System-level Optical Models of 3-D Laser Projection Systems using Micromirror Arrays

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ABSTRACT

In this paper the development of system-level optical models that can be used e.g. for simulating entire laser projection systems is presented. After a short motivation ray optics as the simplest theory of optics is introduced. For the general case of tracing non-meridional rays in 3-D space the vector forms of the law of refraction and the law of reflection are provided, which lend themselves especially to implementation in math tools. The more advanced concept of beam optics is presented then. The simulation results obtained with the implemented models are provided and compared to the behavior of the real-world panoramic projector.

Keywords: Micromirror array, laser scanner, system-level modeling, geometric optics, Gaussian beam

INTRODUCTION

Several papers on MEMS/MOEMS components for free-space laser beam deflection and scanning purposes have been published within the last few years, some of them focusing on micromirrors, e.g. [1] and —from the Chemnitz University of Technology— [2], [3]. In order to efficiently support the (computer-aided) design of entire systems containing such components (as the laser projection system presented here) appropriate system-level models of all their components are needed. They provide behavioral system simulation capabilities for creating a virtual prototype of the whole system to predict the influence of certain parameters (geometry, resonant frequencies etc.) on the system performance.

A first laser projection system was built up using two 1-D micromirror arrays (mirrors can be tilted about one axis by applying an electrostatic force) as depicted in figure 3 and another one where a panoramic projection can be achieved using a 2-D micromirror and an additional circularly symmetric reflector, see figures 9 and 6. Recently a new silicon bulk-micromachined 2-D micromirror array of 7×7 mirror cells (see figure 2) was designed to replace the single mirror. With the small mirrors having a resonant frequency $10\times$ higher than that of the single mirror it will be possible to increase the number of image lines tenfold.

To do joint system-level simulations of such a projection system also the optical domain has to be considered, i.e. first of all the resultant beam path from the laser to the screen has to be traced. This paper deals with the system-level modeling of such components.

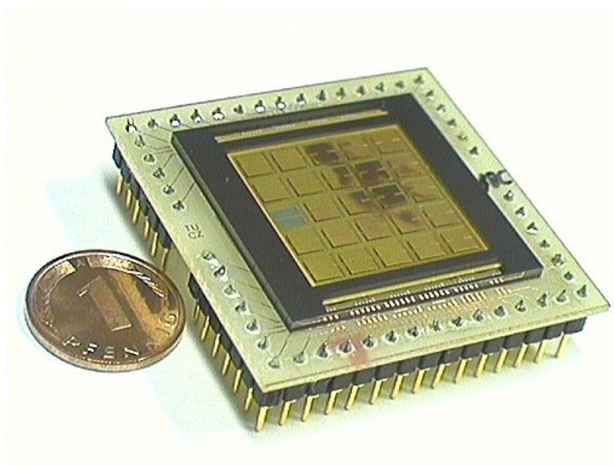

Figure 1: 1D mirrorarray mounted on a chip carrier

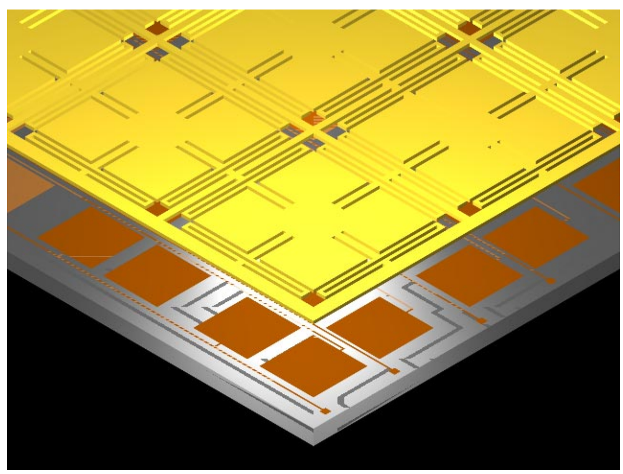

Figure 2: Rendered detail of the 2-D micromirror array

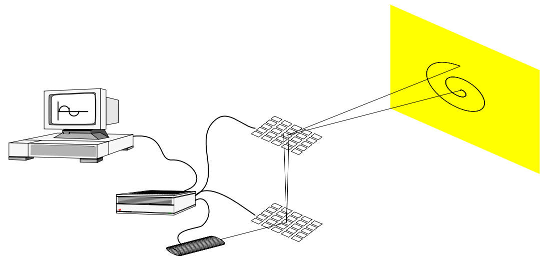

Figure 3: System Diagram of the Laser Projection System

OPTICAL MODELING

A key feature to be modeled is the path of the laser beam that travels through the system and their spatial modulation as a function of time due to the dynamic behavior of the microelectromechanical actuator(s). *Ray optics* (also called *geometrical optics*) being the simplest propagation model for optical signals is sufficient for a first approximate simulation of the dynamic system behavior.

Ray Optics

In this theory light is described by rays that travel in different optical media governed by a set of geometrical rules which can be derived from the postulates of ray optics. Thus, light rays are refracted at the boundary between two media (with refractive indices n_1 and n_2, respectively) according to Snell's Law, $n_1 \sin\varepsilon = n_2 \sin\varepsilon_2$, with the refracted ray lying in the plane of incidence. Modeling the set-up of the projection system usually requires tracing of non-meridional (and non-paraxial) rays in 3-D space—which, by the way, precludes the use of popular ray-transfer matrix methods. In this general case we prefer using the law of refraction in vector form [4] because it eases the representation and lends itself to an implementation in math tools. The direction of the incident and refracted rays is given in terms of the unit vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, respectively, the surface normal is represented by the unit vector $\boldsymbol{n}$. If the sine function in Snell's law is replaced by the magnitude of vector products, $\sin\varepsilon_1 = |\boldsymbol{s}_1 \times \boldsymbol{n}|$ and $\sin\varepsilon_2 = |\boldsymbol{s}_2 \times \boldsymbol{n}|$, we obtain the vector form of the law of refraction

$$n_1(\boldsymbol{s}_1 \times \boldsymbol{n}) = n_2(\boldsymbol{s}_2 \times \boldsymbol{n}) \tag{1}$$

Applying some theorems of vector algebra this equation can be transformed so that the direction of the refracted ray is separated:

$$\boldsymbol{s}_2 = \frac{n_1}{n_2}\boldsymbol{s}_1 - \boldsymbol{n}\left\{\frac{n_1}{n_2}(\boldsymbol{n}\cdot\boldsymbol{s}_1) - \sqrt{1-\left(\frac{n_1}{n_2}\right)^2\left[1-(\boldsymbol{n}\cdot\boldsymbol{s}_1)^2\right]}\right\} \tag{2}$$

A similar expression can be obtained for the law of reflection, which states that the reflected ray lies in the plane of incidence with the angle of incident ray and the that of the reflected ray being related through $\varepsilon_1 = -\varepsilon_2$, both angles measured with respect to the surface normal. It can be shown that the law of reflection is just a special case of the law of refraction and its vector form can be formally derived from the vector form of the law of refraction (1) when the refractive index n_2 of the second medium is set to $n_2 = -n_1$ and the unit vector in the direction of the reflected ray $\boldsymbol{s}_2$ to $-\boldsymbol{s}_2$ [4]:

$$\boldsymbol{s}_1 \times \boldsymbol{n} = \boldsymbol{s}_2 \times \boldsymbol{n} \tag{3}$$

By applying a few transformations the direction of the reflected ray can be separated:

$$\boldsymbol{s}_2 = \boldsymbol{s}_1 - 2(\boldsymbol{n}\cdot\boldsymbol{s}_1)\boldsymbol{n} \tag{4}$$

These formulae together with the classical means of analytical geometry provide the basis for tracing arbitrary rays through complex optical systems, see figure 5. Rays are generally represented as lines in 3-D space $\boldsymbol{r} = \boldsymbol{r}_0 + \lambda\boldsymbol{s}$, planar boundaries between media e.g. in a parameter-free equation of a plane $\boldsymbol{n}\cdot\boldsymbol{r} + p = 0$, spheres and other 2nd order surfaces by their respective equations. The points of incidence of rays with these surfaces are determined by solving the appropriate systems of algebraic equations. For these non-planar surfaces, additionally, the surface normal (tangential plane) at the point of incidence, i.e. the first derivative of the 2nd order surface equation with respect to all coordinates, has to be calculated.

Of course, the very simple approximation of ray propagation does not consider phase, polarization, wavelength or intensity, and the wave nature of light precludes the existence of spatially confined rays without angular spread, but in the following we will see that light can take the form of beams that come as close as possible to this idealization.

Beam Optics

Laser sources generate wavefronts making small angles to the optical axis, their TEM_{00} mode can be exactly described using the *Gaussian beam approximation* [5]. Gaussian beam is one solution to the paraxial Helmholtz equation

$$\nabla_T^2 A - \mathrm{j}2k\frac{\partial A}{\partial z} = 0 \tag{5}$$

where $\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse part of the Laplacian operator and $A = A(\boldsymbol{r})$ is the complex envelope of a paraxial wave $U(\boldsymbol{r}) = A(\boldsymbol{r})\exp(-\mathrm{j}kz)$. An expression for its complex amplitude can be derived from the other simple solution of (5), the paraboloidal wave, by shifting the center of the wave to a purely imaginary point $\xi = -\mathrm{j}z_0$ (z_0 is known as the Rayleigh range):

$$U(\boldsymbol{r}) = A_0 \frac{W_0}{W(z)} \exp\left\{-\frac{\rho^2}{W^2(z)} - \mathrm{j}\left[kz - k\frac{\rho^2}{2R(z)} + \zeta(z)\right]\right\} \qquad (6)$$

where $\rho = \sqrt{x^2+y^2}$ is the radial distance and the parameters beam radius $W(z)$, wavefront radius of curvature $R(z)$, waist radius W_0 ($2W_0$ is called the spot size) and excess phase $\zeta(z)$ are given as follows (see also figure 4):

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}\,, \qquad R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$

$$\zeta(z) = \arctan\frac{z}{z_0} \qquad , \qquad W_0 = \sqrt{\frac{\lambda z_0}{\pi}}\,. \qquad (7)$$

Moreover, the optical intensity, which is a crucial parameter of a projection system,

$$I(r) = |U(\boldsymbol{r})|^2 = |A_0|^2\left(\frac{W_0}{W(z)}\right)^2 \exp\left(\frac{-2\rho^2}{W(z)^2}\right) \qquad (8)$$

can be calculated. The Gaussian beam diverges at an angle of $\theta_0 = \lambda/(\pi w_0)$.

Transmission of a Gaussian beam though different optical components can also be calculated (see [5]) resulting only in a change of the beam parameters while maintaining its general Gaussian characteristics.

Applying the Gaussian beam propagation model the system designer can more exactly predict the projection pattern (e.g. spot size) and the influence of the various optical components on the path from the laser source to the screen (beam-shaping lenses, differently shaped mirrors) without imposing much computational overhead compared to simple geometric propagation models, which must not be neglected in a more or less interactive CAD environment.

For (misaligned) complex optical systems it has been suggested to still model the propagation of the center of the beam using the simple ray model and add the respective equations for the transformation of the intensity and beam waist, but care must be taken because that approximation is restricted to lossless systems and fails e.g. when beams are clipped by optical components [6], [7].

EXPERIMENTS AND RESULTS

We have implemented ray-optical models according to the above described equations (2) and (4) (see figure 5) together with electromechanical models of 1-D and 2-D micromirrors (and arrays of mirrors) in MATLAB® enabling us to carry out transient simulations e.g.

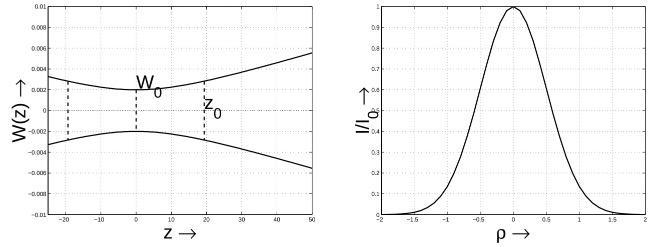

Figure 4: Width and normalized intensity of a Gaussian beam

of the entire panoramic laser projector whose virtual prototype is depicted in figures 9 and 6, see [8] and [9]. The real experimental projection system currently consists of a laser diode, the electrostatically driven 2-D micromirror, an additional rotation-symmetric reflector (a metal-coated aspherical lens) and a Perspex tube serving as the cylindrical screen. When excited appropriately the micromirror will perform a circularly polarized oscillation in resonant mode eventually leading to a helical trajectory of the laser spot on the screen, see figure 6. The simulation results we obtained (see figure 9) exhibit very good correspondence with the behavior of the real-world system providing verification of our models. With a slight extension of the models to also account for bundles of marginal rays of the laser beam we have provided a first approximation of the beam diameter transformation in the system. Moreover, we could study the effects of beam focussing, see figure 9. Currently the Gaussian beam propagation model is incorporated into our models to get a better beam approximation.

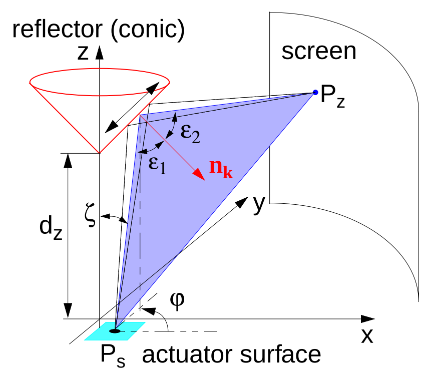

Figure 5: Modeling laser beam propagation using ray optics

CONCLUSION, OUTLOOK

Appropriate system-level models of MEMS components that consider only the behavioral aspects relevant behavioral aspects of a virtual system prototype provide an efficient means to the early microsystem design steps for they enable a fast trade-off analysis and refinement of the component specification. Future work will focus on modeling other complex systems, e.g. a free-space optical switch, and taking into account further effects such as diffraction that gain more importance as the dimensions of the microstructures shrink. Besides, the calculation of reverse beam propagation will be tackled.

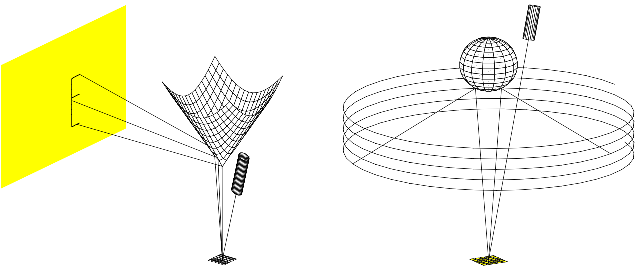

Figure 6: Laser projection system using a conic or spherical reflector

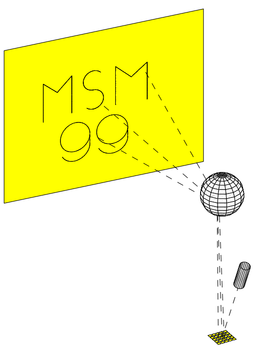

Figure 7: Simulation of vector graphics mode

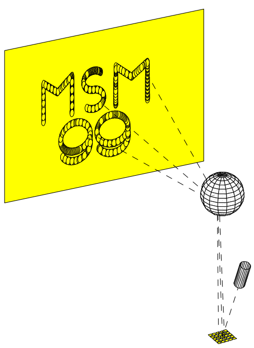

Figure 8: Bundles of marginal rays

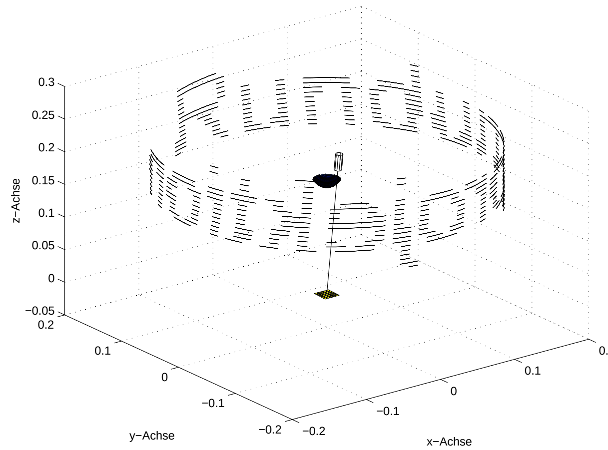

Figure 9: The simulated panoramic projector

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