Modeling of the Piezoelectric Micropump for Improving the Working Parameters

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ABSTRACT

In this paper a modeling for the flow of viscous liquids and a way to improve the piezoelectric micropump efficiency are described, in order to find the most convenient set of hydraulic parameters. The main goal is to characterize the flow rate as function of these parameters and to find the best configuration related to head loss. The considered parameters are inlet and outlet pressures, length of internal channels, frequency of power supply applied on the piezoelectric material; transversal piezoelectric pressure. The important results are: a theoretic model for the flow; an evaluation of flow rate as function of parameters; an analysis for the dependence of head loss and efficiency of micropump as function of most important parameters of the micropump. Also, the behavior of piezoelectrically actuated micro-membrane using analytical solutions and FEM simulation (ANSYS) has been investigated.

INTRODUCTION

Micropumps are essential in microliquid handling systems. In the last years, different micropumps have been developed, using two principle: with moving part valves and with passive valves. As materials, silicon and glass are preferred, due to the property of being resistive to aggressive media. For obtaining good performances (precise delivery of volumes, accurate flux control, reliability) is necessary to choose a certain set of working parameters. That means i) to find the relations between these parameters and outputs and ii) to optimize the setting of parameters in order to reduce the loss, to increase the fezability and efficiency.

DESCRIPTION OF THE MICROPUMP

The micropump is presented in the figure below. The dimensions of pump body are: 20 mm diameter and 6mm height. The operation of the micropump is based on the piezoelectric effect: the voltage difference applied to the piezoelectric disk induces a radial strain, which causes the deflection of the membrane. The discharge of the micropump is controlled by the amplitude of the alternating voltage difference across the piezoelectric disk.

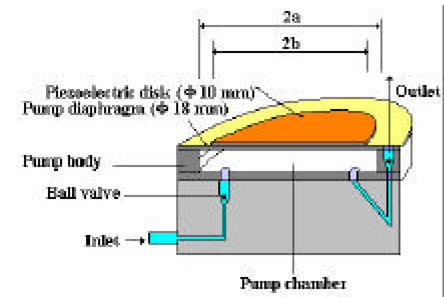

Fig. 1 Micropump configuration

FLOW MODELLING

This work analyses for the first time the influence of the fluid viscosity to fluid flow through micropump channels taking into account the following:

- the fluid characteristics;
- membrane deformation;
- small dimensions of the micropump channels.

For the theoretic model of the flow we used the Bernoulli equation separated for the inlet channels and outlet channels (because the flow is not continuos), in order to obtain the average velocity in the channels. With this velocity, the flow rate and the head losses can be calculated. For inlet flow, the Bernoulli equation on a streamline is:

$$\frac{1}{g}\int_{s1}^{s2}\frac{\partial v}{\partial t}ds+\frac{v_M^2}{2g}+\frac{p_2}{\rho g}+h=\frac{v^2}{2g}+\frac{p_1}{\rho g},\quad t\in[0,\ T/2]\quad(1)$$

where s is the curvilinear abscissa. $M_1(s1)\in S_1$, $M_2(s2)\in S_2$, (S_1 - the cross section at the inlet, S_2 – the "free" surface), p_1 - the inlet pressure, p_2 – pressure in the pump chamber. We consider the streamline corresponding to the average velocity. On the membrane we have:

$$S(t)=\int_0^R\int_0^{2\pi}\sqrt{1+\left(\frac{\partial w}{\partial r}\right)^2}\ r\ d\theta dr=2\pi\int_0^R\sqrt{1+\left(\frac{\partial w}{\partial r}\right)^2}\ r\,dr\quad(2)$$

$$V_M(t)=\frac{1}{S(t)}\int_{S(t)}\vec{v}_m(x,y,t)dA\qquad(3)$$

The mean velocity V ("mean" related to the cross section) in the channel depend on time by trigonometric function:

$$V=V_{max}\sin(\omega t)\ ;\ \text{then}\ \frac{\partial V}{\partial t}=\omega V_{max}\cos(\omega t)\qquad(4)$$

With T the period of a cycle and f - frequency, the pumped volume in the interval of T/2 is:

$$Vol = S_C V_{\max} \int_0^{\pi/\omega} \sin(\omega t)\,dt = \frac{2 S_C V_{\max}}{\omega} = \frac{S_C V_{\max}}{\pi f} \qquad (5)$$

The flow rate will be:

$$Q = \frac{\omega}{2\pi} \int_0^{\pi/\omega} Q(t)\,dt = \frac{S V_{\max}}{\omega} \qquad (6)$$

In order to calculate the velocity, it is necessary to evaluate the head loss. The head loss h along the streamline is caused by two factors: friction and changes in velocity or direction of flow (minor losses). The loss due to the friction is proportional to the square of the average velocity and to ratio between the length and square of diameter of channel:

$$h_{fi} = f\,\frac{l_i}{d_i^2}\frac{V^2}{2g}\ ;\ \mathrm{f} = \frac{64}{\mathrm{Re}} \ \Rightarrow\ \mathrm{h} = \frac{64\mu}{2\rho \mathrm{g}} V\Big(\sum_i \frac{l_i}{d_i^2}\Big) + h_s$$

$$h = \frac{64\mu}{2\rho \mathrm{g}} V\Big(\sum_i \frac{l_i}{d_i^2}\Big) + \frac{6\mu}{\rho g R} V \qquad (7)$$

The last term denotes the loss on the surface of the ball valve: $h_s = 6\mu V / g\rho R$; V is the average velocity and R is the radius of ball.

The losses due to the corners or to the transition between the channels with different diameters are:

$$h_{mi} = k_i \frac{V^2}{2g},\quad \mathrm{i} = \overline{1{,}4} \Rightarrow h_m = \frac{V^2}{2g}\sum_{i=1}^{4} k_i = C_2 V^2 \qquad (8)$$

where k_i is a specific coefficient for each type of change in geometry.

The flow model for the flow from the pump chamber to outlet can be described in an analogue mode.

Regarding the viscosity, it is possible that the intermolecular forces on the wall to be different from the forces in the volume of fluid; this fact could influences the viscosity near of wall. This influence increases with the decreasing of dimension of the channel. In the literature it is defined the viscosity on the wall, μ_w and the apparent viscosity μ_a, which is larger than viscosity in the volume of fluid (at high distances from the wall), μ_∞. The apparent viscosity is not a material property but a characteristic of flow.

Usually, viscosity is function of temperature T, and pressure p. Thompson gives a representation of viscosity as function of normal distance y from the wall for obtaining a qualitative relation between μ and y, where the oscillations have been mediated. So,

$$\mu_a = \mu_\infty + (\mu_w + \mu_\infty) e^{-(y/\delta)^m} \qquad (9)$$

where δ is scale of length range and m is a scaling factor. Thompson has investigated the flow of a fluid, considering three different types of interaction with the wall: i) the same attraction between the wall and liquid; ii) double attraction between the wall and liquid; iii) no attraction between the wall and liquid.

In our model we considered a double attraction between the wall and liquid. In this case, for a certain distance y from the wall, we have:

$$\mu_a = \mu_\infty (1 + e^{-(y/\delta)^m}) \qquad (10)$$

The used value for μ_a is calculated for $y=y_0$ corresponding to a streamline for the average velocity in the channel, and for m=1/4, according to a smaller decreasing of μ_a from μ_w to μ_∞.

Model of membrane deflection

The voltage difference applied to the piezoelectric disk induces a radial strain, which causes the deflection of the membrane. The volume stroke can be calculated from the function, which describes the membrane deflection. Taking into account that the real volume of fluid can be smaller than the volume stroke, due to the fluid inertia and viscosity, the evaluation of volume stroke as function of frequency and elastical properties of the membrane is very important in order to find the pump efficiency.

The radial analysis of the membrane was started from the equation for the slope in radial coordinates for a circular plate (thickness a), loaded with radial stress. First, for the case of compressive stress, we have [1]:

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} + \left(\frac{k^2}{a^2} - \frac{1}{r^2}\right)\varphi = 0 \qquad (11)$$

The solution of (12) has the following form:

$$\varphi = C_1 J_1\left(k\frac{r}{a}\right) \qquad (12)$$

where: $N = q_{piezo} h_2$ and $k^2 = \dfrac{Na^2}{D}$.

J_1 is the Bessel function of first kind and first order. The constant is determined from the boundary conditions. We have:

$$\varphi(r) = -\frac{dw(r)}{dr} \qquad (13)$$

By integration we obtain:

$$w = C_1 \frac{a}{k} J_0\left(k\frac{r}{a}\right) + C_2 \qquad (14)$$

Solution for micropump structure is obtained considering the superposition of solutions for:

- A circular membrane (radius a) with a central hole (radius b), clamped on the edges.
- A circular membrane (radius b).

The solution for the center membrane (0≤r≤b) is:

$$w_1^c = C_1^c \frac{b}{k_e} J_0\left(k_e \frac{r}{b}\right) + C_2^c \qquad (15)$$

$$k_e^2 = \frac{Nb^2}{D_e} \qquad (16)$$

The solution for the membrane with central hole (b≤r≤a) is:

$$w_2^c = C_3^c \frac{a}{k_m} J_0\left(k_m \frac{r}{a}\right) - C_4^c \int J_2\left(k_m \frac{r}{a}\right) dr + C_5^c \qquad (17)$$

where $$k_m^2 = \frac{Na^2}{D_m} \qquad (18)$$

For the case of tensile stress, the equation of the slope will be:

$$\frac{d^2 j}{dr^2} + \frac{1}{r}\frac{dj}{dr} - \left(\frac{k^2}{a^2} + \frac{1}{r^2}\right) j = 0 \qquad (19)$$

The slope will be characterized by the modified Bessel function. The solution for the center membrane (0≤r≤b) is:

$$w_1^t = -C_1^t \frac{b}{k_e} I_0\left(k_e \frac{r}{b}\right) + C_2^t \qquad (20)$$

The solution for the membrane with central hole (b≤r≤a) is:

$$w_2^t = -C_3^t \frac{a}{k_m} I_0\left(k_m \frac{r}{a}\right) - C_4^t \int I_2\left(k_m \frac{r}{a}\right) dr + C_5^t \qquad (21)$$

The constants are determined from the boundary conditions at r=a (w_2=0 and dw_2/dr=0) and from the continuity conditions at r=b ($w_1 = w_2$ and $dw_1/dr = dw_2/dr$).

NUMERICAL RESULTS

Using these theoretical considerations, the flow rate, the head loss and the membrane deformation were calculated, as function of inlet and outlet pressures, dimensions of internal channels, frequency of membrane oscillations and piezoelectric pressure. The calculus was made for two types of viscous fluid: water and Fomblin Z25 oil. The flow rate increases with velocity, but also the head loss increases. For obtaining the best configuration we defined a new parameter named "inefficiency ratio" which represents the ratio between head loss and flow rate, and must be minimized in order to obtain a maximum efficiency of the micropump.

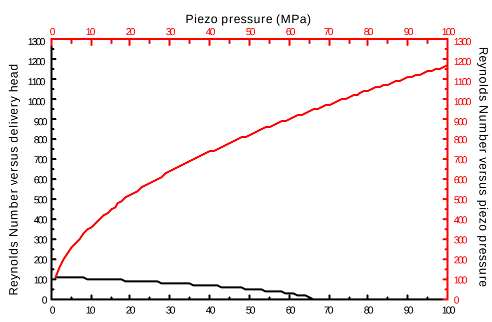

Fig. 2 Reynolds number for water vs delivery head and frequency

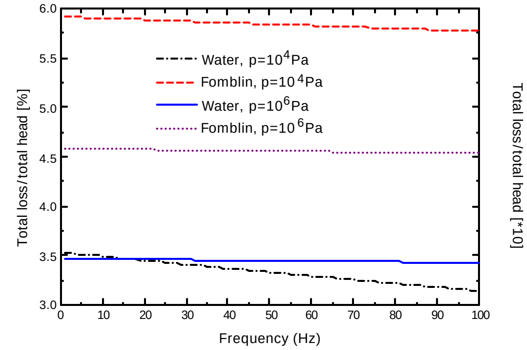

Fig.3 Head loss/head ratio versus frequency

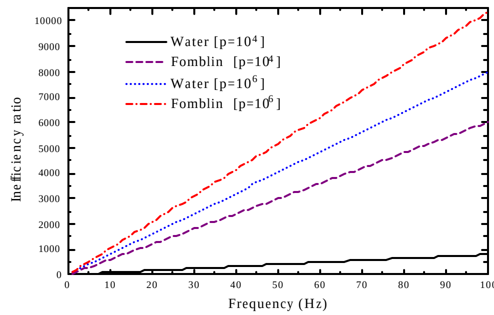

Fig.4 The inefficiency ratio versus frequency

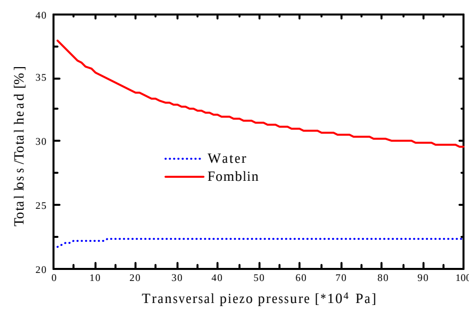

Fig. 5 Head loss/head vs piezo pressure

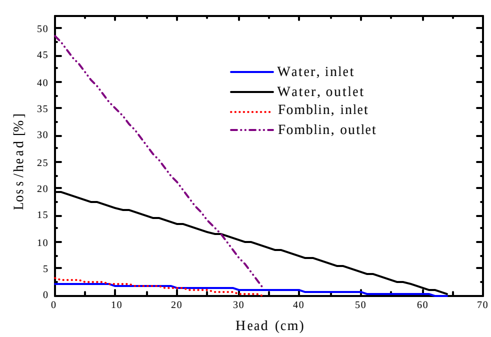

Fig. 6 The inefficiency ratio vs suction/delivery head

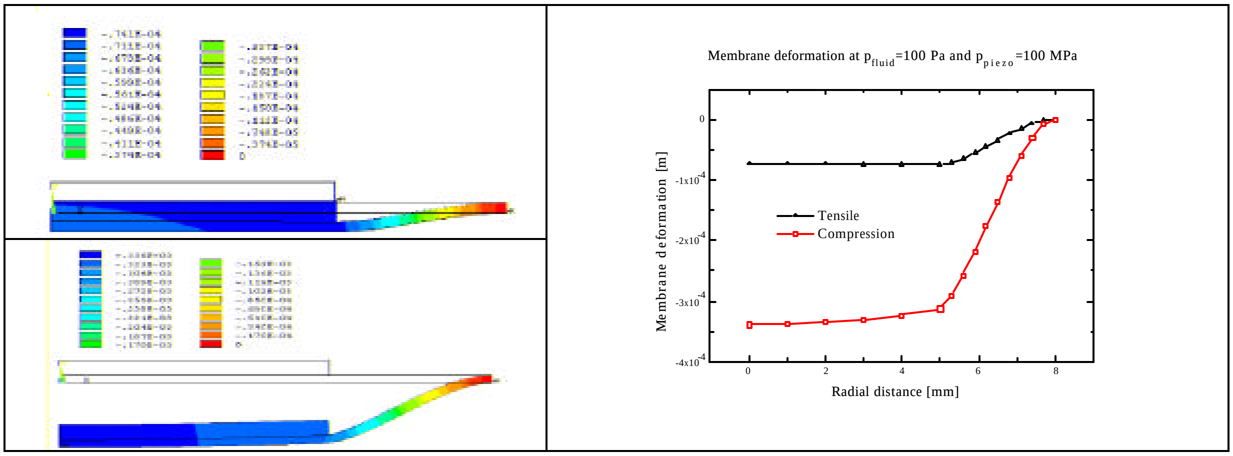

Fig. 7 Tensile and compressive deformation of the membrane at p_{fluid}=100 Pa and p_{piezo}=100 MPa

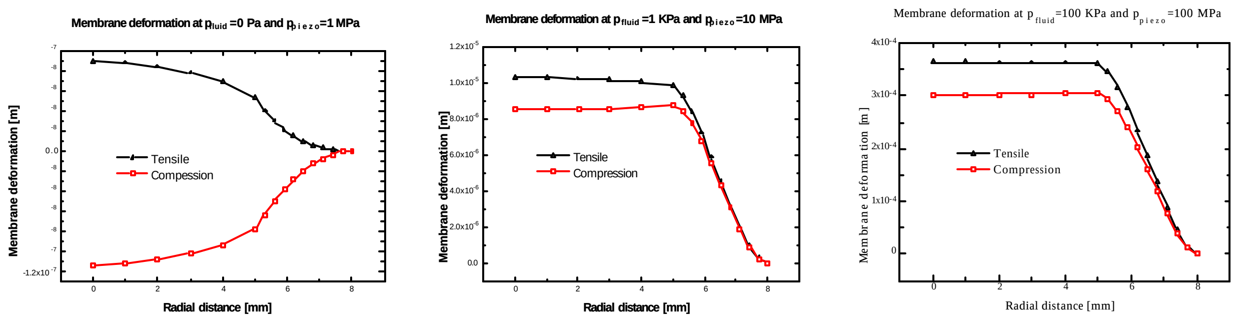

Fig. 8 Tensile and compressive deformetion of membrane for different cases

DISCUSSIONS AND CONCLUSIONS

There are two points of view regarding the volume pumped in a cycle. For an ideal efficiency, the pumped volume is equal to the volume stroke from the brass membrane. The total pressure on the brass membrane is composed by the piezoelectric pressure, atmospheric pressure and the pressure of fluid inside the pump chamber. The sum of fluid pressure and the atmospheric pressure is a static pressure, which causes a static transversal deformation of the membrane, which is, in fact, a “lost volume”. The real volume stroke is caused from the radial strain applied on the membrane by the piezoelectric actuator, as result of alternating applied voltage V.

The second point of view regards the real condition for the flow, which depends on the inlet and outlet parameters. When the frequency increase, the response time of the fluid decreases, due to the inertial and viscosity effects, as well as the flow rate will be smaller. The influence of suction head and delivery head on the flow rate is bigger then influence of frequency. So, the frequency permits a fine control of the flow rate around a desired value, as function of head.

The flow remains laminar in the entire regim of working mode. The Reynolds number for water increase to 1200 for high piezo pressure and decrease with delivery head. For Fomblin Z25, the value of Reynolds number is much lower (of order of units), because the kinematic viscosity is higher.

The results show that the pump efficiency is higher for smaller frequencies. The strong decreasing of efficiency as function of frequency is caused by two facts: decreasing of mean velocity and increasing of volume stroke, when the frequency increases.

A comprehensive ANSYS analysis for membrane deformations in various conditions for fluid and piezo pressure has been performed; a practical range for piezo pressure has been appreciated and a polynomial regression was applied to fit the shape of deformed membrane in order to calculate the stroke volume.

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