A Design Tool for Inductive Position and Speed Sensors via a Fast Integral Equation Based Method

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ABSTRACT

This paper describes using 3D integral equation based simulation in combination with simulation managment to provide a design tool for an eddy-current inductive position and speed sensor under development at CSEM Microsystems. Both the frequency dependence and the position dependence of the inductance is explored to understand the operation and sensitivity of the sensor. The frequency dependence of the inductance can be captured in a low order model which can be used for higher level system simulation.

Keywords: Magnetic sensor, integral equation, simulation, eddy currents

1 INTRODUCTION

Macroscopic inductive sensors have been developed for many years as a means of noncontact position sensing. More recently, micromachined, planar-coil inductive sensors have been successfully fabricated for a higher level of integration, miniturization, and lower manufacturing cost. With miniturization and higher requirements on precision, simple ideal models and 2D models become inaccurate and full 3D modeling is necessary. Finite Element tools are available for 3D modeling but unfortunately such tools require a mesh of all of free space both around and in the conductor geometry. Such an approach leads to large matrix systems which require long computation time. In fact, mesh refinement requirements are so high to capture skin effect that solution can be intractable. Another disadvantage is that the mesh of space around the counductors hinders geometry design exploration. For instance, to explore the response of a position sensor, the user would need to produce a new mesh for each position.

In this paper we describe using 3D integral equation based simulation in combination with simulation managment to provide a design tool for an eddy-current inductive position and speed sensor under development at CSEM Microsystems. The integral equation approach is based on the Partial Element Equivalent Circuit (PEEC) method [1] and is advantageous in that only the interior of conductors needs to be meshed. Thus for exploring the response of this displacement sensor, the multiple simulations can be easily automated since for each new position, the space between the sensor and device does not have a mesh. Combining the integeral equation approach with the fast multipole method [2] as is done in [3] gives a tool capable of simulating the tens of thousands of elements necessary to model complicated 3D geometries and phenomenon such as skin and proximity effects. With simulation management, the user can extract the relevant inductance parameters automatically over the range of positions to be sensed or over a range of design parameters such as the frequency and dimension.

We begin in the next section by describing the PEEC approach, then in Section 3 we briefly describe the sensor geometry and operation. Section 4 explores the response of the sensor to variations in both position and frequency of operation. Finally, in Section 5 we conclude.

2 3D INDUCTANCE COMPUTATION

In the area of interconnect analysis, perhaps the best known integral equation approach is the Partial Element Equivalent Circuit (PEEC) method [1]. To model current flow in the PEEC method, the interior of conductors is divided into volume *filaments*, each of which carries a constant current density along its length. In order to capture skin and proximity effects, the cross section of each conductor is divided into bundles of filaments.

The interconnection of the filaments, plus sources, at the terminal pairs, generates a "circuit" whose solution gives the desired inductance and resistance parameters. These concepts are illustrated in Figure 1 where a single bent wire is divided into sections and those sections are each divided into a bundle of filaments. In the figure, each section is divided into only four filaments, but in practice they are divided into many more with thin filaments near the surface to capture the concentration of current there due to the skin effect at high frequencies.

To derive a system of equations for the filament currents we start by assuming the applied currents and voltages are sinusoidal, and that the system is in sinusoidal steady-state. The PEEC method uses a Galerkin approach applied to the governing integral equation and derives a relation between the filament currents and

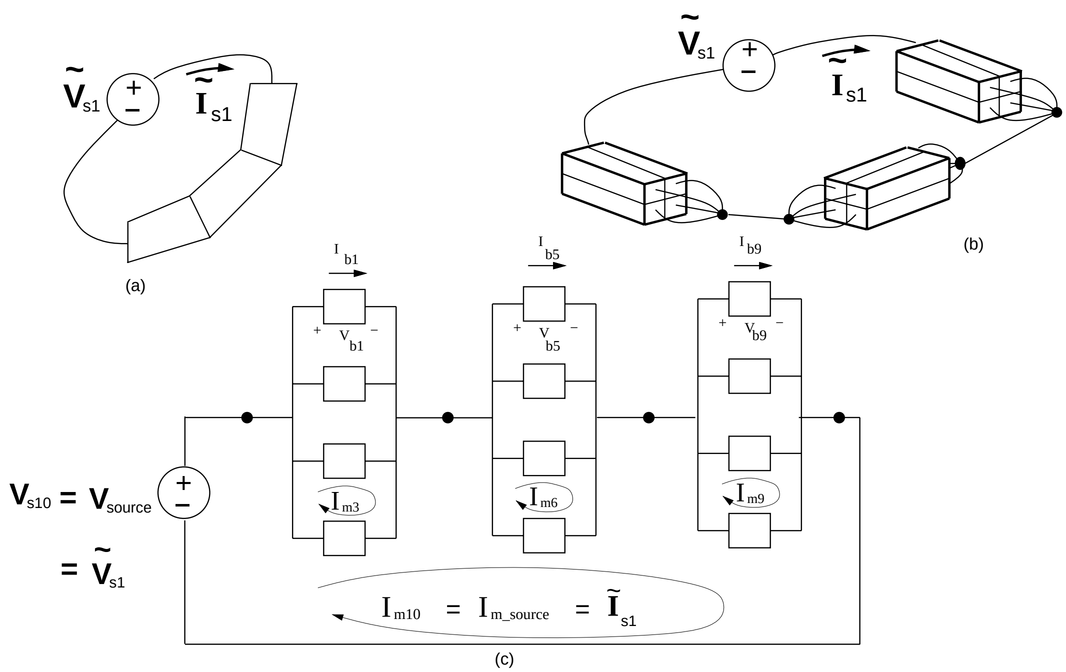

Figure 1: One conductor, (a) as piecewise-straight sections, (b) discretized into filaments, (c) modelled as a circuit.

voltages

$$\boldsymbol{Z}\boldsymbol{I}_b = \boldsymbol{V}_b, \tag{1}$$

where $\boldsymbol{V}_b,\ \boldsymbol{I}_b \in \mathbb{C}^b$, b is the number of branches (number of current filaments), and $\boldsymbol{Z} \in \mathbb{C}^{b\times b}$ is the complex impedance matrix given by

$$\boldsymbol{Z} = \boldsymbol{R} + \boldsymbol{j}\omega\boldsymbol{L}, \tag{2}$$

where ω is the excitation frequency. The entries of the diagonal matrix $\boldsymbol{R} \in \mathbb{R}^{b\times b}$ represent the DC resistance of each current filament, and $\boldsymbol{L} \in \mathbb{R}^{b\times b}$ is the dense matrix of partial inductances [4]. This is a dense matrix since every filament is magnetically coupled to every other filament in the problem.

Extracting the equivalent inductance, l, and resistance, r, for the single conductor example of Figure 1 involves applying a voltage, V_{s1}, and then using (1), plus Kirchhoff's laws to solve the circuit to compute the current, I_{s1}, through the source. The equivalent r and l are then given by

$$r + j\omega l = \frac{V_{s1}}{I_{s1}}. \tag{3}$$

This approach directly extends to the multiconductor case by applying vectors of sources, $\boldsymbol{V}_s$, and computing the vector of currents $\boldsymbol{I}_s$ to compute the equivalent inductance and resistance matrices.

Note that to solve the circuit equations by Gaussian Elimination would require $\mathcal{O}(n^3)$ operations since $\boldsymbol{L}$ is dense. Such cost makes solving problems with more than a few thousand filaments intractable. To improve the situation, MemHenry follows the approach in [3] which uses a preconditioned GMRES iterative algorithm [5] to solve the circuit equations using the circuit technique known as Mesh Analysis. In general, the cost for each iteration of an iterative algorithm applied to solving the circuit equations is $\mathcal{O}(n^2)$. However, it is possible to approximate computation with $\boldsymbol{L}$ to $\mathcal{O}(b)$ operations using a hierarchical multipole algorithm [2]. Such algorithms also avoid explicitly forming $\boldsymbol{L}$, and so both the computation time and memory required is nearly $\mathcal{O}(b)$. With such an approach, geometries with tens of thousands of filaments can be solved in a matter of hours. In the remainder of this paper, we discuss the results of applying MemHenry to compute the response of the inductive position sensor.

3 SENSOR GEOMETRY AND OPERATION

The sensor consists of a large primary coil enclosing four smaller secondary coils and is used to detect the position of the sensor over a periodic target such as the square teeth shown in Figures 2 and 3. The primary is excited with a current source and the difference of the voltage induced on adjacent secondary coils is used to sense the position. Specifically, as the sensor passes over the teeth, eddy currents are induced in the teeth which oppose the field produced by the primary as shown from simulation in Figure 4. The image current reduces the flux through the coils and the magnitude of this change

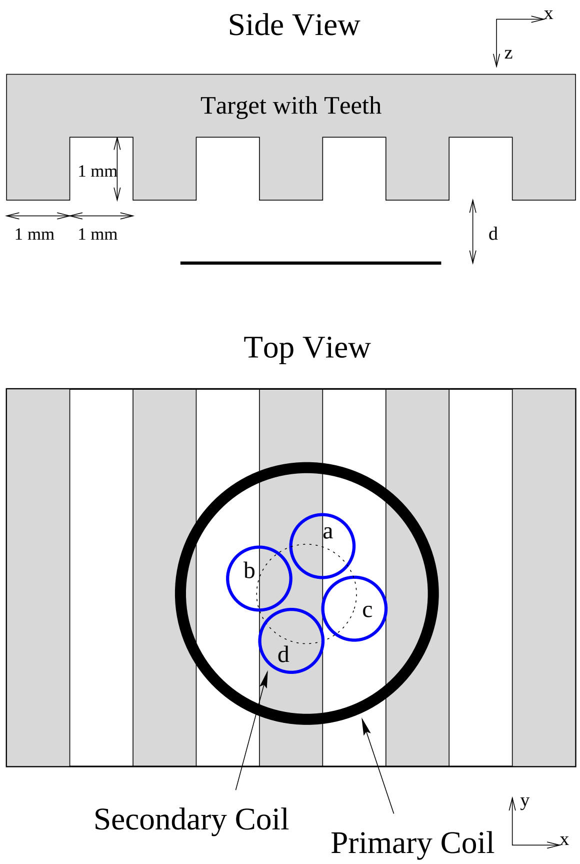

Figure 2: Sensor Geometry. The outer radii of the primary and secondary coils are 2 mm and 0.5 mm, respectively

is dependent on the position of the teeth relative to the coils: the closer the tooth, the greater the reduction in flux which can be measured as a change in the mutual inductance between primary and secondary coils.

The primary and secondary coils are copper and aluminum, respectively, and the target is copper. The outer diameter of the secondary coils is 1 mm and is set equal to the width and spacing of the teeth. The primary consists of 62 turns, and the secondary coils, 90 turns. To sense the teeth moving in the x-direction, the center of the four secondary coils are placed 0.5 mm (one quarter of the tooth period) apart as shown in Figure 2. For a sensor response less dependent on the distance, d, to the target, coil A and coil B as well as C and D are connected differentially.

4 RESULTS

To simulate the response of the sensor with the MEMCAD implementation of the tool, called MemHenry, the sensor geometry was created and volume meshed into 1710 brick elements. Since all the turns of the coil are similar in structure, for simplicity, only one turn of each coil was modelled.

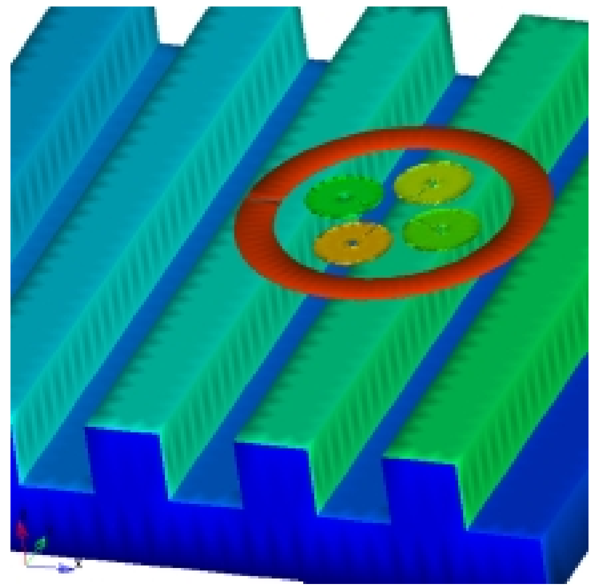

Figure 3: Sensor Geometry, 3D view. Single turns are shown wide for illustration.

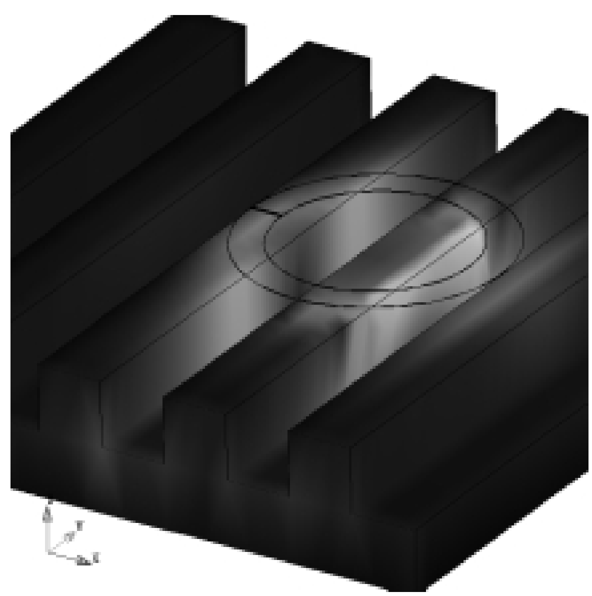

Figure 4: Induced current density on target

The geometry was then automatically refined to roughly 13,000 elements by MEMHENRY in order to capture skin and proximity effect up to the specified 100 kHz operation. In this section we use this model and discretization to explore the response of the sensor under variations in both position and drive frequency. For all the simulations in this section, for a single position of the sensor and single frequency, the computation of the resistance and inductance matrices took no more than one half hour on a Sun Ultra 30.

4.1 Frequency Dependence

The sensitivity of the sensor to the presence of the teeth of the target is highly dependent on the magnitude of the induced eddy currents. Because the target is made of a finite conductivity material, the magnitude of these eddy currents is strongly dependent on the frequency of excitation of the primary. Therefore, in the design of the sensor, understanding the frequency dependence is of great importance.

For a fixed position of the sensor over the target, the mutual inductance between the primary and coil a was computed from DC up to 10 MHz for $d = 250\mu m$. The system of 13,000 unknowns resulting from the circuit of Section 2 was solved for one frequency point per decade and the computation took under an hour on an Ultra 30. These results, labelled as 'Exact Solve' in Figure 5, show that up to 1 kHz, the eddy currents in the target do not significantly perturb the inductance and the sensor does not "see" the target at all. After 1 kHz, where the skin depth is on the order of the dimension of the structure, the eddy currents begin to reduce the mutual inductance to the secondary coil and thus the sensor now sees the target. As the frequency is raised, the inductance drops and thus for the higher frequencies the sensitivity of the sensor is greater. If the frequency is raised too high however, the structure eventually self resonates due to parasitic capacitances. To design considering the tradeoffs between these competing phenomenon, plus the interaction of the sensor to its surrounding circuitry, we require a model of the response in Figure 5 for a higher level system simulation such as SPICE or SABER. This model must capture the *frequency dependence* of the inductance.

To automatically generate such a model, one approach might be to use a fitting algorithm to fit the frequency data to a rational function which can then be incorporated into the simulator. Such an approach involves first computing the inductance at many frequency points, which can be expensive, and then developing a robust approach to fit the data to a rational function. Instead, we can construct a model *directly* from the PEEC discretization of Section 2 by using the numerically robust model reduction scheme described in [6]. The advantage of such an approach is that the computation of the entire model costs as much as roughly one frequency point computation. The results for a $30th$ order model are shown in Figure 5 and were computed in half an hour on an Ultra 30.

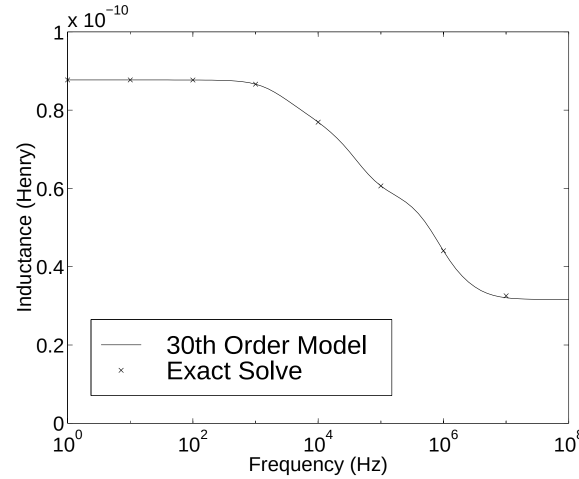

Figure 5: Frequency dependence of the mutual inductance between single turns of primary and secondary coils. The exact data results from solving the PEEC circuit at multiple frequency points. The 30th order model results from applying model reduction to generate a model for system simulation.

4.2 Sensing Position

Section 4.1 explored the frequency response of the sensor at *one* position only. In this section we describe using Simulation Management to automatically extract the response of the sensor with position.

To compute the response of each of the coils of the sensor as the teeth move by, the mesh for the target was moved through 2 mm, or one period of the teeth. Since we are using an integral equation approach which requires only a mesh of the conductors, moving the position of the target relative to the sensor involves only moving the mesh of the target. This movement and simulation was automatically performed for 17 positions with Simulation Management in the MEMCAD environment. The response of coil b for three different frequencies is shown in Figure 6. It is known from experiment that this response is roughly sinusoidal as the simulation shows. Also note that the inductance at 10 kHz is much less sensitive to position than at 100 kHz as predicted in Section 4.1.

For 100 kHz, the response of all four coils is shown in Figure 7. Notice that since the coils are spaced one quarter of a period apart in the x-direction, the sinusoids are roughly 90 degrees out of phase. Also, the secondary coils have cylindrically symmetric placement with respect to the primary, but they are not symmetric with respect to both the teeth and the primary. This

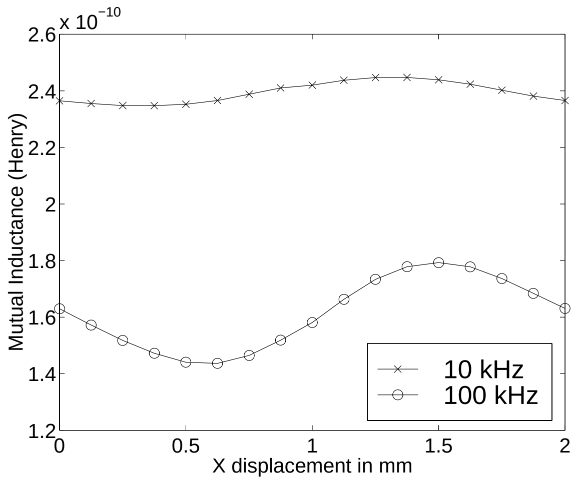

Figure 6: Response of Coil B as teeth move through one spatial period. Note higher sensitivity at 100 kHz.

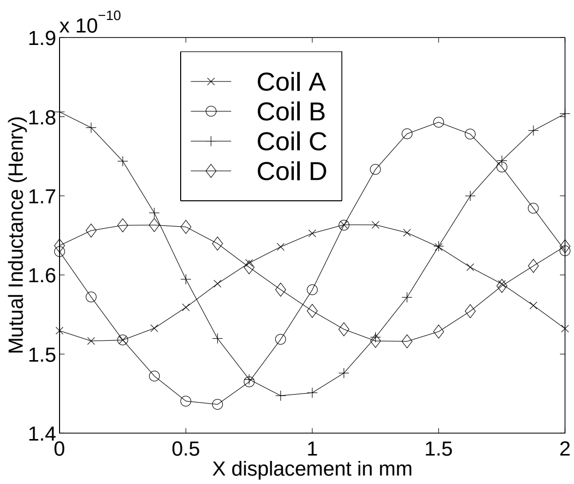

Figure 7: Response of the four coils at 100 kHz. The coils are spatially one quarter of a period apart as reflected in the figure.

difference can only be captured with a 3D solver and appears in the less-than-90 degrees phase difference and large magnitude difference in the responses of coils a and b.

With the results of Figure 7 generated automatically through simulation management, it would be possible to automatically generate SPICE or SABER models of the data for use in higher level system simulation and design.

5 SUMMARY AND FUTURE WORK

In this paper we have demonstrated the use of integral equation based techniques in combination with simulation management to rapidly and automatically compute the output of an inductive position and speed sensor. The dependence on both frequency and position was explored. Higher level system models were automatically generated which capture the frequency dependence. All of the above capabilities were available in the commercial release of MEMCAD 4.5 in the first quarter of 1999.

As mentioned at the end of Section 4.2, automatic generation of system models for variation of position are possible since the data is generated automatically. Parametrizing and fitting this movement automatically is under development.

The next phases of the tool involve first the computation of forces on conductors to enable coupled magnetomechanical analysis and second the ability to handle materials of permeability other than that of free space.

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