Modeling the Electro-Thermal Response of Thermally Isolated Micromachined Distributed Structures

Eran Socher*, Ofir Bochobza-Degani*, David Elata+ and Yael Nemirovsky*

*Department of Electrical Engineering, Technion – Israel Institute of Technology
socher@tx.technion.ac.il
+Department of Mechanical Engineering, Technion – Israel Institute of Technology

ABSTRACT

This work analyzes the electro-thermal behavior of thermally isolated micromachined resistive structures. Micromachined resistive microbridges are modeled using a quasi-1D heat balance equation taking into account the resistance dependence upon local temperature. A novel numerical scheme for solving the coupled electro-thermal problem is suggested. The scheme is based on applied temperature rather than applied current or voltage. The novel scheme can capture the thermal instabilities under both voltage and current control, for various types of materials. The model is compared with experimental data measured on surface micromachined polysilicon structures, showing good agreement.

Keywords: Electro-thermal modeling, thermal runaway, micro-electro-thermal devices, electro-thermal instability, MEMS.

1 INTRODUCTION

Joule heating is employed in a wide range of thermally isolated *MEMS* devices for sensing (e.g., gas-sensors [1]) or actuation [2] purposes. In some micro-sensors (e.g., microbolometers [3]) Joule heating is a byproduct of the sensing mechanism. The Joule heating in the microstructure is induced either by a voltage or a current source (Fig.1). The applied electrical power at any given point along the suspended structure depends on the local temperature. This is due to the temperature-dependent electrical resistivity of the structural material. For sufficiently low applied voltage (current), the generated electrical power is balanced by thermal conduction and the system reaches a stable steady state. Increasing the applied voltage (current) above a specific value may result in unbounded increase of the generated electrical power. In this case the temperature of the device is unbounded and no steady state is reached. The effect is similar to the *Pull-in* phenomena in electro-mechanical devices.

Due to the possible temperature loss of stability, it is important to accurately model the behavior and instability of thermally isolated *MEMS* to ensure proper design and operation.

Currently available commercial *CAD*-tools for *MEMS* iteratively solve the coupled electro-thermal problem assuming a fixed voltage (current) [4]. However, to the best of the authors knowledge many of these *CAD*-tools assume either fixed resistivity or linear models for temperature-dependent resistivity, thus they are unable to accurately capture the pphysical phenomena. Moreover, it was recently shown [5] that the convergence rate of the relaxation scheme used in common iterative solvers vanishes as the instability point is approached. This leads to inaccurate results.

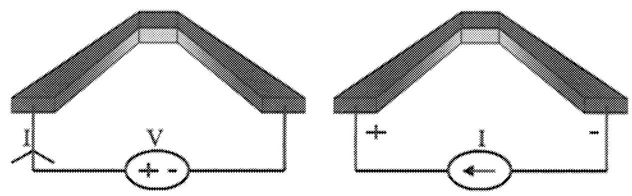

Figure 1: A suspended microstructure heated by either a voltage or a current source.

The novel scheme presented here is motivated by the stabilization technique used in the *DIPIE* scheme. The proposed solver fixes the temperature at a pre-chosen node on the structure rather than the applied voltage (current). By fixing the temperature, the unstable problem is replaced by a series of equivalent stable problems, similar to the *DIPIE* approach. This ensures the convergence of the numerical scheme at any steady state, stable or unstable.

2 ELECTRO-THERMAL MODELING

The modeling and novel numerical scheme methodology are demonstrated here for the case of a suspended resistive bridge using a distributed quasi-1D thermal balance equation:

$$\frac{d^2T}{dx^2} + r_T r_e(T) I^2 = 0 \qquad (1)$$
$$T(0) = T(L) = T_0$$

where T is the local temperature across the bridge, r_T is the thermal resistance of the bridge per unit length, r_e is the electrical resistance per unit length (which depends on the local temperature) and I is the uniform current in the bridge.

The electrical resistance dependence upon temperature is non-linear when considering a wide temperature range and depends upon the bridge material. In metals, the dependence is dominated by the mobility temperature dependence, which can be described using [6]:

$$r_e(T) = r_e(T_0)\left(\frac{T}{T_0}\right)^{\beta} \tag{2}$$

where β is a material dependent parameter.

In intrinsic semiconductors, such as vanadium oxide that is used in microbolometers, resistance temperature dependence is dominated by thermal generation of charge carriers. The resistance temperature dependence can be described as [6]:

$$r_e(T) = r_e(T_0)exp\left(\frac{\Delta E}{k_B T} - \frac{\Delta E}{k_B T_0}\right) \tag{3}$$

In doped semiconductors, both the charge carrier mobility and the charge carrier density depend upon temperature. Typically the resistance increases with temperature for medium temperatures and decreases at high temperatures when intrinsic carrier density exceeds the extrinsic one. The dependence in n-type silicon or polysilicon can be approximated by [6]:

$$r_e(T) = r_e(T_0)\left(\frac{T}{T_0}\right)^{\beta}\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \left(\frac{2n_i}{N_D}\right)^2}\right]$$
$$n_i(T) = n_i(T_0)\left(\frac{T}{T_0}\right)^{1.5} exp\left(\frac{E_g}{2k_B T_0} - \frac{E_g}{2k_B T}\right) \tag{4}$$
$$E_g(T) = 1.205 - 2.8 \cdot 10^{-4} T\,[eV]$$

3 NUMERICAL SCHEME

The temperature distribution across the bridge can be calculated by simultaneously solving Eq. 1 and one of Eq. 2, 3 or 4 depending on the bridge material. Existing relaxation schemes impose a bias current or voltage and iteratively solve the temperature distribution and the resistance distribution across the bridge. Conventional schemes suffer from convergence problems when approaching instability points due to the unbounded nature of the temperature. Moreover, for a given current (voltage) bias two solutions may be possible (one stable and one unstable) or no steady state solution at all. This is illustrated in Fig. 2, which shows the thermal power and the electrical power as functions of temperature for increasing bias current. As a result convergence to the right solution may require prior information and assumptions given to the solver.

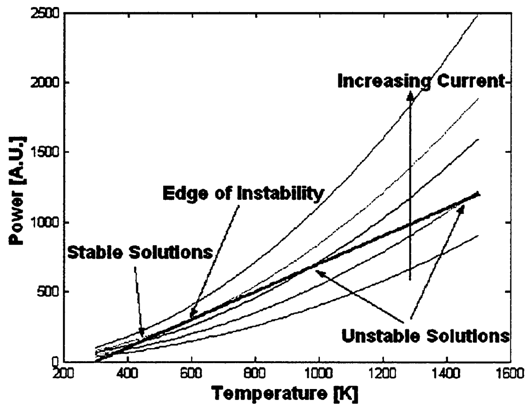

Figure 2: Thermal power (bold black) and electrical power for increasing currents vs. temperature

Our numerical approach is based on imposing the temperature rather than the electrical bias. The numerical solution uses the finite difference method. The solver first imposes the temperature at a specific node, e.g. the node of maximal temperature. Based on the boundary conditions and thermal diffusion an initial temperature distribution is found. Local electrical power is calculated according to the heat balance equation (Eq. 1). Based on the calculated local resistance a postulated "local" current is evaluated. In the numerical solution the applied current is estimated by the average of these "local" currents. The process is then iteratively repeated until the current converges (all local currents become equal) and thus the correct temperature distribution is reached.

Three types of microbridge materials were simulated using the novel scheme. Fig. 3 shows the normalized I-V curve of a metal type microbridge. The temperature increases along the curve, thus increasing the resistance and saturating the current. The point of maximum current is also the edge of stability, after which the solution is unstable. This means that for an applied current higher than the maximum, the structure will heat up uncontrollably and burn out.

Fig. 4 shows the normalized I-V curve of a narrow bandgap intrinsic semiconductor microbridge, such as vanadium oxide. Again the temperature increases along the curve, but now the resistance decreases, so the voltage saturates. For the case depicted in Fig. 4 the voltage is bounded but there is no maximum. Approaching the boundary voltage small changes in voltage causes a rapid increase in temperature thus risking thermal runaway in the microbridge.

Fig. 5 shows a normalized I-V curve of a doped semiconductor type microbridge. At medium temperatures resistance increases with temperature due to the decrease in mobility and the current saturates. However, when sufficiently high temperatures are reached, the increase in thermally generated charge carriers is more dominant and current increases rapidly, this time with a definite voltage maximum defining the thermal runaway instability point.

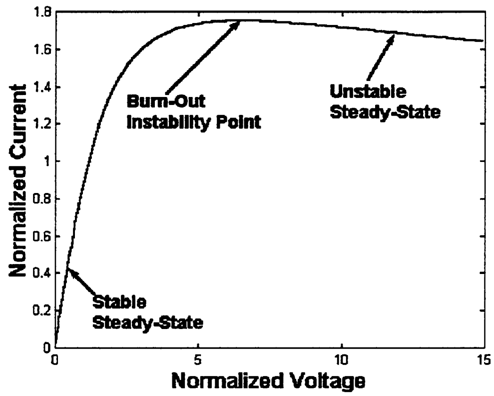

Figure 3: Normalized I-V curve of a metal type microbridge.

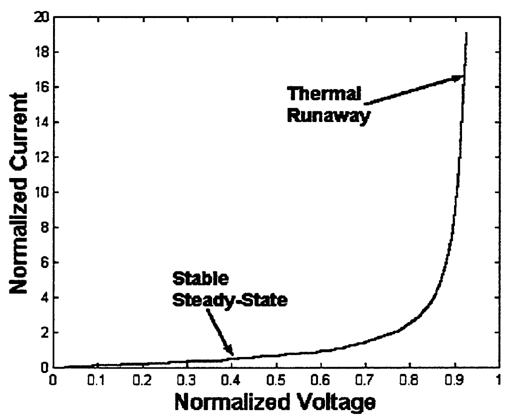

Figure 4: Normalized I-V curve of a narrow bandgap semiconductor type microbridge.

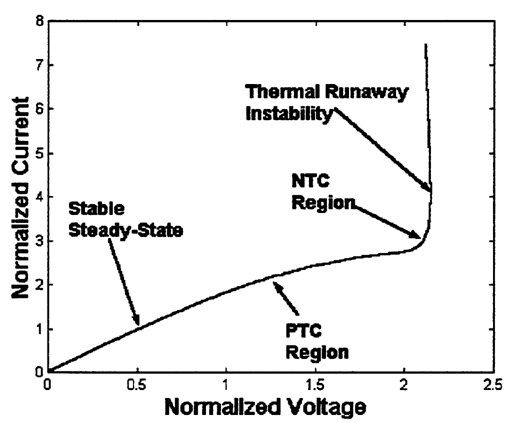

Figure 5: Normalized I-V curve of a doped semiconductor type microbridge

It can be seen that imposing the temperature instead of the current or the voltage allows accurately modeling both current and voltage instabilities. The metal type bridge material case was chosen to illustrate the convergence characteristics of the numerical scheme. Fig. 6 shows the decrease of the residual error with the number of iteration for three steady states: a stable state, the point of maximal current (the edge of instability) and an unstable state. Even the unstable state is modeled because the temperature is fixed and bounded. It can be seen that increased accuracy raises the number of iterations approximately logarithmically. The number of iterations increases only slightly when modeling instability compared with a stable state.

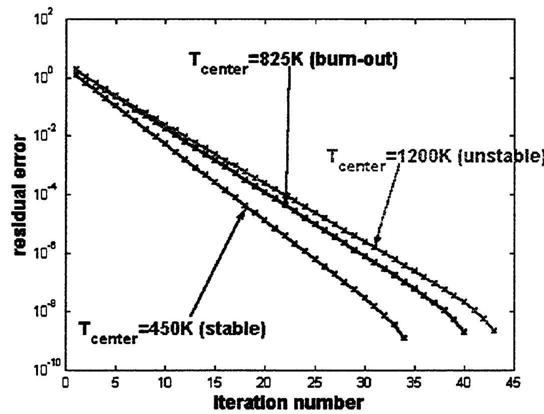

Figure 6: Convergence to various steady state solutions

Another important characteristic of the numerical scheme is the influence of the mesh refinement. The normalized temperature at the burn-out point is found with increasing accuracy for larger meshes. It can be seen in Fig. 7 that the convergence behavior is regular, allowing extrapolation of the result for an infinite mesh. Increasing the accuracy does not change this behavior and the result itself does not change by more than the defined accuracy. Fig. 8 shows the convergence to the critical current of the point of burn-out. The regular behavior makes it also possible to extrapolate the asymptotic value with respect to both the meshing and the accuracy.

4 COMPARISON WITH MEASUREMENT

In order to validate the numerical approach, the simulated results are compared with experimental measurements. To this end, suspended microbridges made of n-doped polysilicon were fabricated using surface micromachining. Fig. 9 shows a sample device with a microbolometer type suspended structure. The holes in the membrane are used to speed up the underetching of the underlying sacrificial silicon dioxide.

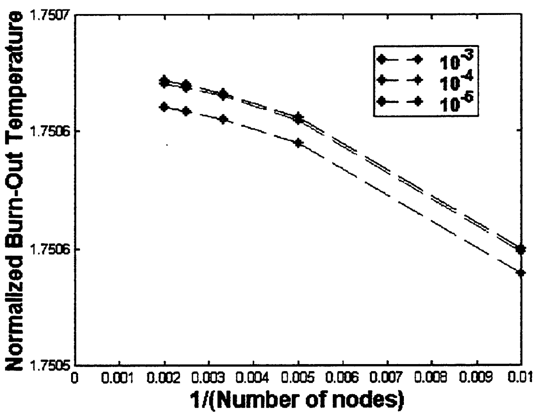

Figure 7: Convergence characteristics of burn-out temperature with mesh refinement

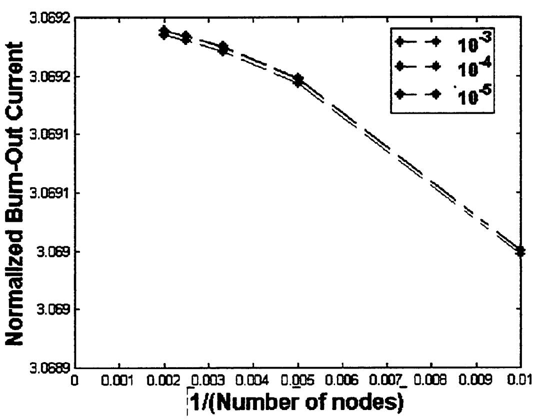

Figure 8: Convergence characteristics of burn-out current with mesh refinement

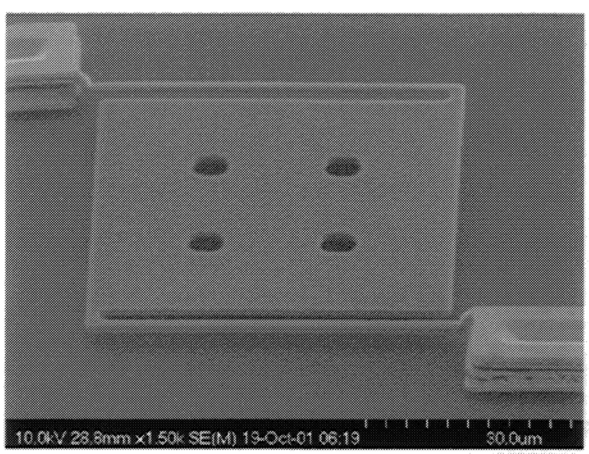

Figure 9: A micromachined highly doped polysilicon microbolometer.

The device is modeled as a quasi-1D distributed structure. Fig. 10 shows the measured results, and the simulation results with fitted parameters of temperature dependence. These fitted parameters are: doping density of $N_D=2.5*10^{19}$ cm^{-3}, thermal conductivity of 30 W/mK, mobility temperature power of -0.8 and thermal conductivity temperature power of 0.6. The simulation captures the two regions of operation – the positive temperature coefficient of resistance at lower temperature and the thermal runaway at higher temperatures. The device was indeed burnt-out when a higher voltage was applied.

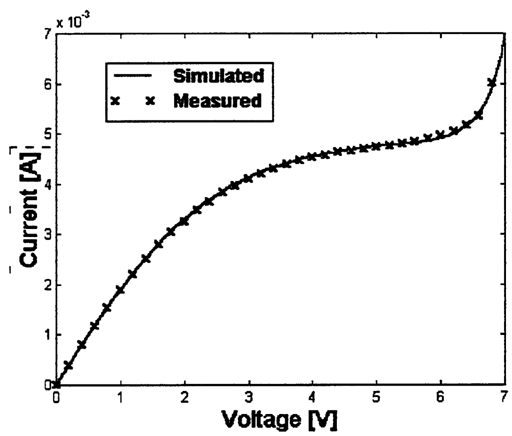

Figure 10: Measured vs. simulated I-V curves of a sample device.

5 SUMMARY

A quasi-1D model is suggested for suspended resistive microbridges. A novel numerical scheme is used for analyzing both stable and unstable steady states, especially the point of the edge of instability. The numerical approach is demonstrated for three cases of common types of resistive materials. The numerical scheme consistantly converges to both stable and unstale steady states either current or voltage bias. Good agreement was shown between measured and simulated results for surface micromachined doped polysilicon structures.

REFERENCES

[1] D. Briand et. al., JMEMS Vol. 9, 303-308, 2000.

[2] L. Que et. al., JMEMS Vol. 10, 247-254, 2001.

[3] E. Iborra et. al., JMEMS Vol. 11, 322-329, 2002.

[4] Coventorware, Coventor Inc., see www.coventor.com or Intellisuite, Intellisense Inc., see www.intellisense.com

[5] O. Bochobza-Degani et. al., MEMS2002, pp. 200-203.

[6] J. P. McKelvey, Solid State and Semiconductor Physics, Krieger Publishing, 1986.