Analytical Modelling for Accelerometers with Electrically Tunable Sensitivity

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ABSTRACT

Results of the analysis and modelling of a pendulum type of accelerometer in an electrostatic field are presented. A common-mode voltage is used to yield an electrostatic positive feedback that amplifies the mechanical sensitivity. The externally applied electrostatic field enables the tuning of both sensitivity and spectral selectivity. The electromechanical coupling is analyzed both analytically and numerically, in terms of electrostatic shear forces and bending momenta. The results are used to design and fabricate accelerometers for mechanical spectrum analysis.

Keywords: accelerometer, electromechanical feedback, spectrum analyzer

INTRODUCTION

The sensitivity of an accelerometer structure is primarily set by the seismic mass and stiffness of the suspension beams. The stiffness results from the mechanical and geometric parameters of the device, which are often overdimensioned for reliability. The resulted mechanical sensitivity is increased, in both open and closed-loop systems, by an electronic gain. Closed-loop operation is often realised using electrostatic feedback [1]. The mechanisms involved in such feedback action offer more potential than actually exploited in conventionally operated servo accelerometers. For noise considerations it would be of advantage to use the electrostatic field to implement gain in the mechanical domain. For this reason the properties of electrostatic positive feedback on an encastrated inverted pendulum are examined. This electrostatic field replaces the gravitational field that determines the behaviour of the conventional inverted pendulum.

Inverted pendulum

The conventional clamped inverted pendulum in the gravitational field, which induces the positive feedback, is illustrated in Figure 1. The weigth of the seismic mass m is assumed to be concentrated at the top: $F_v = ma_v = mg$. When an horizontal acceleration field acts upon m, a displacement δ of the unclamped bar end will perturb the static equilibrium of the vertical beam.

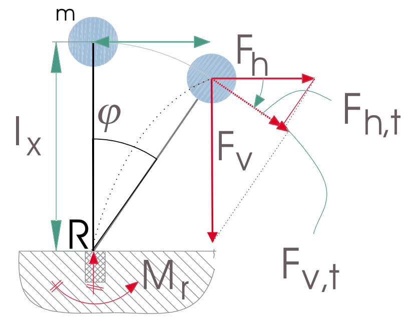

Figure 1: Clamped inverted pendulum

As a consequence, the gravitational force will now have a shearing component $F_{v,t}$ which *will amplify* the rotational effect of the horizontal force F_h.

This positive feedback effect could be interpreted either geometrically (Figure 2) or algebraically (Figure 3). Small displacements δ are assumed all over, in order to simplify the qualitative understanding. The geometrical representation of the equilibrium point clearly shows an increase in the actual rotation angle φ_1 compared with the case for which the vertical force would not act, φ_0. The key factor is the dependence of the active bending momentum on the angle φ. From an input viewpoint, it is like the external horizontal force is equilibrated by a smaller effective resistive momentum $M_{r,eff}(\varphi) < M_r(\varphi)$. The algebraic view helps with a quantitative characterization of the positive feedback. The block diagram shows that the gain φ/a_h can be controlled by the vertical acceleration component $a_v = g$. The same principle remains valid if the vertical gravitational field is replaced with an electrostatic one. The added value is in the electronic control of this field, and thus the potential of dynamic tuning of both sensitivity and spectral selectivity with respect to the input horizontal acceleration.

ELECTROMECHANICAL COUPLING

The prototype device is presented in Figure 4. A clamped vertical beam of length l_{x0} supports at its top a rigid horizontal arm. The entire structure is electrically conductive and biased at zero volts. Each side of the horizontal arm acts as a movable plate for the corre-

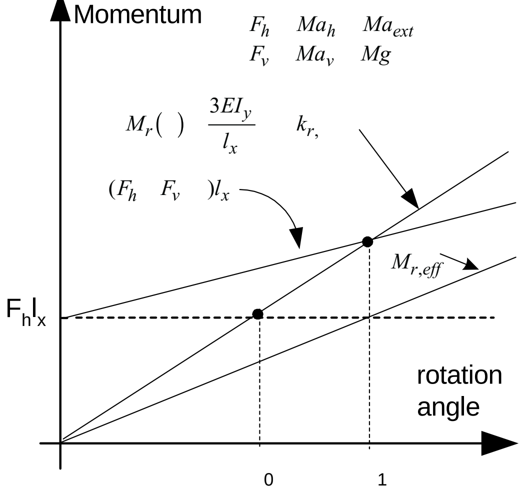

Figure 2: Geometrical interpretation of the positive feedback mechanism

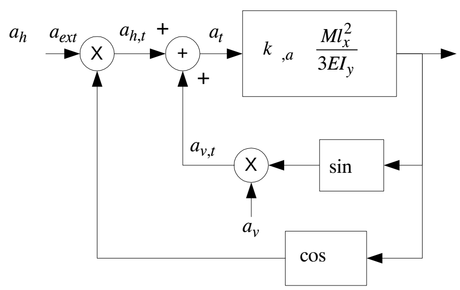

Figure 3: Algebraic view — block diagram

sponding capacitor, $C_1(\varphi, \delta)$ or $C_2(\varphi, \delta)$. Traditionally, electrostatic force feedback is used for counteracting the inertial force in null measurement systems. In the device presented here a different approach is pursued, based on using a common-mode voltage for actuation. The generated electrostatic forces will thus cause amplification of the bending induced by an external horizontal acceleration field a_{ext}.

So far linear bending has been assumed, which is far from realistic. The purpose of the oversimplified model was to offer qualitative understanding. In this section a

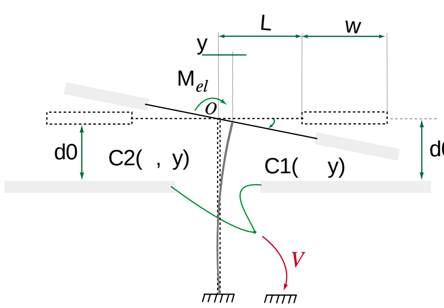

Figure 4: Prototype device for electrostatic momentum feedback

more thorough analysis is introduced, in terms of generalized coordinates and generalized forces. The first problem is the choice of a suitable generalized coordinate-force set. In the electrical domain, the obvious choice is the pair electric charge Q – electric voltage V, or (Q_1, V_1), (Q_2, V_2) in the case of a differential capacitive system. The generalized coordinates needed for a complete characterization of the mechanical state are more difficult to establish. To simplify, the movements out of the plane x-y will be neglected. A further reduction will be to separate the mechanical movable structure into two submodules: the vertical beam (which bending acts as an energy storage mechanism) and the horizontal arm. This last component is considered as a solid rigid (which can be assured by design), rigidly attached to the vertical pillar. If one neglects the inertial body forces along the pillar, then the equivalent mechanical scheme will correspond to a (rectangular) beam loaded at the top with a shearing force F_y (inertial plus electrostatic force resultant in the horizontal direction) and with a bending momentum M_{el} (due to the electrostatic forces between capacitors plates). The deflection equation for such a loading gives the solution:

$$v(x) = \frac{x^2\left[3M_{el} + F_y\left(3l_{x0} - x\right)\right]}{6EI_y} \quad (1)$$

$$\varphi\left(x\right) = v'\left(x\right) = \frac{x\left[2M_{el} + F\left(2l_{x0} - x\right)\right]}{2EI_y} \quad (2)$$

where $v(x)$ denotes the deflection along the pillar in the horizontal direction, φ — its derivative, E — the elastic modulus, and I_y— the inertia modulus.

The feedback is generated through the dependence of M_{el}, F_y on $\delta \triangleq v(l_{x0}), \varphi_0 \triangleq v'(l_{x0})$. The question is now if the knowledge of δ, φ_0, completely determines the spatial position of the deformed device. Because the horizontal arm is by assumption rigid (no deformation), then its position in 0xy plane is completely determined by 3 parameters: $\delta_y = \delta$ (translation in 0y direction), δ_x (translation in 0x direction) and φ_0 (rotation around O). We implicitly assumed the horizontal arm rigidly fixed on the pillar, on a perpendicular direction. As usual for the case of small bending approximation, δ_x is neglectible, so the previous two parameters completely characterize the mechanical position (the mechanical state) of the horizontal arm.

To completely characterize the mechanical state of the pillar, the bending curve (elastica) must be known, to give the information regarding the energy stored in mechanical stress. Neglecting for the time being the distributed inertial forces, from the formula above one sees that the curve $v\left(x\right)$ is a 3rd order polynomial in x : $v(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$. There are 4 coefficients to be determined, and 4 boundary conditions are known: $v\left(0\right) = 0$, $v'\left(0\right) = 0$, $v(l_{x0}) = \delta$, $v'\left(l_{x0}\right) = \varphi_0$. This means that δ and φ_0 can be taken as generalized coordinates to completely characterize the mechanical state of the system. For each of them, there is an associated generalized force:

- δ corresponds to a generalized top force in $0y$ direction F_δ, with O as acting point. F_δ acts as a top shear force for the pillar and as $0y$ translation force for the horizontal arm
- φ_0 corresponds to a generalized momentum around O, M_φ. It is a bending momentum for the vertical beam and a rotation momentum for the horizontal arm.

The dependency $C_{1,2}(\varphi_0, \delta)$ yields the associated generalized forces generated by electrostatic action. It is useful to rewrite Eqns.(1),(2) in terms of φ_0, δ:

$$v(x) = \frac{x^2}{l_{x0}}\left[3\frac{\delta}{l_{x0}} - \varphi_0 + \frac{x}{l_{x0}}\left(\varphi_0 - 2\frac{\delta}{l_{x0}}\right)\right] \quad (3)$$

$$v'(x) = \frac{x}{l_{x0}}\left[6\frac{\delta}{l_{x0}} - 2\varphi_0 + 3\frac{x}{l_{x0}}\left(\varphi_0 - 2\frac{\delta}{l_{x0}}\right)\right] \quad (4)$$

ANALYTICAL MODEL

Assuming a perfect coupling between the mechanical and the electrical domain, the energy balance equation is:

$$v_1(\partial Q_1) + v_2(\partial Q_2) = \frac{1}{2}\partial(v_1 Q_1) + \frac{1}{2}\partial(v_2 Q_2) + F_\delta(\varphi_0, \delta)\cdot\partial\delta + M_\varphi(\varphi_0, \delta)\cdot\partial\varphi_0 \quad (5)$$

The terms $v_1(\partial Q_1), v_2(\partial Q_2)$ represent the variation in the generated electric energy; $\frac{1}{2}\partial(v_i Q_i)$ are the variations of stored electrical energy components, and $F_\delta(\varphi_0, \delta)\cdot\partial\delta + M_\varphi(\varphi_0, \delta)\cdot\partial\varphi_0$ gives the variation in the generated mechanical energy. The variation of the electrostatic state can be done either at constant voltage $v = \text{const}$, or at constant charge $Q = \text{const}$. It is straightforward to compute the electrostatic generalized forces corresponding to a variation in either δ or φ_0. Figure 5 illustrates the geometrical parameters involved. The results (for one side of the horizontal arm) are presented in Table 1. The linearizations were performed around the zero value for all the generalized coordinates δ_x, δ_y and φ respectively.

The prototype device considered also satisfies: $\delta_{x,1} = \delta_{x,2}$, $\delta_{y,1} = -\delta_{y,2}$, and $\varphi_1 = -\varphi_2$. From the table it results that a constant-voltage excitation is to be preferred in the case of a momentum feedback. In such a case, the electrostatic shear forces are constant and independent of the displacement δ_y, so their difference will be zero. The total bending momentum due to electrostatic action is:

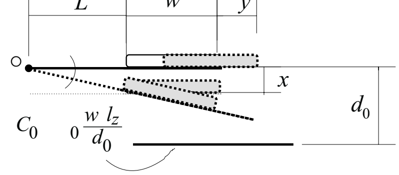

Figure 5: Generalized coordinates

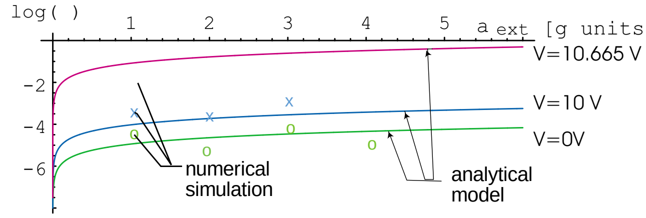

Figure 7: Rotation angle as a function of external acceleration, for different control voltages ($V_{\rm cr} = 10.67V$)

$$M_\varphi = M_{\varphi,1} - M_{\varphi,2} = 4C_0V_0^2\frac{L^2 + L\cdot w + w^2/3}{d_0^2}\varphi_0 \quad (6)$$

The equivalent block diagram of the electromechanical coupling is given in Figure 6. Here, N_a represents the total number of horizontal arms, to amplify the electrostatic action (see Figure 8). Although the positive feedback loop is at the level of the rotation angle φ_0, the supplementary bending momentum will also have an influence on the maximum bending $\delta = v(l_{x0})$. This happens because the two chosen generalized coordinates δ, φ are independent, but not orthogonal. The block diagram allows a determination of the critical voltage V_{cr}, corresponding to the stability limit of the positive feedback loop:

$$V_{cr}^2 = \frac{d_0^2}{L^2 + L\cdot w + w^2/3}\frac{1}{4N_aC_0}\frac{EI_y}{l_{x0}} \quad (7)$$

NUMERICAL SIMULATIONS

In order to compare the analytical model with numerical simulations, a finite element analysis of the device was performed using PDEase2D package [2]. For values of the common voltage V close to V_{cr}, the convergence problems were difficult to overcome, but the general results of the finite element analysis agreed with the analytical model. In Figure 7 the analytical model and FEM simulation results are confronted.

FABRICATION TECHNOLOGY

A set of devices was designed, according to the analytical model. A surface micromachining process was

Table 1: Generalized electrostatic forces

$W_0 = \frac{1}{2}C_0V_0^2$	F_x	F_y	M_φ
$Q = Q_0 = \text{const}$	$\frac{W_0}{d_0}$	$\frac{W_0}{w}\left(1 - 2\frac{\delta y}{w}\right)$	$W_0\left(\frac{w+2L}{d_0} + \frac{w^2}{3d_0^2}\varphi\right)$
$v = V_0 = \text{const}$	$\frac{W_0}{d_0}(1 + 2\frac{\delta x}{d_0})$	$\frac{W_0}{w}$	$W_0\left(\frac{w+2L}{d_0} + 4\frac{L^2+Lw+w^2/3}{d_0^2}\varphi\right)$

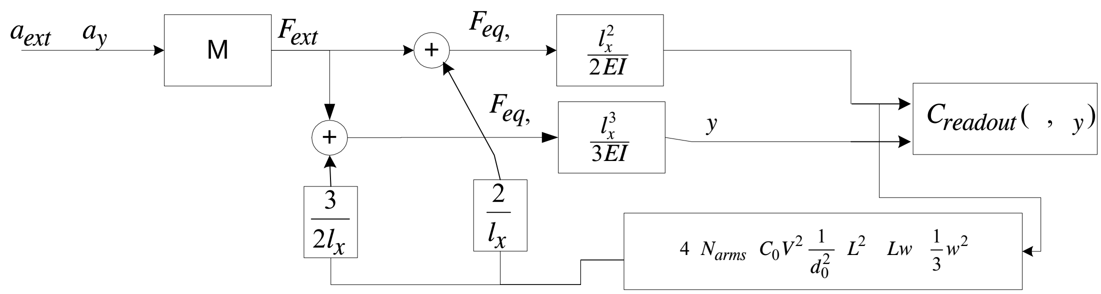

Figure 6: Block diagram of electrostatic momentum feedback

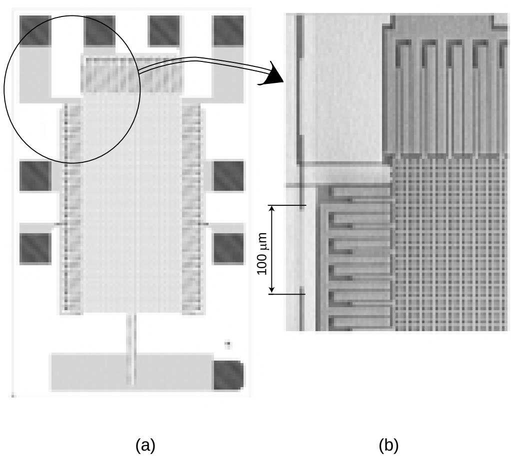

Figure 8: (a) Layout and (b) SEM-detail of a fabricated accelerometer

used, with a deposited polysilicon layer of $10\,\mu m$ thickness. A RIE process is used to pattern the polysilicon layer, followed by a wet-etching release step. A layout image is presented in Figure 8a, and a SEM photo of part of the structure in Figure 8b.

APPLICATIONS

The technique presented enables dynamic sensitivity tuning of accelerometers with a relatively low mechanical sensitivity, by using an external DC common-mode voltage. A specific feature is that the gain is set at the interface between the mechanical and electrical domains, and thus offers the possibility of an improved signal-to-noise ratio.

A step further is to implement a frequency-selective positive feedback loop. If the DC control voltage is replaced with a common-mode voltage of constant frequency ω_0 (much lower than the fundamental mechanical resonant frequency), then the positive feedback will be effective only for the components in external acceleration having frequencies multiple of $2\omega_0$. Simple electronic signal processing is sufficient to realize a vibration spectrum analyzer, which is a promising alternative to FFT using a DSP in a microsystem. The functioning do not rely on the resonant frequencies of an array of beams, but rather on sweeping the excitation voltage through the part of the mechanical spectrum of interest.

CONCLUSIONS

A dynamically programmable accelerometer is presented, based on electromechanical feedback through a momentum bending action. The amount of positive feedback is controlled by a common-mode voltage; its amplitude determines the loop gain, and its frequency the selectivity of the device to the input acceleration spectrum. An analytical model for the quasistatic operation is derived, and compared with the results of finite element simulations. The concept is applied to design an accelerometer aimed for peforming spectral analysis, in a condition monitoring system.

REFERENCES

[1] R.P. Kampen, M.J. Vellekoop, P.M. Sarro and R.F. Wolffenbuttel, "Application of electrostatic feedback to critical damping of an integrated silicon capacitive accelerometer", Sensors and Actuators, A43, 100–106, 1994

[2] —, PDEase2D Reference Manual, 1996