An efficient Adaptive Single-Mode Pull-In extraction algorithm for computer aided design of electrostatic MEMS devices

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ABSTRACT

A novel method for extracting the Pull-In parameters of electrostatic MEMS devices is presented. This method is up to 3000 times faster than the prevalent Voltage Iteration method and up to 30 times faster than the recently presented Displacement Iteration scheme. In contrast to prevalent methods, when searching for the Pull-In state, the Adaptive Single Mode (*ASM*) method does not invest unnecessary computational effort to calculate other equilibrium states. To this end, a single-mode approximation of the actuator deformation is used to rapidly estimate the Pull-In parameters. Unlike common reduced order modeling approaches, increased accuracy is achieved not by additional predetermined approximation modes but rather by repeatedly adapting a single mode to better capture the deformation at the Pull-In state. To illustrate the method the clamped-clamped beam actuator is simulated.

Keywords: Pull-In, Electrostatic actuator, Numerical scheme, Adaptive Single Mode

1 INTRODUCTION

Electrostatic actuation is a prevalent means of driving microstructures and it can be found in most MEMS fields of application. A fundamentally important feature of electrostatic actuation is the inherent Pull-In instability that results from the nonlinear nature of electrostatic forces. In many applications Pull-In is utilized to achieve binary mode operation (e.g., TI-DMD™ [1]). In many other applications where a large travel range is required Pull-In should be avoided (e.g., scanning micromirror [2]).

In either case, it is crucial to accurately estimate the Pull-In parameters of the device in the design process. To achieve optimal design through parametric analysis highly efficient Pull-In modeling tools are essential.

To this end, many modeling methods for extracting the Pull-In parameters of electrostatic actuators have been proposed in literature [3-12], and specialized Pull-In extraction modules are available in commercial MEMS design tools. These methods can be divided into two main categories: local mode discretization approaches [3-8], and reduced-order models [9-11]. In both approaches, the Pull-In parameters can be extracted using either the prevalent Voltage-Iteration (VI) scheme [3-7], the pseudo-arclength continuation method [12], or the recently suggested Displacement-Iteration Pull-In Extraction (DIPIE) scheme [8]. Other extraction methods for single-mode reduced order models include: direct extraction [10] and effective suspension coefficient [11].

The discretization approaches are flexible and capable of handling actuators with general configurations. Typically, these methods yield accurate results for which the convergence of the computation can be readily verified by discretization refinements. However, these methods require extensive computational effort in terms of memory and run-time. In contrast, reduced order modeling approaches can rapidly extract the Pull-In parameters. However, the derivation of the approximation modes may require a considerable computational effort in itself. These methods are less flexible in the sense that each configuration has a different set of optimal approximation modes. Moreover, it is more difficult to verify convergence with respect to the number of modes and hence these methods may be less accurate. Among the reduced order methods, the single-mode approximations are the most rapid but the least accurate in extracting the Pull-In parameters.

In all the above mentioned methods, much numerical effort is invested in calculating the equilibrium states evenhough only the Pull-In state may be of interest.

In the present work a novel Adaptive Single Mode (ASM) method for extracting the Pull-In parameters of general electrostatic actuators is proposed. This method aims at directly approaching the Pull-In state with out calculating other equilibrium states of the actuator. In this sense the ASM method is computationally more efficient, as will be demonstrated in the following.

The ASM method uses a single mode approximation that is repeatedly corrected using a discretization scheme, to extract the Pull-In parameters. This is in contrast to common reduced order modeling approaches in which the approximation modes are predetermined prior to the analysis, and in which an increasing number of modes is required to improve the accuracy.

The novel ASM approach maintains the flexibility, accuracy and convergence of the discretization approaches but it is shown to be more efficient than the previously mentioned Pull-In extraction methods.

In the following section the algorithms of the VI, DIPIE and ASM methods are described. To illustrate the ASM method an example problem including stress-stiffening nonlinearity is solved in section 3. The performance and results of the ASM are compared with those obtained from the VI and DIPIE methods.

2 PULL-IN EXTRACTION ALGORITHM

2.1 VI and DIPIE algorithms

The equilibrium states (stable and unstable) of a general electrostatic actuator are presented schematically in Fig 1. The Pull-In state is the equilibrium state for which the applied voltage is maximal [8].

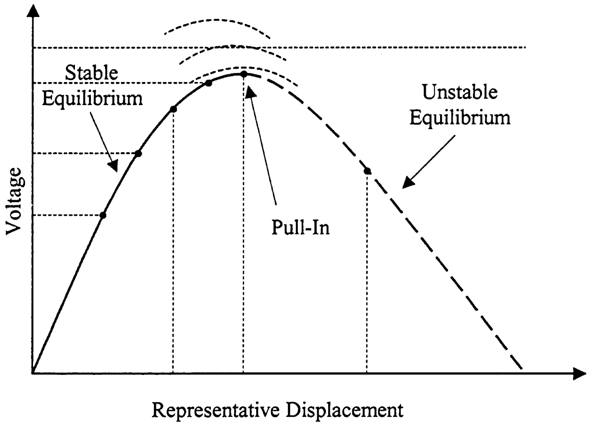


Figure 1: Schematic description of the equilibrium states of an electrostatic actuator. The horizontal and vertical dashed lines describe the search strategy of the *VI* and *DIPIE* schemes, respectively. The dashed arcs describe the *ASM* equilibrium curves for various single modes.

In the Voltage Iterations Pull-In extraction scheme [3-7] the applied voltage is iteratively modified between two limits, above and below the Pull-In voltage. For each applied voltage, convergence to the equilibrium state indicates a stable equilibrium and the lower limit is increased, whereas lack of convergence indicates non-equilibrium and the upper limit is decreased (horizontal dashed lines in Fig. 1). This calculation is repeated until the two limits are within the required accuracy.

In the recently proposed DIPIE scheme [8], the displacement of a pre-chosen node is iteratively modified within the range defined by the unloaded and collapsed states of the actuator (vertical dashed lines in Fig. 1). The deformation of the actuator is calculated by solving the voltage-free electromechanical equilibrium equations. This ensures that the reaction force at the pre-chosen node vanishes and that the voltage is uniform over the electrodes [8]. The voltage associated with this equilibrium is then calculated. Next, the deflection of the pre-chosen node associated with the maximum applied voltage is searched to extract the Pull-In state.

In both the VI and DIPIE schemes much numerical effort is invested in the calculation of many equilibrium states of the actuator while searching for the Pull-In state.

2.2 The ASM algorithm

The motivation behind the novel Adaptive Single Mode algorithm is to extract the Pull-In state of the actuator without investing unnecessary numerical effort to converge to any other equilibrium state.

In many electrostatic actuators the deformation mode at the Pull-In state is a good approximation of the deformation mode at equilibrium states in the vicinity of Pull-In. This is to say that the deformation mode changes smoothly around the Pull-In state.

This motivates the use of a reduced order model using a single-mode approximation of the deformation to extract the Pull-In parameters. For a given single-mode approximation the equilibrium curve near the Pull-In state is described by an arc in Fig. 1. If the single-mode approximation is corrected such that it approaches the Pull-In deformation mode, the equilibrium curve will approach the exact equilibrium curve in the vicinity of Pull-In.

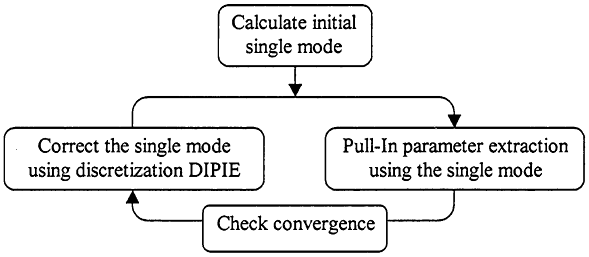


Figure 2: Flowchart of the Adaptive Single Mode algorithm

This adaptive single mode algorithm, described in Fig 2, is used to correct the single mode approximation to approach the Pull-In deformation. The initial single approximation mode is computed by solving the mechanical problem with a constant distributed force using any discretization scheme. This distributed force is the electrostatic force induced by applying a unit voltage to the unloaded actuator. Next, the Pull-In state is extracted using a reduced order model in which the deformation is proportional to this single mode. Finally, the DIPIE scheme is used to correct the single approximation mode. To this end, the deflection of a pre-chosen node is fixed at its previously calculated Pull-In value. The deformation is corrected to approach the equilibrium state of the actuator associated with the deflection of the pre-chosen node.

The last two steps are repeatedly executed such that each new mode further approaches the exact Pull-In deformation. The process is terminated when the Pull-In parameters converge within a given criterion.

Unlike the VI and DIPIE schemes, the ASM method does not attempt to solve the exact equilibrium states of the actuator. In this respect, the ASM is computationally more efficient as will be demonstrated in the example problems.

3 EXAMPLE PROBLEM

To demonstrate the performance of the ASM method, the Pull-In state of the clamped-clamped beam actuator is calculated. The equilibrium equation of the actuator shown in Fig. 3, is given by [8]

$$\frac{d^4\tilde{y}}{d\tilde{x}^4} - 12\frac{\sigma_r}{E}\left(\frac{L}{t}\right)^2 \frac{d^2\tilde{y}}{d\tilde{x}^2} - 6\frac{g^2}{t^2}\left[\int_0^1 \left(\frac{d\tilde{y}}{d\tilde{x}}\right)^2 d\tilde{x}\right]\frac{d^2\tilde{y}}{d\tilde{x}^2} =$$

$$= \begin{cases} \dfrac{1}{(1-\tilde{y})^2}\left(1+\dfrac{2g}{\pi W}(1-\tilde{y})\right)\tilde{V}^2 & \alpha < \tilde{x} < (1-\alpha) \\ 0 & else \end{cases}$$

where $\tilde{y} = \frac{y}{g}$, $\tilde{x} = \frac{x}{L}$ and $\tilde{V}^2 = \frac{\varepsilon_0 W L^4}{2g^3 E^* I} V^2$. Here x, y, g, L, and t are geometrical parameters shown in Fig. 3, W is the width of the beam, σ_r is the effective residual stress, ε_0 is the permitivity of free-space, I is the second moment of the beam cross-section and E^* is the effective elastic modulus.

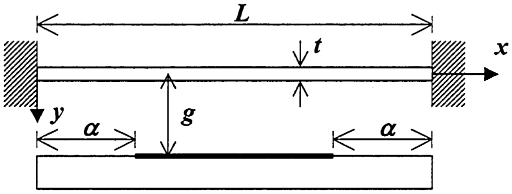
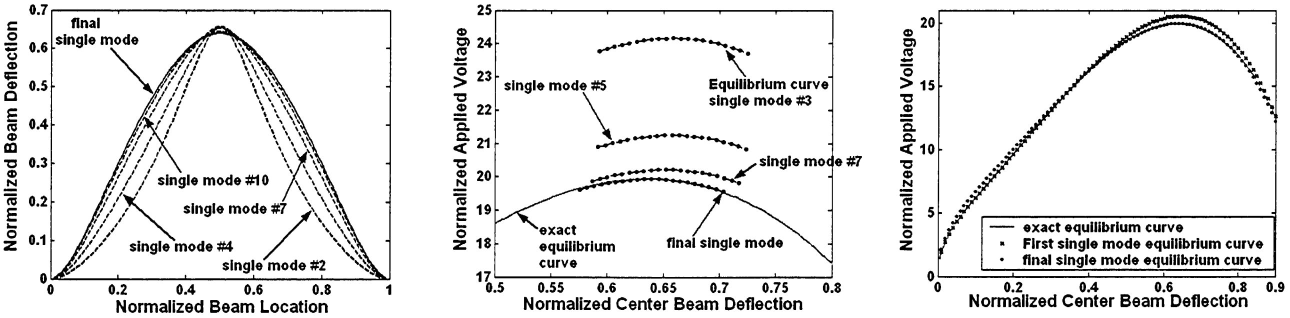


Figure 3: The clamped-clamped beam actuator.

The Pull-In parameters of the actuator were calculated using the ASM, DIPIE and VI methods, and the results are compared in the following. The solution process of the ASM method is illustrated in Fig. 4. Figure 4a describes the single-mode evolution during the solution process from the first to the final single-mode approximations. The equilibrium curves for selected single-mode approximation are shown in Fig. 4b and are compared to the exact equilibrium curve calculated by the DIPIE method. As can be seen the approximated equilibrium curves approach the exact curve as the mode is corrected. Figure 4c compares the first and last equilibrium curves with the exact curve. The equilibrium curve of the first single-mode approximation is a good estimate of the exact curve in the vicinity of the unloaded state but is inaccurate in the vicinity of Pull-In. On the other hand, the equilibrium curve of the last single-mode approximation is a good estimate of the exact curve in the vicinity of Pull-In but it is inaccurate for lower applied voltages. This demonstrates that the ASM aims to properly describe the equilibrium states in the vicinity of Pull-In with disregard of other equilibrium states.

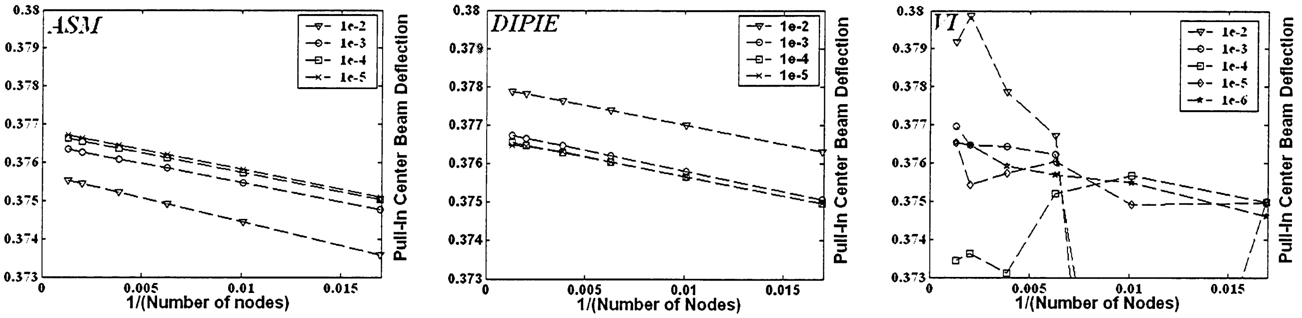
The convergence of the ASM method with refined accuracy and discretization is compared to the convergence of the DIPIE and VI schemes in Fig. 5. The ASM method shows consistent convergence similar to that of the DIPIE scheme, in contrast to the inconsistent behavior of the VI scheme [8]. This consistent behavior suggests that the Pull-In parameters can be well predicted using few coarse discretizations. To examine the accuracy of the extracted Pull-In parameters of the ASM method, a parametric analysis with increasing stress-stiffening nonlinearity is conducted. Figure 6 compares the Pull-In parameters extracted by the ASM method (solid lines) to those extracted by the DIPIE scheme ('x' marks). The difference in the extracted Pull-In voltage and center beam deflection is below 0.1% and 1%, respectively. For moderate stress-stiffening (g/t<5) the difference in the Pull-In center beam deflection is below 0.1%.

The significant advantage of the ASM method is evident in Fig. 7, where the run time required for Pull-In parameters extraction is compared with the DIPIE and VI schemes. The ASM method is 10÷30 times faster than the DIPIE method and 200÷3000 times faster than the VI method.



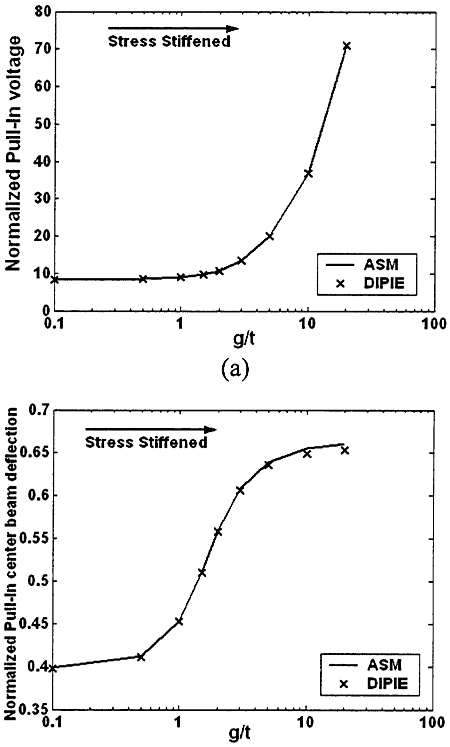
(a) (b) (c)

Figure 4: The solution process of the ASM method. (a) Evolution of the single-mode approximation. (b) Equilibrium curves for selected single-mode approximation. The exact equilibrium curve is calculated by the DIPIE method. (c) Comparison of the equilibrium curve based on the first and last single mode approximations.



(a) (b) (c)

Figure 5: Convergence of the Pull-In deformation with mesh and accuracy refinement – comparison between the ASM method and the DIPIE and VI schemes [8].



(b)

Figure 6: The Pull-In parameters for a clamped-clamped beam actuator with increasing stress stiffening.

4 SUMMARY

A novel Adaptive Single Mode method for extracting the Pull-In parameters of electrostatic actuators was presented. The method aims to directly find the Pull-In state without investing unnecessary numerical effort to reconstruct other equilibrium states of the actuator, and is therefore computationally efficient. In solving the clamped-clamped beam actuator with considerable stress stiffening nonlinearity, the ASM is 200÷3000 times faster than the conventional VI method.

The Pull-In parameters extracted by the ASM method agree well with those extracted by the DIPIE simulations for increasing stress stiffening. In the ASM module that searches for the Pull-In state, the proportion between the N degrees of freedom is fixed defining a deformation mode shape. The slight deviation in the results of the ASM emanates from neglecting the mode shape variations in the vicinity of Pull-In. Nevertheless, the small deviation (<0.1% In the Pull-In voltage) even in highly nonlinear stress stiffening cases indicates that this assumption is reasonable. However, this slight error can be eliminated by performing additional DIPIE calculations using the final results of the ASM calculation as initial conditions. With this error elimination calculation, the ASM method is still more efficient than the full DIPIE calculation.

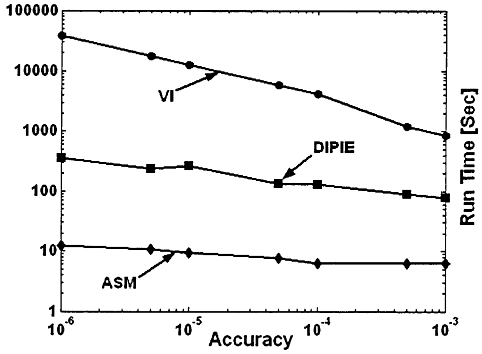


Figure 7: Required *CPU* time for extracting the Pull-In parameters with increasing accuracy

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