Modeling of Piezoelectric MEMS using the Finite Cloud Method

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ABSTRACT

In this paper we present a meshless based Finite Cloud Method (FCM) for the numerical solution of partial differential equations (PDEs) governing piezoelectricity Using a point distribution, FCM constructs interpolatio functions without assuming any connectivity between th points. A collocation approach is employed to obtain a solution at every point within the domain.

The coupled mechanical and electrical equations of a piezoelectric material are solved by an iterative FCM. This approach has been verified on two static two-dimensional piezoelectric problems with exact analytical solutions.

Keywords: meshless methods, piezoelectrics, MEMS

INTRODUCTION

The sensor and actuation properties of piezoelectric materials have led to a number of applications in the world of microelectromechanical systems (MEMS) [1]. Applications of piezoelectric materials include pressure sensors, strain gauges, accelerometers, ultrasonic motors, and micro-actuators.

Currently, modeling of these devices requires generation of a two or a three-dimensional mesh to numerically solve the coupled mechanical and electrical equations of piezoelectricity. The creation of a mesh can be computationally very expensive, especially when the geometry of the device is complicated. In this paper, we present a new meshless technique, which is referred to as a Finite Cloud Method (FCM). Instead of a mesh, FCM requires only a distribution of points over the domain to solve the coupled electromechanical equations.

The governing equations are satisfied at every point using a point collocation approach. Starting with an initial assumption of zero mechanical displacements, the electrical equation is solved for the electric potential. The electric potential solution is then used in the mechanical equation to solve for the displacements. The electrical equations are solved again with the newly computed displacements, and the procedure is repeated until a self-consistent solution is obtained.

The iterative FCM is employed to solve a pair of two-dimensional static piezoelectric strips with different loading conditions. The calculated solution to each problem agrees with the analytical results given in th literature.

GOVERNING EQUATIONS

The constitutive equations for a piezoelectric material can be expressed in terms of the strains and the electric field [2]:

$$\begin{aligned} \sigma_p &= c^E_{pq}\varepsilon_q - e_{kp}E_k \\ D_i &= e_{iq}\varepsilon_q + \xi^\varepsilon_{ik}E_k \end{aligned} \qquad (1)$$

where σ_p, ε_q, D_i, and E_k are the stress tensor, the strain tensor, the electric displacement vector, and the electric field vector, respectively. c^E, e, and ξ^ε are the elastic stiffness, piezoelectric, and dielectric constants, respectively. Superscript E and ε represent coefficients measured at constant electric field and strain, respectively. The constitutive equations can also be written in terms of the stress and the electric field:

$$\begin{aligned} \varepsilon_p &= s^E_{pq}\sigma_q + d_{kp}E_k \\ D_i &= d_{iq}\varepsilon_q + \xi^\sigma_{ik}E_k \end{aligned} \qquad (2)$$

where s^E, d, and ξ^σ are the elastic compliance matrix, piezoelectric matrix, and dielectric constants, respectively. Superscript σ represents coefficients measured at constant stress. Relationships between (c^E, e, ξ^ε) and (s^E, d, ξ^σ) are given in [2].

Strains are related to displacements b

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad (3)$$

and electric field is related to electric potential b

$$E_i = -\phi_{,i} \qquad (4)$$

The governing mechanical equilibrium is

$$\sigma_{ij,j} + f_i^B = 0 \qquad (5)$$

and the governing electrostatic equilibrium is

$$D_{i,i} = 0 \qquad (6)$$

NUMERICAL APPROACH

The interpolation functions employed by the FCM are based on the reproducing kernel technique [3]. In two-dimensions, the reproducing kernel technique for an approximate solution can be written as

$$u^a(x,y) = \int_\Omega \overline{w}_d(x-s, y-s)u(s)ds \qquad (8)$$

where u^a is an approximation of u, and $\overline{w}_d(x\text{-}s,y\text{-}s)$ is the corrected kernel function. In discrete form, equation (8) can be expressed as

$$u^a(x,y)=\sum_{I=1}^{NP} N_I(x,y)u_I \tag{9}$$

where NP is the number of points in the domain, $N_I(x)$ is the interpolation function at point I, and u_I is a nodal value at point I. The interpolation function $N_I(x)$ is defined as

$$N_I(x,y)=C(x-x_I,y-y_I)w_d(x-x_I,y-y_I)\Delta V_I \tag{10}$$

where $C(x\text{-}x_I,y\text{-}y_I)$ is the correction function, $w_d(x\text{-}x_I,y\text{-}y_I)$ is the kernel function centered at the node (x_I,y_I), and ΔV_I is a measure of the domain surrounding point I. The two-dimensional kernel function, $w_d(x\text{-}x_I,y\text{-}y_I)$, is taken as the product of two one-dimensional cubic spline functions.

The correction function for the two dimensional piezoelectric equations has the following form:

$$\begin{aligned} C(x-x_I,y-y_I) &= c_0(x,y)+c_1(x,y)(x-x_I)+c_2(x,y)(y-y_I) \\ &+c_3(x,y)(x-x_I)^2+c_4(x,y)(y-y_I)^2+c_5(x,y)(x-x_I)(y-y_I) \end{aligned} \tag{11}$$

The coefficients of the correction function are found b satisfying the appropriate reproducing conditions for particular order derivative [4].

Compute K_e
    LU factor $K_e \rightarrow L_e\, U_e$
Compute K_m
    LU factor $K_m \rightarrow L_m\, U_m$

given $u^{(0)} = 0$
for(i = 1,2,…until convergence)
{
  compute $f_e(u^{(i-1)})$
  solve $L_eU_e\, \phi^{(i)} = f_e$

  compute $f_m(\phi^{(i)})$
  solve $L_mU_m\, u^{(i)} = f_m$
}
end

Algorithm 1: An iterative approach for solving the governing electromechanical equations of piezoelectricit

A collocation approach for the electromechanical equations using the FCM interpolation functions gives

$$K_e\phi = f_e \tag{12}$$

$$K_m u = f_m \tag{13}$$

where K_e is an NP x NP matrix of electrical coefficients, ϕ is an NP x 1 vector of unknown potentials, f_e is an NP x 1 vector of electrical forcing terms dependent on the mechanical displacements, K_m is a 2NP x 2NP matrix of mechanical coefficients, u is a 2NP x 1 vector of unknown displacements, and f_m is a 2NP x 1 vector of mechanical forcing terms dependent on the electric potential.

The coupled electromechanical equations can now be solved using the iterative technique detailed in Algorithm 1. The first step is to compute K_e and K_m. Since the coefficient matrices are not electromechanically coupled, they are onl computed and LU factored once. Making an assumption of zero mechanical displacements, f_e is calculated, and the electrical system of equations can be solved for the unknown $\phi^{(i)}$. Once $\phi^{(i)}$ is known, f_m can be computed, and the mechanical system can be solved for the unknowns $u^{(i)}$. The displacements will be used in the next iteration to compute f_e, and the algorithm will continue until ϕ and u converge to a self-consistent solution.

RESULTS

Using the iterative Finite Cloud Method, two static piezoelectric problems were solved. The same material (PZT-5) was considered for both problems. PZT-5 properties along with important dimensions are summarized in Table 1 [5]. Both examples assume the material is transversely isotropic.

Table 1: Material Properties and Dimensions

S_{11}	16.4 E-6 mm^2/N	d_{31}	-172 E-9 mm/V
S_{12}	-7.22 E-6 mm^2/N	d_{33}	374 E-9 mm/V
S_{22}	18.8 E-6 mm^2/N	d_{15}	584 E-9 mm/V
S_{55}	47.5 E-6 mm^2/N	ξ_{11}	-1.5135 E-7 N/V^2
σ_o	-5.0 N/mm^2	ξ_{33}	-1.5135 E-7 N/V^2
σ_1	20.0 N/mm^3	V_o	1000 V
L	1.0 mm	h	0.5 mm

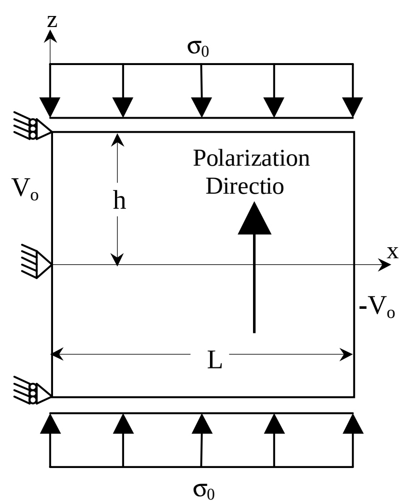

Figure 2: Piezoelectric strip subjected to a uniform stress and an applied voltage

The first problem considers a 1.0 mm by 1.0 mm piezoelectric strip, polarized in the vertical direction, subjected to a uniform stress in the y direction and an applied voltage as shown in Figure 2. In this example, the applied electric field is perpendicular to the polarization of the material resulting in a shear strain due to the piezoelectric effect. Under the action of the compressive stress, the piezo-strip experiences a negative strain in the z-direction and expands slightly in the x direction due to the Poisson effect. Plots of the computed mechanical displacements are given in Figure 3 and Figure 4. The grid in the background of Figure 5 represents the original shape of the strip before any loading. The computed potential is given in Figure 6. Both the mechanical displacements and the potential distribution match with the exact solution given in [6].

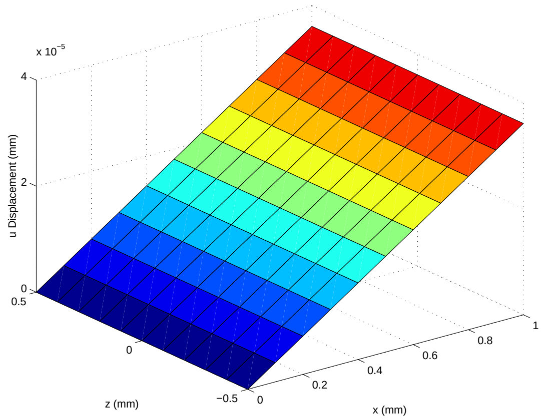

Figure 3: u displacement

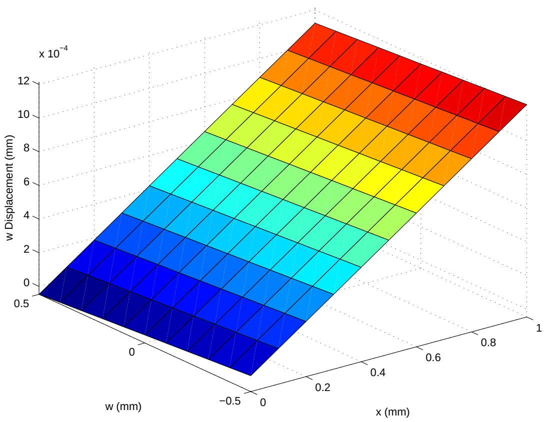

Figure 4: w displacement

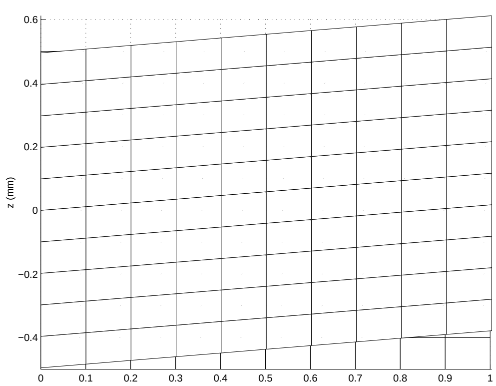

Figure 4: Mechanical deformation

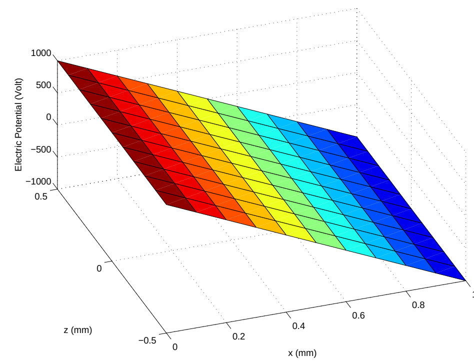

Figure 6: Potential distribution

The second example, as shown in Figure 7, consider another 1 mm by 1mm piezoelectric strip subjected to an applied voltage and a linearly varying stress.

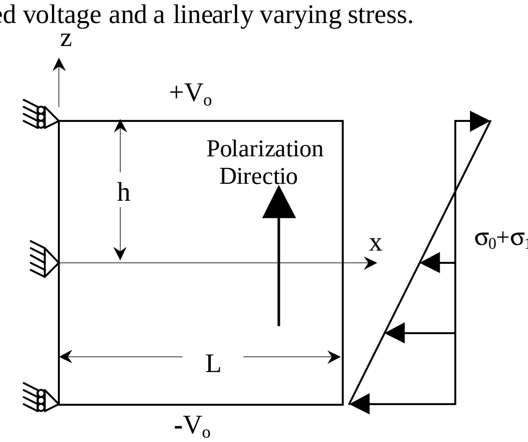

Figure 7: Piezo-strip subjected to a linearly varying stress and an applied voltage

Because the electric field is in the opposite direction of the polarization, the piezo-strip will contract in the z-direction and expand along the x-direction. The shape of the applied stress will cause bending in the strip. The mechanical displacements are given in Figure 8 and Figure 9. The grid in the background of Figure 10 represents the original shape of the piezo-strip before applying any voltage or stress. Figure 11 shows a plot of the potential distribution. Both the mechanical displacements and potential distribution match with the exact solution given in [6].

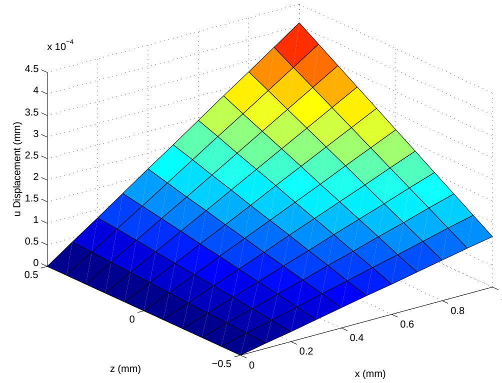

Figure 8: u displacement

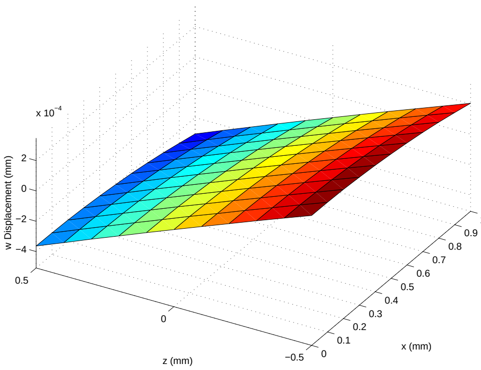

Figure 9: w displacement

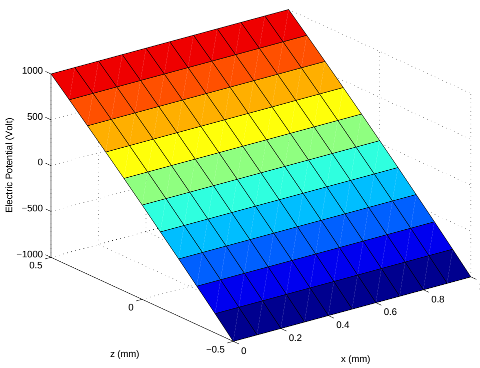

Figure 10: Potential distribution

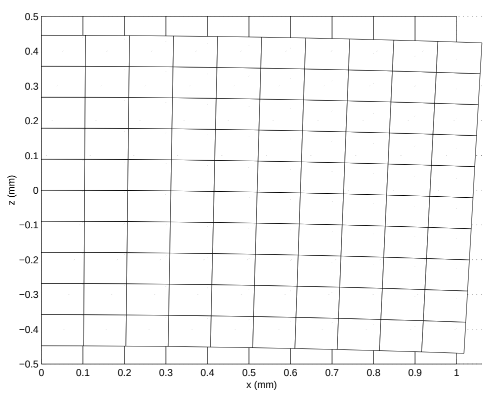

Figure 11: Mechanical deformation

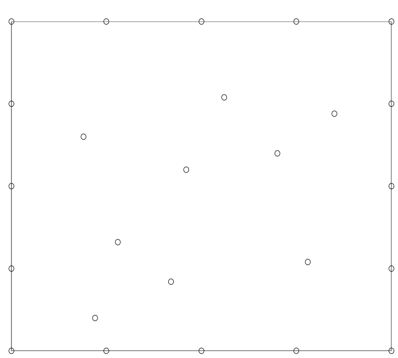

Figure 12: Point distribution with randomly placed interior nodes

To illustrate the flexibility of the meshless FCM method, example 2 was also solved using a 5 by 5 grid with randomly placed interior points. A plot of the point distribution is given in Figure 12 and the computed mechanical deformation is given in Figure 13. The results again match with the exact solution. This exampl illustrates that FCM can generate accurate results regardless of the point distribution used.

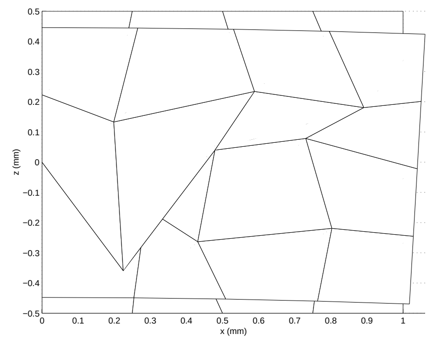

Figure 13: Mechanical deformation using a random distribution of points

CONCLUSION

The Finite Cloud Method has been applied to solve the coupled partial differential equations governing piezoelectricity. Results from the two static piezoelectric problems indicate that the method can accurately model the static behavior of single layer piezoelectric devices subjected to a variety of loading conditions. FCM was able to solve both examples using a simple distribution of points rather than having to generate a mesh of the domain. It was also shown that FCM can easily handle random point distributions without any special considerations. In summary, the Finite Cloud Method is a promising new method for modeling piezoelectric devices.

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REFERENCES

[1] H. S. Kim et. al., *Materials for Mechanical and Optical Microsystems*, Boston, MA, pp 161-166, 1996.

[2] IEEE Standard on Piezoelectricity, ANSI/IEEE Std 176, pp 9-15, 1987.

[3] W. K. Liu et. al., *Comp. Mech.*, Vol 18, pp 73-111, 1996.

[4] N. R. Aluru, "A Point Collocation Method Base on Reproducing Kernel Approximations", to appear in *Comp. Mech.*

[5] N. N. Rogacheva, *Piezoelectric Shells and Plates* , London, pp 9, 1994.

[6] V. Z. Parton et. al., *Appl'd Mech.: Soviet Reviews; Vol 2: Electromagnetoelasticity*, pp 1-58, 1989.