1$, $i = j = 1$), we have:

$$\Delta^1_{out,perp} = \alpha_{1,perp} - c_1 \beta_{11,perp}$$

$$= \overline{A^T}\left(\overline{d_1} - c_1 \overline{d_2}\right) \tag{5}$$

where A is unit force vector at the output port in direction perpendicular to the desired motion desired and $\overline{d_1}$ and

$\overline{d_2}$ are displacement vectors corresponding to input $\overline{F_{in}}$ and output $\overline{F_{out}}$ $(\overline{F_{out}} \perp \overline{A})$ force vectors respectively. Now writing the Lagrangian function for $\Delta^1_{out,perp}$:

$$\begin{aligned} L &= \Delta^1_{out,perp} \\ &= \overline{A^T}\left(\overline{d_1} - c_1\overline{d_2}\right) - \overline{\lambda}[\underbrace{\overline{\overline{K}}(\overline{d_1} - c_1\overline{d_2}) - (\overline{F_{in}} - c_1\overline{F_{out}})}_{0}] \end{aligned} \tag{6}$$

where $\overline{\lambda}$ is Lagrange multiplier vector and $\overline{\overline{K}}$ is the global stiffness matrix. Differentiating (6) with respect to element relative density ρ_i, we have:

$$\left.\begin{aligned} \frac{\partial L}{\partial \rho_i} &= (\overline{A^T} - \overline{\lambda}\overline{\overline{K}})\left(\frac{\partial \overline{d_1}}{\partial \rho_i} - \frac{\partial c_1}{\partial \rho_i} - c_1\frac{\partial \overline{d_2}}{\partial \rho_i}\right) \\ &\quad -\overline{\lambda}\left(\frac{\partial \overline{\overline{K}}}{\partial \rho_i}\left(\overline{d_1} - c_1\overline{d_2}\right) + \frac{\partial c_1}{\partial \rho_i}\overline{F_{out}}\right) \end{aligned}\right\} \tag{7}$$

To avoid calculating expensive displacement vector gradients, coefficient of first term on RHS of (7) is equated to 0:

$$(\overline{A^T} - \overline{\lambda}\overline{\overline{K}}) = 0 \tag{8}$$

This equation is solved in commercial code ANSYS to find λ values which are substituted in second term of RHS of (7) to get sensitivity of $\Delta^1_{out,perp}$:

$$\frac{\partial L}{\partial \rho_i} = \frac{\partial \Delta^1_{out,perp}}{\partial \rho_i} = -\overline{\lambda}\left(\frac{\partial \overline{\overline{K}}}{\partial \rho_i}\left(\overline{d_1} - c_1\overline{d_2}\right) + \frac{\partial c_1}{\partial \rho_i}\overline{F_{out}}\right) \tag{9}$$

The sensitivities of $\overline{\overline{K}}$ and c_1 are derived in [8].

6 NUMERICAL IMPLEMENTATION

A software tool has been developed for continuum-based structural topology optimization of compliant mechanisms. The design domain is discretized using plane stress (with thickness) 4-noded elements. Commercial code ANSYS is customized and called as a subroutine for obtaining linear FEA solution. Globally convergent MMA [9] is used for obtaining design variable (ρ_i) solution in the optimization step. FEA, sensitivity analysis and optimization solution are done repetitively [8] till convergence is achieved.

To circumvent the problem of checkerboards and to achieve minimum feature size control, two techniques have been implemented:

1. Mesh independency heuristic filter on element sensitivities [8].
2. Constraining the slope of density [10].

To avoid the dependence of resulting topologies on the choice of optimization parameters and initial guess, continuation method [3] is implemented wherein the penalty value (for SIMP material interpolation model) is varied from 1.0 to 3.0 in increments of 0.1.

7 RESULTS

The topology optimization software has been validated using numerous test problems and the designs obtained are analyzed in ANSYS and validated against the results in literature. Here, results for multiple output formulation and perpendicular direction constraint are shown:

7.1 Multiple Outputs

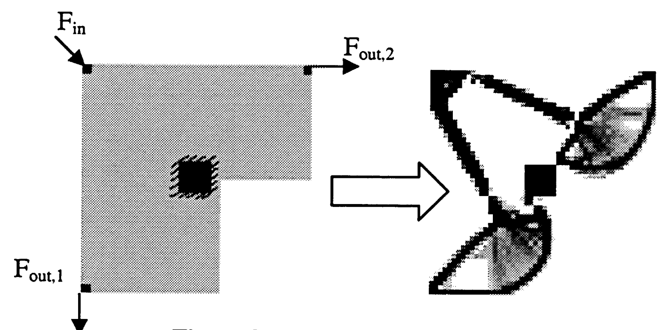

Figure 2: Two output ports example

Simultaneous multiple outputs formulation is demonstrated in Figure 2 wherein weighted sum of mechanical advantage (MA) at the two output ports is maximized under maximum and minimum input port displacement constraint. Mesh independency heuristic filter is used and problem seems to have converged after a total of 100 iterations. The MA value at port 1 is 0.233 and at port 2 is 0.21 and input port displacement is 7μms. The problem is not perfectly symmetric and hence the difference in MAs at output ports.

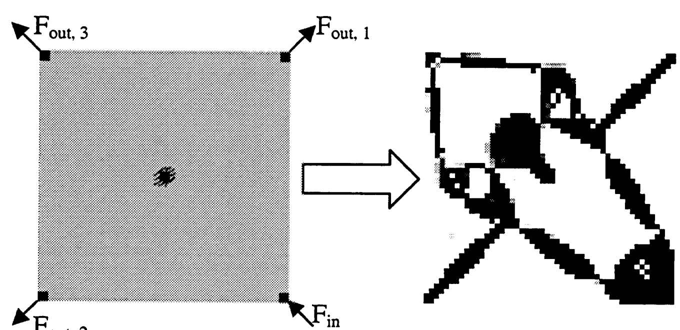

Figure 3: Three output ports example

In Figure 3 again, weighted sum of mechanical advantages at the three output ports is used as the objective function and input port displacement is constrained. The square box in the middle is fixed. The final value of objective function obtained is 0.2166 (at output ports 1 and 2, MA = 0.1772 and at port 3, MA=0.293) and input port displacement is 5μms.

7.2 Perpendicular motion constraint at output port:

Here, a typical microgripper design (Figure 4) is first shown without any constraint on motion in direction perpendicular to the desired motion direction at the output port and then the constraint is included.

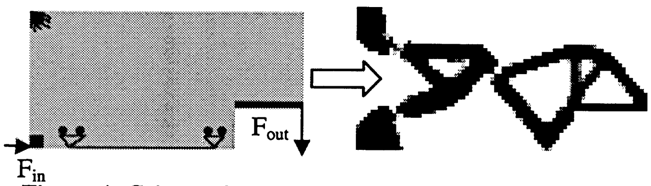

Figure 4: Gripper design without perpendicular constraint

Mechanical Efficiency was used as the objective function and the constraints imposed were maximum and minimum input port displacement, $MA \geq 0$ and $GA \geq 0$. The converged mechanical efficiency value was 0.831 and input port displacement was 6.45μms.

When perpendicular constraint is included, all other inputs remaining the same, following topology (Figure 5) is obtained:

Figure 5: Gripper design with perpendicular constraint

Here perpendicular motion is constrained to be between -1.5 and 1.5. The converged objective function value is 0.814, input port displacement is 6.2μms and displacement in perpendicular direction at output port is 1.499μms. This design compares very well with the design published in Pedersen et al. [7] obtained using path generating objective function and nonlinear FEA.

7.3 Experimental Validation:

Following gripper design was obtained with high stiffness workpiece at the output port in the direction perpendicular the desired motion direction. This gripper was fabricated using SOI DRIE etched process and measurements were done using automatic probe station. The displacement at output port in X-direction is 10μms and in Y-direction is -0.8μms as predicted by the topology optimization software.

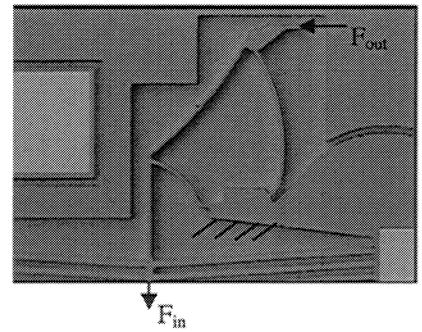

Figure 6: Optical microscope image of gripper

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REFERENCES

[1] G. K. Ananthasuresh, S. Kota, and Y. Gianchandani, "A methodical approach to the design of compliant micromechanisms." Solid-State Sensor and Actuator Workshop, 189-192, 1994

[2] M. P. Bendsoe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method." Comp. Meth. Appl. Mech. Eng., 71, 197-224, 1988.

[3] T. Buhl, C.B.W. Pedersen and O. Sigmund, "Stiffness design of geometrically nonlinear structures using topology optimization," Struct Multidisc Optim, 19, 93-104, 2000

[4] M. Frecker, N. Kikuchi and S. Kota, "Topology Optimization of Compliant Mechanisms with Multiple Outputs," accepted by Structural Optimization.

[5] U. D. Larsen, O. Sigmund, S. Bouwstra, "Design and Fabrication of Compliant Micromechanisms and Structures with Negative Poisson's Ratio," Journal of Microelectromechanical Systems, 6, 99-106, 1997.

[6] G. K. Lau, H. Du and M. K. Lim, "Use of functional specifications as objective functions in the topological optimization of compliant mechanism," Comput. Methods Appl. Mech. Engrg. 190, 4421-4433, 2001.

[7] C. B. W. Pedersen, T. Buhl and O. Sigmund, "Topology synthesis of large-displacement compliant mechanisms," Intl. Jour. Numr. Meth. Engg., 50, 2683-2705, 2001.

[8] O. Sigmund, "On the Design of Compliant Mechanisms using Topology Optimization," Danish Center for Applied Mathematics and Mechanics, Report 535, 1996.

[9] K. Svanberg, "A globally convergent version of MMA without linesearch," Struct Multidisc Optim, 9-15, 1995.

[10] M. Zhou, Y. K. Shyy and H. L. Thomas, "Checkerboard and minimum member size control in topology optimization", Struct. Multidisc. Optim., 21, 152-158, 2001