Tilt-Angle Stabilization of Electrostatically Actuated Micromechanical Mirrors

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Abstract

Recently proposed optical subsystems utilizing microelectromechanical system (MEMS) components are being developed for use in optical crossconnects, add-drop multiplexers, and spectral equalizers. Common elements to these subsystems are electrostatically actuated micromechanical tilt mirrors that steer optical beams to implement the subsystem functions. In the past, feedback control methods are commonly used to obtain precise mirror orientations to minimize loss through optical switch fabrics or to stabilize attenuation through spectral equalizers. However, the usual feedback schemes do not counteract the pull-in effect and the mirror tilt angle range is limited because of inherent instability beyond a critical tilt angle (pull-in angle). In this paper, we introduce a closed-loop feedback-control method to overcome the pull-in instability of electrostatically actuated micromirrors. A voltage slightly larger than the pull-in voltage is first applied when the mirror is at zero position, and the voltage is then linearly reduced as the mirror approaches the desired position. Experimental measurements, showing that tilting angle beyond the pull-in point can be achieved, are in good agreement with theoretical analysis.

Keywords: MEMS mirror, feedback control, electrostatic pull-in effect.

1 Introduction

MEMS technology has over the past few years been widely and vigorously probed for possible application in lightwave communication systems. Various MEMS components and subsystems, such as tunable lasers and filters, reconfigurable wavelength-add-drop multiplexers, dynamically gain-equalizers, tunable chromatic dispersion compensators, polarization controllers, and optical cross connects have been demonstrated. One of the most commonly used devices in these applications is electrostatically actuated micromirrors. The use of electrostatic actuation for tilting micromirors is attractive because of the high energy densities and large forces available in microscale. The mirror tilting angle is controlled by a balance between an electrostatic attractive torque and a mechanical resorting torque. In static equilibrium, the electrostatic torque and the mechanical torque are equal, resulting in a stable condition of the mirror. As the voltage is increased, the electrostatic torque increases and eventually overcomes the mechanical torque, resulting in instability or a collapse condition, where a contact between the mirror electrodes and the substrate is formed. Such a "pull-in" phenomenon prevents mirror electrodes from being stably positioned over a large tilting range.

A method to extend the mirror's stable tilting range is highly desirable. Such a method will increase the tunable range of tunable lasers and filters, optical attenuators, and dynamic gain-equalizers [1, 2], etc. This method will also enable the design of large-port-count (and thus high capacity) and highly scalable optical cross-connects which have emerged as a critical network element for constructing next-generation mesh-based optical networks [3]. In this paper, we introduce a closed-loop feedback-control method to enable operation of electrostatic micromirrors beyond the critical pull-in angle. A voltage slightly larger than the pull-in voltage is first applied when the mirror is at zero position, and the voltage is then linearly reduced as the mirror approaches the desired position. Arbitrary tilting angles within device geometry constraint can be achieved with this method. Experimental measurements, showing that tilting angle beyond the pull-in-point can be achieved, are in good agreement with theoretical analysis.

2 The Pull-In Problem

In this section, we first briefly discuss the pull-in effect. We then present a feedback-control method for stabilizing the mirror beyond the pull-in angle.

2.1 Pull-in Analysis

Fig. 1 shows a schematic view of an electrostatic rectangular torsion mirror including the movable plate, the stationary electrode, and the needed parameters for the pull-in analysis. Before pull-in, the mechanical torque $M_r = K_\alpha \alpha$ (K_α is the mechanical spring constant) equals the electrostatic torque M_e. M_e can be calculated as [4]

$$M_e = \frac{\varepsilon_0 V^2 b}{2\alpha^2}\left[\frac{1}{1-\frac{\beta\alpha}{\alpha_{max}}} - \frac{1}{1-\frac{\gamma\alpha}{\alpha_{max}}} + \ln\left(\frac{\alpha_{max}-\beta\alpha}{\alpha_{max}-\gamma\alpha}\right)\right] \quad (1)$$

where V and ε_0 are the applied voltage and the dielectric constant of air, respectively. The maximum constrained tilt angle $\alpha_{max}=d/a_3$, the electrode length ratio $\beta=a_2/a_3$, and the reduced electrode edge location $\gamma=a_1/a_3$.

In typical designs, a_1 is much smaller than a_2 or a_3, thus $\gamma \approx 0$. At the pull-in point, the mechanical spring constant K_α is equal to the electrostatic spring constant (i.e., $\mathrm{d}M_e/\mathrm{d}\alpha$). Thus differenting (1) with respect to α, multiplying by α, and substrating (1) yields [5],

$$\frac{1}{1-\beta\theta_{pin}} - 1 + \ln(1-\beta\theta_{pin}) - \frac{\beta\theta_{pin}}{3(1-\beta\theta_{pin})^2} + \frac{\beta\theta_{pin}}{3(1-\beta\theta_{pin})} = 0 \quad (2)$$

where θ is the fractional deflection of the rectangular plate α/α_{max}, and θ_{pin} is the fractional deflection at pull-in.

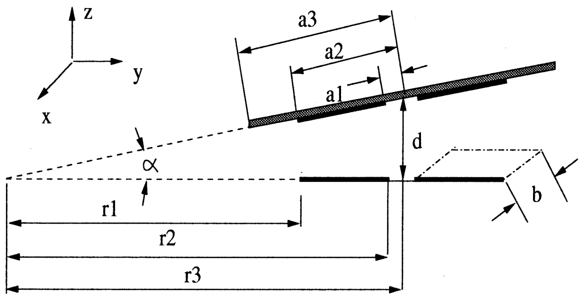

Figure 1: A schematic diagram of an electrostatic torsion mirror where a_1 is the distance between the rotation axis to the nearest edge of the fixed electrode, a_2 is the distance to the end of the electrode, a_3 is the distance to the end of movable plate, b is the electrode width, d is the vertical separation distance and α is the tilting angle. $a_3 = 100\mu m$, $a_2 = 88.08\mu m$, $a_1 = 0\mu m$, $b = 300\mu m$, $d = 10.4\mu m$, and $K_\alpha = 1.95 \times 10^{-8}$Nm/rad.

Solving equation (2) yields [5]

$$\beta\theta_{pin} \approx 0.4404. \tag{3}$$

When the electrode length ratio β=1, $\theta_{pin} \approx$ 0.4404, which means that the full plate electrode design gives about 44.04% travel range.

The pull-in effect can be also graphically explained. In Fig. 2, the electrostatic torque under different applied voltages as well as the mechanical torque are plotted. As can be seen from the figure, for sufficiently low voltages, the angle of the mirror exhibits two equilibrium positions, where only the lower tilting angle equilibrium position is stable and the other is unstable. For a certain voltage, the two equilibrium positions intersect at a single unstable equilibrium position. Such a voltage is called the pull-in voltage and the corresponding angle is called the pull-in (snap-down) angle. For voltages above the pull-in voltage, the electrostatic torque is greater than the mechanical torque for any angle, and thus there are no equilibrium positions at all. For the device shown in Fig. 2, $V_{pin} = 100$V, $\alpha_{pin} = 2.98^o$, and $\alpha_{max} = 5.96^o$.

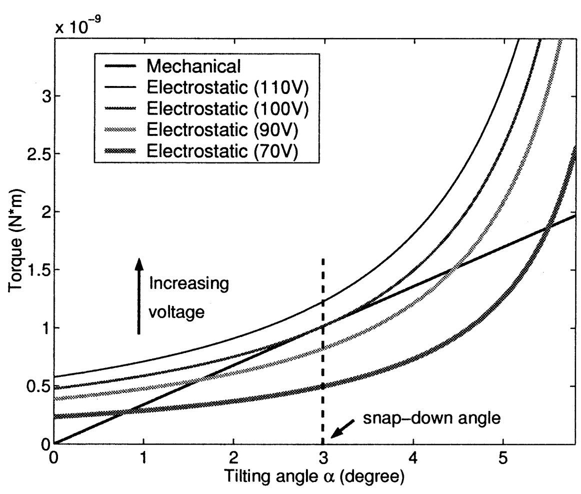

Figure 2: Electrostatic and mechanical torques as a function of mirror tilt angle. The electrostatic torque is shown for different applied voltage.

2.2 Tilt-angle Stabilization through Feedback Control

To overcome the pull-in effect and thus to obtain stable mirror positions at large angles, the shape of the electrostatic torque curve must be altered. First, the electrostatic torque curve needs to have an intersection with the mechanical restoring torque curve at an angle of α_{tilt} larger than the pull-in angle α_{pin}. Secondly, the electrostatic torque curve must intersect with the mechanical restoring torque curve from the side of upper half-plane, meaning that the electrostatic torque needs to be smaller than the mechanical resorting torque when α exceeds α_{pin}. The second requirement ensures that the equilibrium position α_{tilt} is a stable equilibrium position. By doing so, arbitrary tilting angles within the device geometry constraint can be achieved.

The simplest way to satisfy these two requirements is to choose a constant electrostatic torque $M_e(\alpha) = K_\alpha \alpha_{tilt}$ as shown by the dotted line in Fig. 3. The constant electrostatic torque curve intersects with the mechanical resorting torque curve at a single stable position of α_{tilt}. The voltage that is required to maintain such a constant electrostatic torque can be derived by substituting $M_e(\alpha) = K_\alpha \alpha_{tilt}$ into Eq. (1) and solving for V. Note that instead of a constant voltage, the voltage applied to the mirror device needs to be time-dependent and is a function of the tilting angle α. The dotted line in Fig. 5 shows the actual voltage shape. A constant electrostatic torque curve in principle is simple. However as can be seen from the figure, with a constant electrostatic torque, the voltage that is required when the mirror tilts at smaller angles is much larger than the pull-in voltage.

To overcome this high voltage requirement, an electrostatic torque curve as shown by the dash-dot line in Fig. 3 is proposed. The plot implies that a voltage V_{inti} which is slightly above the pull-in voltage is first applied, then as the mirror angle increases, the voltage is reduced to achieve the desired stable equilibrium position. The voltage applied to the mirror device is given by

$$V = \begin{cases} V_{inti} & \text{if } \alpha \leq \alpha_{cross} \\ \sqrt{\dfrac{K_\alpha \alpha_{tilt}}{\frac{\epsilon_0 b}{2\alpha^2}\left[\frac{1}{1-\frac{\beta\alpha}{\alpha_{max}}} - \frac{1}{1-\frac{\gamma\alpha}{\alpha_{max}}} + \ln\left(\frac{\alpha_{max}-\beta\alpha}{\alpha_{max}-\gamma\alpha}\right)\right]}} & \text{if } \alpha \geq \alpha_{cross}. \end{cases} \tag{4}$$

In Eq. (4), α_{cross} is the angle where the electrostatic torque corresponding to a constant voltage V_{inti} equals $K_\alpha \alpha_{tilt}$. α_{cross} can be determined by solving Eq. (1) for α with $V = V_{inti}$ and $M_e = K_\alpha \alpha_{tilt}$. The dash-dot line in Fig. 5 shows the actual voltage shape for generating such an electrostatic torque curve (i.e., the dash-dot line in Fig. 3).

Due to the nonlinear relationship between the electrostatic torque and the applied voltage, in practice, it is often difficult to generate the ideal straight-line electrostatic torque shape. To overcome this, the electrostatic torque is taken to be the form of the dashed line in Fig. 3, which corresponds to applying voltage by linear law as

$$V = \begin{cases} V_{inti} & \text{if } \alpha \leq \alpha_{cross} \\ V_{inti} - k(\alpha - \alpha_{cross}) & \text{if } \alpha \geq \alpha_{cross}. \end{cases} \tag{5}$$

Significant differences between this shape and the ideal shape are expected only at positions far from the stable position.

The mirror itself is strongly damped so oscillations near the stable position will be small thus differences between the torque shapes can be neglected. The dashed line in Fig. 5 shows the voltage shape corresponding to stabilizing the mirror at α_{tilt} by using the linear law. A constant voltage V_{inti} which is slightly larger than the pull-in voltage is first applied when the mirror is at zero deflection, and the voltage is then linearly reduced as the mirror approaches the desired stable position. This is equivalent to artificially create an energy potential well to trap the mirror in a stable deflection angle. In the example simulation (Fig. 5), $V_{inti} = 110$V is chosen, and the corresponding $\alpha_{cross} = 3.27^o$. The slope of the voltage in the linear region is 20.2V/degree, and α_{tilt} is chosen as 4^o. A family of similar curves can be generated for a range of stable positions. And with a linear voltage law, the feedback-control circuitry can be easily implemented.

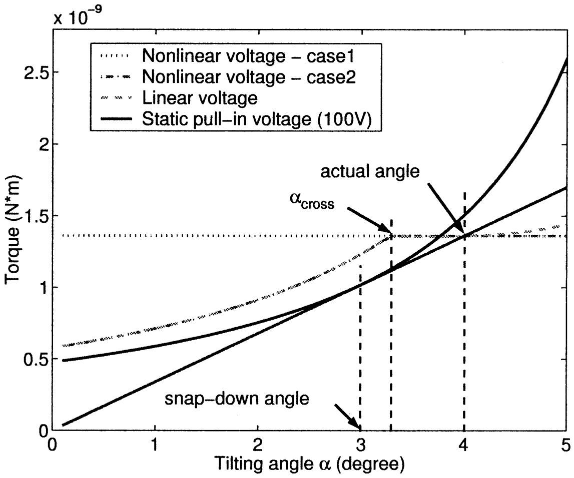

Figure 3: Electrostatic torque shapes applied to the mirror device corresponding to tilting the mirror at a specific angle (4^o).

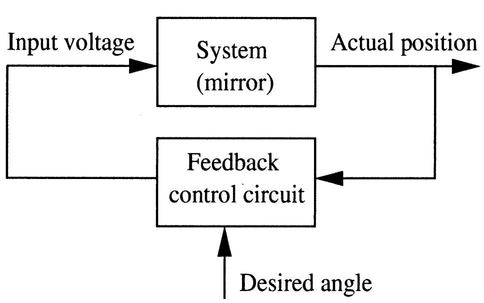

Figure 4: A schematic view for stabilizing the mirror through feedback control.

3 Dynamical Analysis of the System with Feedback Control

In addition to the voltage shape and static tilting angle characteristics, it is critically important to study the mirror dynamic tilting behaviors including overshoot, settling time, etc. For example, one important design criteria is to ensure that the mirror overshoot is within the device geometry tilting constraint α_{max}.

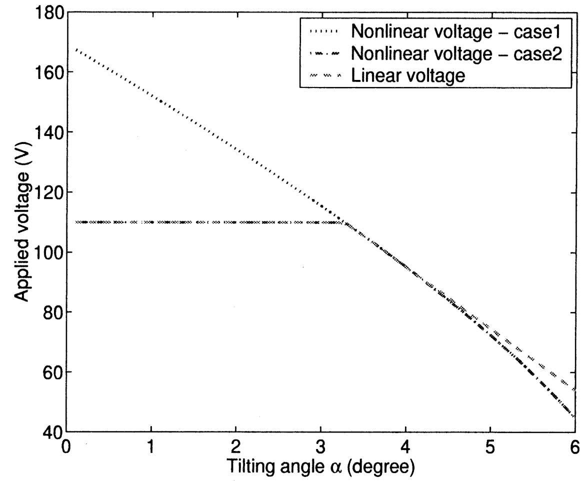

Figure 5: Voltage shapes corresponding to the electrostatic torque curves in Fig. 3.

The mirror dynamical titling behavior can be capture by the following equation:

$$I\ddot{\alpha} + M_d + K_\alpha \alpha = M_e \tag{6}$$

where I is the polar moment of inertia. $I = \frac{m}{12}(b^2 + t^2)$, and m, b and t are the top plate mass, width and thickness, respectively. M_e is the electrostatic torque given by Eq. (1) and M_d is the squeeze-film damping torque.

The damping is mainly from the air that is squeezed underneath the mirror, in the small gap between the tilting top plate and the bottom electrode. The squeeze-film damping torque M_d results from the pressure variation over the mirror surface. The equation governing the pressure distribution in the squeezed film is the nonlinear isothermal Reynold's equation [6]:

$$\nabla \cdot (h^3 p \nabla p) = \frac{12\mu}{1+6K} \frac{\partial(ph)}{\partial t} \tag{7}$$

where $p = p(x, y, t)$ is the gas pressure underneath the mirror; x and y denote the spatial coordinates; h is the squeeze-film thickness and $h = d + x\sin\alpha$; air viscosity $\mu = 1.82$ x $10^{-5} kg/(m \cdot s)$; Kundsen's number $K = \lambda/h$; and the ambient air pressure $p_a = 1.013\times10^5 Pa$. The mean-free path of air $\lambda = 0.064\mu m$.

A finite-difference method is used to solve (6-7). The system begins at α=$\dot{\alpha}$=0. Fig. 6 shows the simulated mirror dynamic tilting responses when driving the mirror with the three different actuation voltages given in Fig. 5. In all the cases, the mirror is stabilized at the desired angle ($\alpha_{tilt} = 4^o$) and the overshoots are below the device geometry constraint. As expected, the transient response corresponding to the constant electrostatic torque (the dotted line in Fig. 3) has the largest overshoot; and the transient responses corresponding to the dash-dot and dashed electrostatic torques in Fig. 3 are almost identical showing the effectiveness of applying voltage by the linear law.

4 Experimental Measurements

The mirror stabilization techniques were tested experimentally using a position-sensing detector (PSD) to monitor the mirror position (tilt angle) and a simple analog

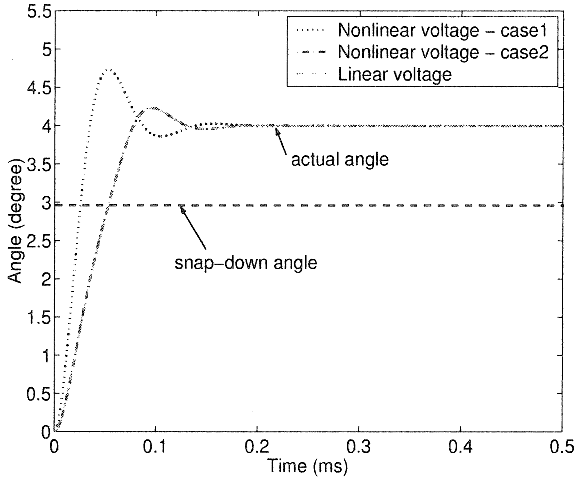

Figure 6: Dynamic tilting responses corresponding the different electrostatic torques in Fig. 3.

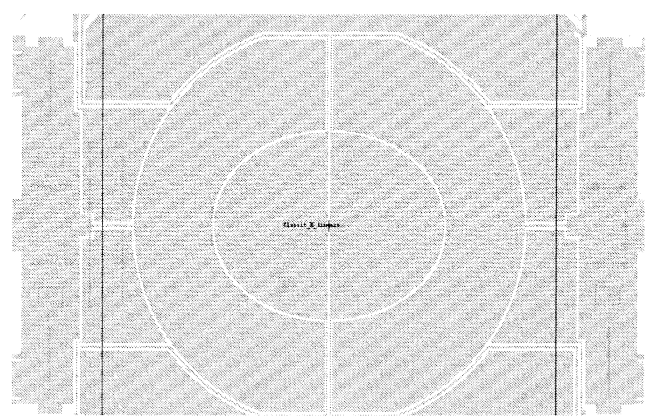

Figure 7: A fabricated circular mirror device. Mirror radius is $250\mu m$, mirror mass is $1.9\mu g$. The resonance frequency of the mirror is 800Hz. Initial gap is $50\mu m$. Snap-down angle is 7.5^o. The electrodes are half ellipses. The radii of the ellipse are $150\mu m$ and $120\mu m$ where $150\mu m$ is in the mirror tilt direction.

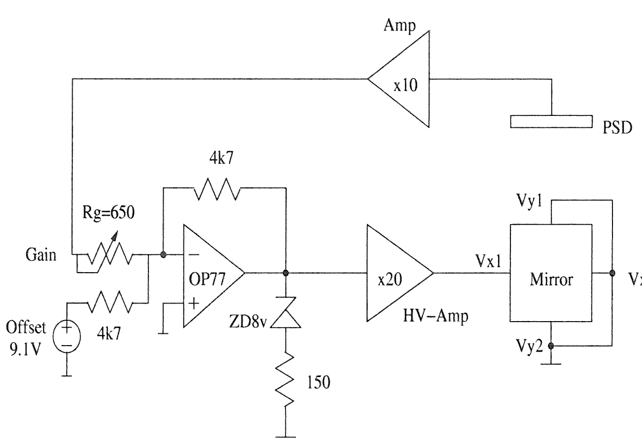

Figure 8: Feedback control circuit for tilting mirror at 9 degree.

circuit to implement the feedback control. In the experiment, a circular mirror shown in Fig. 7 having a snapdown angle of 7.5 degree was stably positioned to a tilt angle of 9 degree [7]. To optically measure the tilt angle of the deflected mirror, a visible 630nm laser light delivered through a single-mode optical fiber is focused onto the mirror. After reflection from the mirror, the beam is divided using a beam splitter to illuminate the position sensitive device (PSD) and to project a spot on a screen for visual assessment of the beam deflection. Fig. 8 shows the feedback circuit schematic. The output voltage from the PSD, which is a directly proportional to the mirror tilt angle, is amplified and then fed to an operational-amplifier circuit (OP77). The adjustable feedback network of this second amplifier stage and a variable voltage source allow setting the gain and offset according to the desired tilt angle. For the example illustrated in Fig. 8, the tilt angle is set to 9 degree, requiring a gain of -7.2 and an offset of -9.1V. After the amplifier, the voltage is clipped at -8V using appropriate Zener diodes. The last amplification stage consists of a high voltage amplifier having fixed gain of 20.

5 Conclusions

A method to stabilize electrostatic micromirror beyond the pull-in angle is developed and experimentally tested. The pull-in effect is overcomed by a closed-loop feedback control system which dynamically adjusts the voltage applied to the micromirror so that a desired electrostatic torque curve can be achieved. For practical reasons, the voltage applied to the mirror device is given by linear law. A voltage slightly larger than the pull-in voltage is first applied when the mirror is at zero position, and the voltage is then linearly reduced as the mirror approaches the desired position. Experimental measurements with a fabricated micromirror, showing that tilting angle beyond the pull-in point can be achieved, are in good agreement with theoretical analysis. Such a mirror stabilization method could greatly increase the scalability of optical switch fabrics and increase the dynamic range of optical attenuators.

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