Analytical Simulation of a 1D Single Crystal Silicon Electrostatic Micromirror

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ABSTRACT

This paper is devoted to the study of the static and dynamic behavior of a 1D torsion single crystal silicon micro mirror. Our aim is to develop a parametric model allowing the use of single calculation software in order to avoid heavy Finite Element Method simulations. The mechanical part was also validated owing to FEM simulations with ANSYS. The most common electrodes are planar ones. We have developed a model of both planar and inclined electrodes micro mirror. MATLAB is used to simulate the steady-state response, the dynamic tilting response to a step load excitation and the frequency response. Global simulation under SABER was carried out in order to compare the micro mirror simulation with experiment.

Keywords: micro mirror, analytical modeling, electrostatic interaction

INTRODUCTION

This micro mirror is developed for a 3D-laser camera [1] concerning a micro robotic application where the deflection of the laser beam is realized by silicon micro mirrors. Satisfy specifications (large angles, lowest driven voltage, high natural frequency and minimum rise time) cannot be obtained by classical planar electrodes micro mirrors and are the results of compromises. This study tries to put in place a modeling to define the best structure in regard of various specifications by taking into account the technological possibilities and to simulate the experimental characteristics before the technological process.

DEVICE MODELING

General overview

The system is composed of two parts (Fig.1): *the micromirror* suspended by two torsion beams, and below *the electrodes* located on each side of the torsion beams axis. Electrostatic forces due to a voltage applied between the mirror and one electrode allows the structure to tilt around the beams. The equilibrium is obtained when the mechanical restoring torque is equal to the electrostatic torque. As the voltage is increased, the electrostatic torque increases and eventually overcomes the mechanical torque. This particular value of voltage is called for electrostatic actuators, the pull-in voltage $V_{pull-in}$ which corresponds to the pull-in tilting angle $\theta_{pull-in}$. Over these values, it is impossible to control the system. The instability threshold is reached.

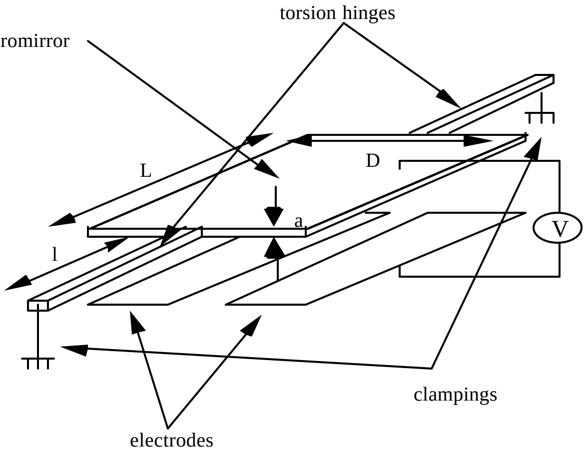

Figure 1: A schematic view of the micro system

The classical dynamical equation of motion of the device is given by (1):

$$I\frac{d^2\theta}{dt^2}+\eta\frac{d\theta}{dt}+M_r=M_e \qquad (1)$$

where I is the moment of inertia, η is the damping factor, M_r the elastic restoring torque and M_e the electrostatic torque.

Moment of inertia I

The moment of inertia about the axis of rotation is given by (2).

$$I=\frac{m}{12}\left(a^2+D^2\right) \qquad (2)$$

where m is the mass of the system, D and a are respectively the width and the thickness of the mirror.

Damping factor h

Factors that give rise to energy loss are: acoustic radiation, internal friction of the system and viscous drag force of the surrounding gas. The acoustic radiation is discussed in [2]. According to Auld [3] and Zener [4] the main contribution of the internal friction in MEMS can be derived from one source: the thermoelastic mechanism that has its origin in thermal energy flow from compressed region to tensile regions. We have estimated by a first order calculation that for torsional strain distribution, such damping is equal to zero. The viscous drag can be calculated by solving Navier-Stokes equation [5]. Squeeze film damping are calculated with initial conditions avoiding air motion in planar micro mirrors with small air gap between electrodes. This is not our case that’s why we have chosen to determine η by experiments.

Restoring torque

For a mirror suspended by two hinges and for small tilting angles, the restoring torque is given by [6]

$$M_r = 2K\theta \qquad (3)$$

where K is the mechanical stiffness of each mono crystalline silicon torsion beam.

Owing to silicon technology we are able to obtain beams with:

- cross rectangular section (Reactive Ion Etching)
- cross trapezoidal section (Bulk micro machining)

Stiffness formula of these beams is given in [7] for an isotropic material. Because we used single crystal silicon we have to develop a mechanical model using the theory of torsion in an anisotropic material:

$$K = \frac{GJ}{l} \qquad (4)$$

where l is the length of the torsion hinge and GJ the factor of torsion which depends on the section of the beam and on the mechanical properties of the silicon. Such model leads to:

$$GJ = 2\iint_{(S)} \Omega(x, y)dxdy \qquad (5)$$

with $\Omega(x, y)$ the solution of the differential equation (6)

$$C_{55}\frac{\partial^2 \Omega}{\partial t^2} + C_{44}\frac{\partial^2 \Omega}{\partial y^2} + 2C_{45}\frac{\partial^2 \Omega}{\partial x \partial y} = 2C_{45} - 2C_{44}C_{55} \qquad (6)$$

which must old over the area of beam cross-section.

Due to the anisotropy of the mechanical properties of the silicon, we used the elastic coefficients defined by the crystallographic orientation of the torsion beams.

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix}$$

The coefficients are indicated below in the classical presentation. Love [8] found an analytical formula for a cross-rectangular section for some particular orientations corresponding to C_{45} equal to zero. With our model, we found the same stiffness values for such orientations also when using Finite Element Method with ANSYS software. In the case of trapezoidal cross section, we have compared results provided by our model and ANSYS simulation. Values are indicated in table 1 for three different beams with the same length of 2 mm.

	Calculus	**ANSYS**
Beam 1	$7{,}65.10^{-10}$ m^4	$7{,}73.10^{-10}$ m^4
Beam 2	$6{,}19.\ 10^{-10}$ m^4	$6{,}27.10^{-10}$ m^4
Beam 3	$4{,}86.10^{-10}$ m^4	$4{,}91.10^{-10}$ m^4

Table I: Comparison Model/ANSYS values for 3 beams

Electrostatic torque

The electrostatic torque is very dependent of the shape of electrodes. The most common electrodes are planar electrodes [9] (Fig. 2).

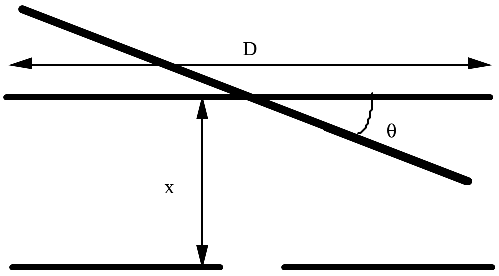

Figure 2:Micromirror with planar electrodes

Solving the Laplace equation for electrostatic potential, V, leads to the expression of the charge per unit area σ. The normal force per unit is given by

$$F = s^2 / 2e \qquad (7)$$

where ε is the dielectric permitivity of the material separating the plates. Then we easily get the electrostatic torque M_e [10]:

$$M_e = \frac{\varepsilon\ V^2 L}{2\theta^2}\left[\ln\left(\frac{x - \frac{D}{2}\sin\theta}{x\cos\theta - d\sin\theta}\right) + \frac{x}{x - \frac{D}{2}\sin\theta} - \frac{x}{x\cos\theta - d\sin\theta}\right]$$

A recent paper [11] validates this model.

We have also developed a model for α inclined electrodes. The same way of calculation is used but in cylindrical coordinates in order to take benefit of the symmetry. For our device (Fig.3), the electrostatic torque is given by:

$$M_e = \frac{\varepsilon \; V^2 L}{2(\alpha-\theta)^2} \ln\left(\frac{x}{d \sin\alpha}\right) \qquad (8)$$

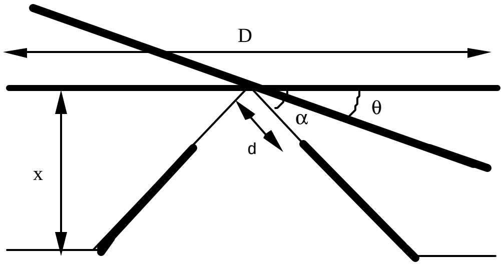

Figure 3: Micro mirror with inclined electrodes

SIMULATIONS

In this section, a comparison analysis of the two types of electrodes is discussed owing to static simulations. Then the global system is simulated with MATLAB software. Finally, SABER allows us to simulate the experimental set up including electronic command circuit.

Static simulations

To compare the structures of electrodes, we have to analyze the steady-state response by setting

$\frac{d^2\theta}{dt^2} = \frac{d\theta}{dt} = 0$ in (1) that becomes $M_e = M_r$.

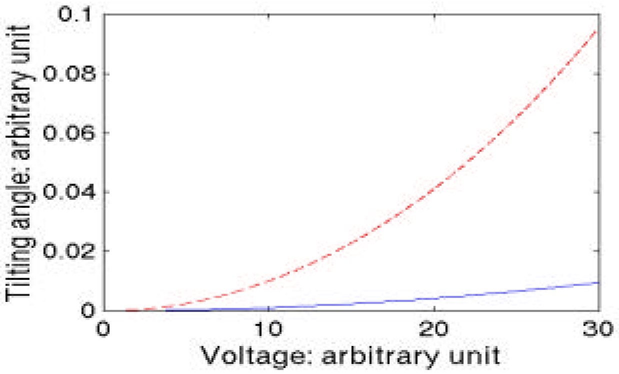

Figure 4: Steady-state tilting angle response for two systems that only differ by the shape of electrodes.
Dashed line: non-planar electrodes (35 degrees)
Solid line: planar electrodes

Figure 4 shows that the use of inclined electrodes reduce the driving voltage. Moreover, we improve the pull-in-tilting angle using inclined electrodes. In the case of planar electrodes, the electrostatic interaction is low. To improve the interaction, we can increase the width D of the mirror. But because $q_{pull-in} \prec D^{-1.5}$, by this way, we also reduce $\theta_{pull-in}$ to a few degrees. Our model shows that the use of inclined electrodes allows reaching values of $\theta_{pull-in}$ of about 10 degrees in the case of 35 degrees. The same observation can be made with a 54 degrees inclined electrodes with $\theta_{pull-in}$ of about 18 degrees but with much higher actuation values.

Static and Dynamic simulations

Once we have developed a model for each element of equation (1), we are able to simulate the micro mirror. We used MATLAB software, which allows us to study the steady-state response (a), the frequency response (b) and the dynamic response to a voltage step (c) (Fig.5).

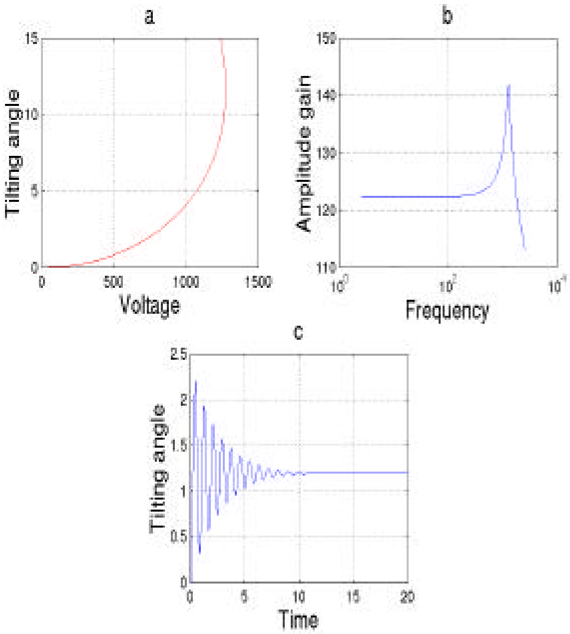

Figure 5: MATLAB simulations

As the micro mirror behavior can be considered as a second order, by fitting the experimental response to a voltage step, the logarithmic decrease of the curve leads to the η value. We obtained $\eta = 9.3 \; 10^{-11}$ Nms/rad for a micro mirror with planar electrodes (bought at Chemnitz University, Germany). We have verified that our model gives exactly same characteristics (static and dynamic). This value of η will be taken now for inclined micro mirror simulations awaiting experimental values from our inclined micro mirror.

SABER simulations

The SABER simulation has been carried out in order to simulate the experiment including the electronic command and test various signal input sequence. The electronic command (i.e. the two voltage amplifier circuits) and blocks describing torsion beams, the two electrostatic actuation, inertia and damping are illustrated on figure 6.

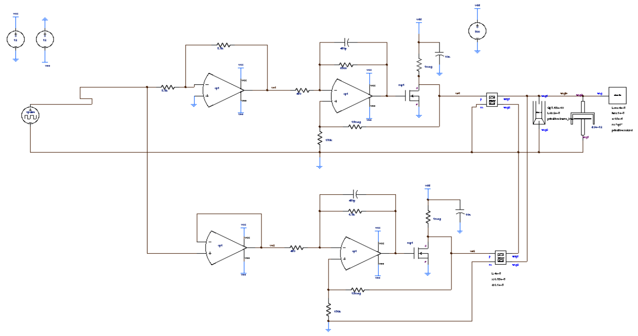

Figure 6: Schematic of the simulated system

An example of a response is given in the figure 7.

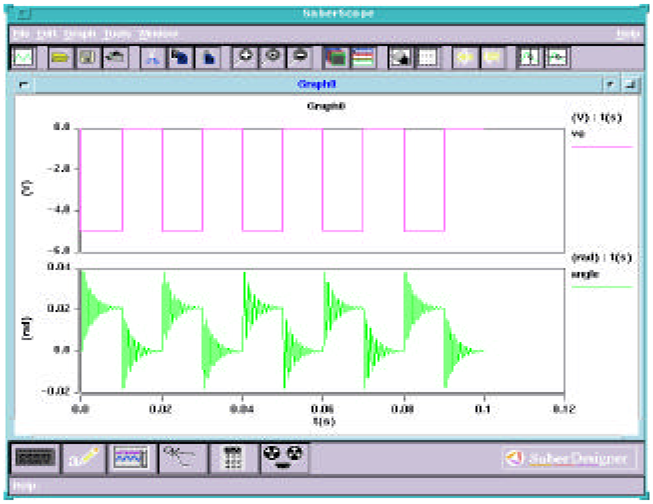

Figure 7: Square stimulus and angular response of the micro mirror

CONCLUSION

A complete modeling of mechanical and electrostatic part of a planar and inclined electrodes micro mirror have been developed. Comparison with experiments in the case of a planar electrode has been performed successfully. The benefit of inclined electrodes in term of angle and voltage is clear. The low damping factor induce to put in place a close loop control to decrease rise time to take advantage of such structure [12]. The two simulations developed, under MATLAB and SABER, are for instant of the same interest. The work done with SABER is a preliminary step toward the development of a MEMS components library in order to describe a micro system as a assembly of elements as it is done today in electronic field.

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