

# Mixed-Level Model for Highly Perforated Torsional Actuators Coupling the Mechanical, the Electrostatic and the Fluidic Domain

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## ABSTRACT

We propose a mixed-level simulation scheme for squeeze film damping effects in microdevices, which makes it possible to include damping effects in system-level models of entire microsystems in a natural, physical-based and flexible way. Our approach allows also for complex geometries, large deflection and coupling to other energy domains. In this work we focus on the coupling with the electrostatic field including the effect of a reduced ground electrode size and fringing fields. Using this coupling scheme we are able to simulate the transient switching behavior of a highly perforated electrostatic microrelay at affordable computational expense. For very small air gaps, which occur if the microrelay is closed, the damping model had to be adequately extended. The predictive simulations could be verified by experimental analysis.

**Keywords:** Electrostatic actuation, squeeze-film damping, reduced order modeling, mixed-level modeling

## 1 INTRODUCTION

Electrostatic attraction between neighboring electrodes is a widely used actuation principle employed in tiltable micromirrors or microrelays. Electrostatic actuation is especially favored for switching applications in mobile telecommunication because of its low power consumption. State-of-the-art commercial software tools are able to analyze the static equilibrium between the electrostatic and the mechanical forces by solving the underlying partial differential equations sequentially in each of the two energy domains involved, but this is a computationally expensive approach for complex devices. Moreover, for switching applications it is primarily the transient behavior that needs to be analyzed. Even if fluidic damping effects are neglected, it is prohibitive to simulate the transient behavior of complex device structures, such as the industrial microrelay depicted in Fig. 1, on the continuous field level in view of the computational cost. On the other hand, since damping can only be neglected under conditions close to vacuum, which is only achievable with an expensive packaging, it is unavoidable to take squeeze-

film damping effects into account in the analysis and design optimization of most MEMS devices.

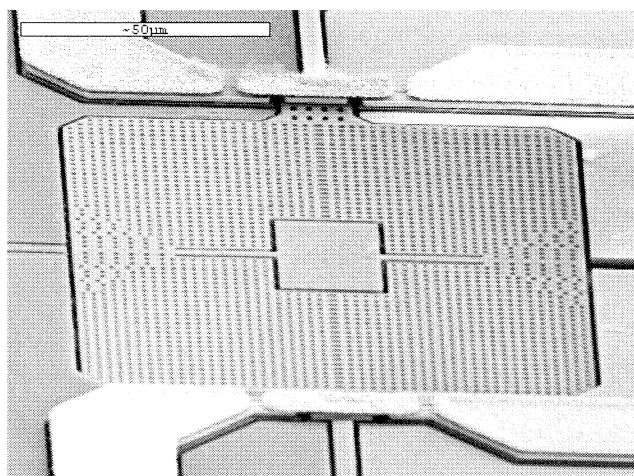


Fig 1: SEM image of an electrostatic microswitch featuring about 3000 etch holes [3].

## 2 MODELING

In order to tackle this problem, we follow a new mixed-level approach, which makes it possible to reduce the model complexity by incorporating the fluidic damping and the electrostatic actuation in system-level models in a physically correct and accurate manner. Besides, our method offers high flexibility with respect to variations of the device geometry and linking to additional other energy and signal domains.

Mixed-level means that discretized continuous field models are combined with compact models. The latter account for corrections of the continuous field model and the elastomechanical and the electrostatic elements of the microdevice.

The fluidic domain is discretized using the Finite Network method. In the terminology of Kirchhoffian network variables, we consider the fluidic massflow (and the mechanical torque, respectively) as generalized flux ("through" variable), which is driven by the pressure gradient in the fluid (or the angular velocity of the mechanical parts, respectively) acting as generalized force

("across" variable). The network elements are described in a hardware description language such as VHDL-AMS.

## 2.1 Mechanical Model

Due to the high stiffness of the polysilicon plate (Fig. 1) the microswitch can be considered as a rigid plate. The tethers where the tiltable plate is fixed to the substrate can be considered as a linear torsional spring, because the angle of torsion  $\varphi$  is small; thus a one-dimensional mechanical model can be applied:

$$M_{tether} = 2 \frac{GI}{L} \varphi \quad (1)$$

$$M_{inertia} = J \ddot{\varphi} \quad (2)$$

where  $G$  denotes the shear modulus,  $I$  the area moment of inertia,  $J$  the mass moment of inertia and  $L$  the length of the torsional spring.

## 2.2 Damping Model

The damping model is principally based on the Reynolds equation, a simplified version of the Navier-Stokes equation; however, this is only valid for those parts of the plate where the lateral dimensions of the structure are much larger than the thickness of the fluid film. In order to account for boundary and perforation effects, corrections have to be added along the edges of the structure and at the edge holes. To this end, special network elements are included as compact models, which are equivalent to fluidic resistances that control the airflow between the cavity under the plate and the environment (Fig. 2).

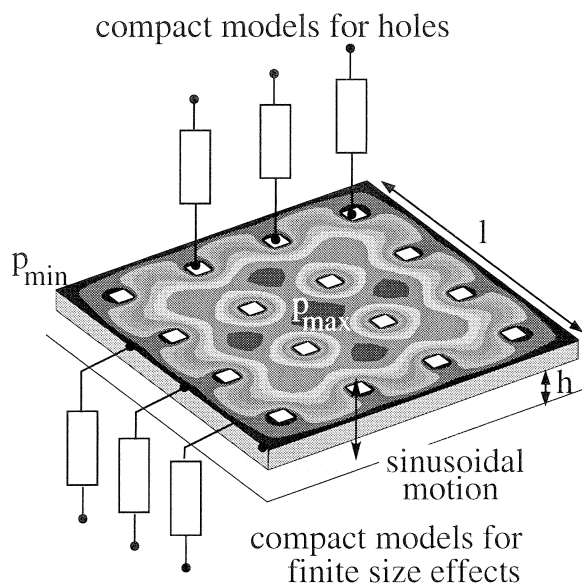


Fig. 2: Corrections for finite and perforated structures.

The basic ideas of this approach and details about these compact models have been presented in [1, 2]. One of the objectives of the present work has been to extend the compact models, which account for the fluidic damping caused by perforations of the mechanical structure, to the case of large displacements. Fig. 3 displays the fluidic resistance as extracted from a FEM analysis of a perforation based on the NSE; evidently the fluidic resistance increases with decreasing squeeze film thickness  $h$ . Hence, assuming the fluidic resistance to be independent of the fluid film thickness (as reported in [1,2]) is a good approximation only, if the fluid-film thickness is not too small. For modeling the pull-in behavior, however, this dependence must not be neglected.

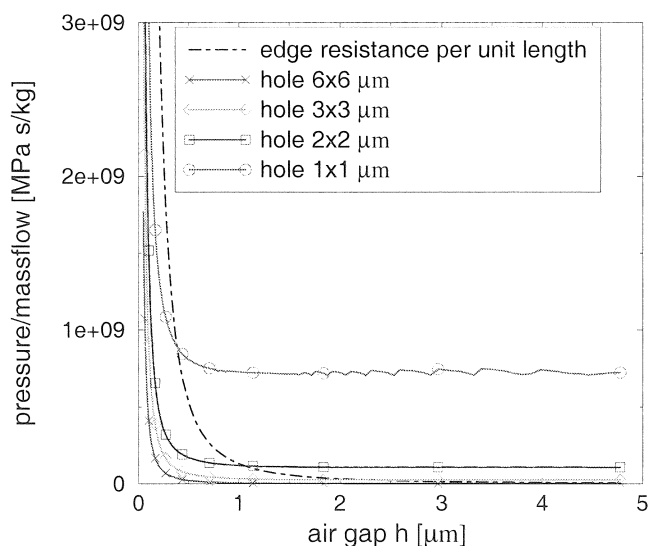


Fig. 3: Fluidic edge resistance per micron and fluidic hole resistance for different hole sizes. The fluidic resistance does not only increase with decreasing hole size, but also with decreasing fluid film thickness.

## 2.3 Electrostatic Model

Another objective has been to properly include the electrostatic force in the mixed-level scheme. It is important to consider the actual size of the ground electrodes.

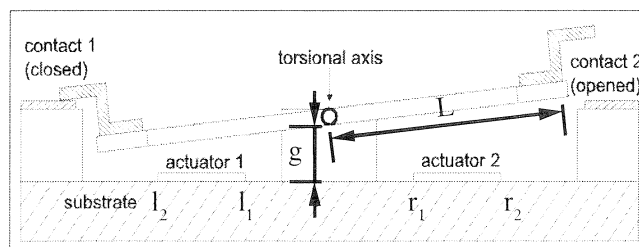


Fig. 4: Schematic cross-section of the torsional actuator. The ground electrodes are named actuator 1 and 2.

If the fringing fields are neglected, the capacitance per width  $C$  can be expressed as:

$$C = \frac{\epsilon_0}{\sin \varphi} \left\{ \log \left( \frac{g + r_2 \sin \varphi}{g + r_1 \sin \varphi} \right) - \log \left( \frac{g - l_2 \sin \varphi}{g - l_1 \sin \varphi} \right) \right\} \quad (3)$$

The parameters are explained in Fig. 4. The electrostatic torque is the derivative of the capacitance (3) with respect to the angle of torsion  $\varphi$ :

$$M = \frac{1}{2} U^2 \frac{\partial C}{\partial \varphi} \quad (4)$$

We assume  $l_1=0$  and that the driving voltage is only applied at actuator 1. Introducing the normalized angle  $\gamma$  as  $\gamma=L \cdot \sin(\varphi)/g$  equation (4) takes the form:

$$M = \frac{\epsilon_0 U^2}{2} \frac{L^2}{g^2} \sqrt{1 - \gamma^2} \frac{g^2}{L^2} \left\{ \log \left( 1 - \left( \frac{l_2}{L} \right) \gamma \right) + \frac{(l_2/L) \gamma}{1 - (l_2/L) \gamma} \right\}$$

The influence of a reduced ground electrode size on the capacitance and the electrostatic torque is depicted in Figure 5. For easy comparison the results are normalized with respect to the values obtained for a ground electrode of the same size as the movable electrode.

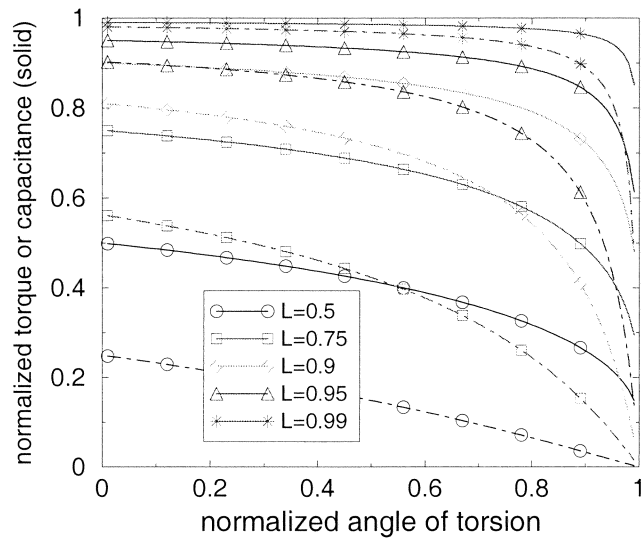


Fig. 5: Normalized capacitance (dotted lines) and electrostatic torque (solid lines) for various lengths related to the length of the movable electrode ( $L$  is the ratio of the length of the ground electrode to the length of the tiltable electrode).

Furthermore, it is important to consider the fringing fields which arise at the edges and near the holes of a highly perforated plate and contribute to the electrostatic torque acting on the mechanical structure. Compact models for the fringing fields around a single hole (Fig. 6)

and near the edges of the plate have been extracted from FEM simulations.

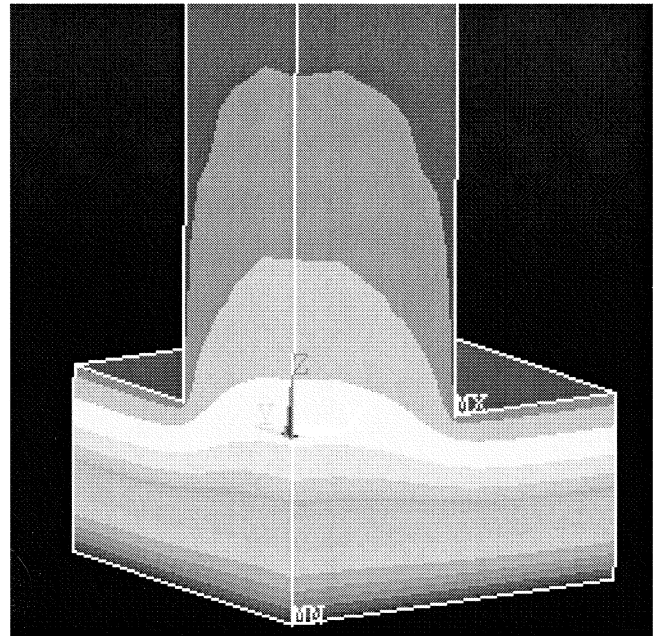


Fig. 6: Impact of fringing fields on the distribution of the electric potential in a square hole (1/4 model).

Now the compact models accounting for the fringing field effects are merged with those accounting for the reduced size of the ground electrode. The result is depicted in Figure 7.

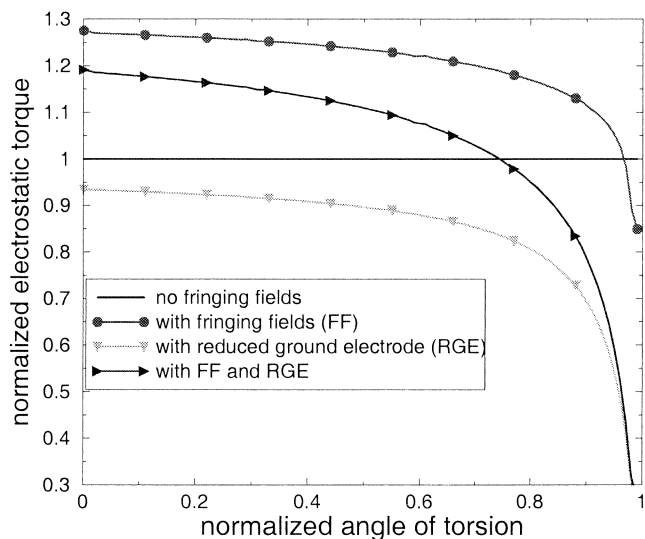


Fig. 7: Influence of fringing fields and a reduced ground electrode size on the electrostatic torque.

### 3 RESULTS

Finally, we compared the device behavior as predicted by the full system model (Fig. 8) with experimental data extracted from measurements on industrial prototypes of the microrelay shown in Fig. 1:

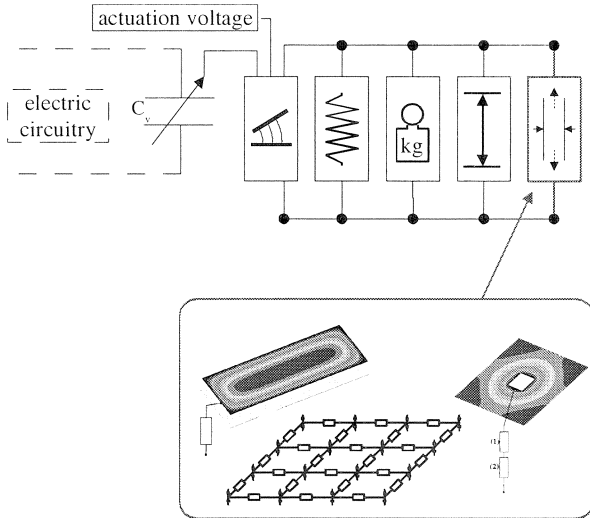


Fig. 8: Full System-level model of the microrelay.

#### 3.1 Electrostatic forces switched off:

The on-to-off switching of three device variants with different perforation density has been measured. The perforation density is reflected in the ratio of the hole area to the total cell area. Fig. 9 demonstrates the excellent agreement of our model with the real device behavior.

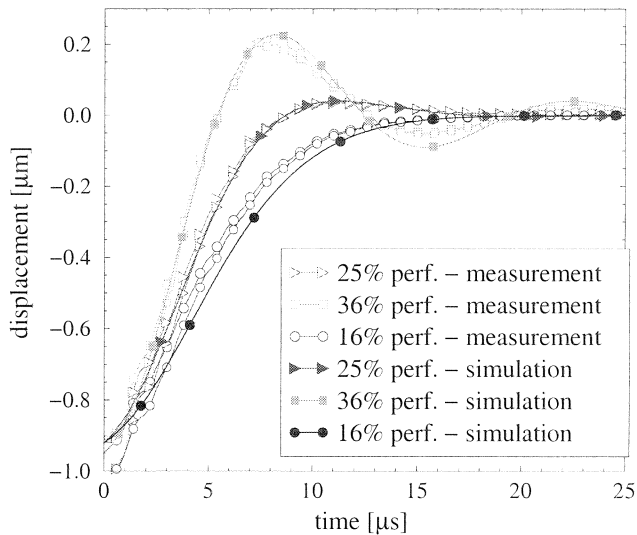


Fig. 9: Transient pull-out behavior for three different degrees of perforation of the movable electrode.

#### 3.2 Electrostatic forces switched on:

Fig. 10 shows the off-to-on switching transients of the same three device variants. The simulation transients are in excellent agreement with the measured ones, which demonstrates the high quality and predictive capability of our modeling approach. This is the first time that the transient behavior of such a complex device, where the coupled problem of the fluidic, the mechanical and the electrostatic domain has to be solved, could be predictively modeled within one hour of computation time.

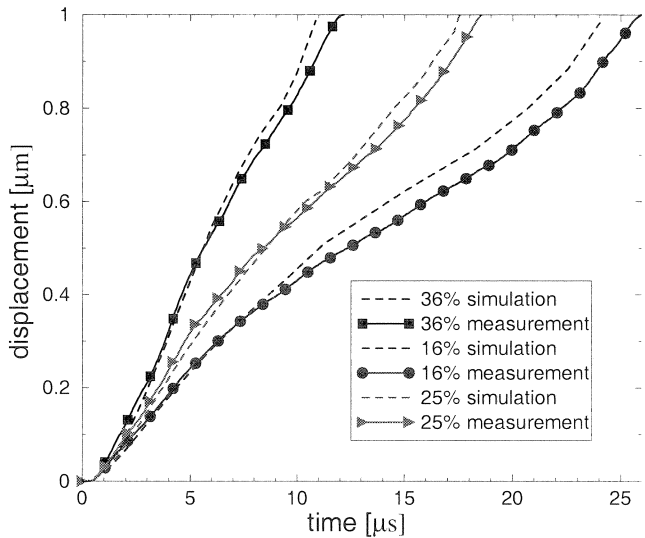


Fig. 10: Transient pull-in behavior for three different degrees of perforation of the movable electrode.

### 4 CONCLUSIONS

We demonstrated a mixed-level model of an electrostatic torsional microrelay based on a physical device description in terms of a standard hardware description language (VHDL-AMS). As physical and geometrical quantities are input parameters of the model, it allows for fast, efficient and reliable design studies.

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