Electro-Mechanical Transducer for MEMS Analysis in ANSYS

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ABSTRACT

The paper introduces an electro-mechanical transducer finite element (EMT) for the strongly coupled simulation of micro electro-mechanical system (MEMS) devices in the commercial finite element analysis (FEA) package ANSYS.

Keywords: micro electro-mechanical system MEMS, finite element analysis FEA, transducer, capacitor, strongly coupled simulation.

INTRODUCTION

The simulation of micro electro-mechanical systems (MEMS) involves numerical analyses in diverging areas of physics, such as solid mechanics, electromagnetics, heat transfer, fluid dynamics and acoustics. The interaction between these individual phenomena can be simulated by a coupled field analysis. The measurement of signals and control of the devices are carried out by electrical circuitry. Therefore, the coupling should incorporate electrical circuit simulation. The coupling can be weak or strong. In a weakly coupled case, the physics domains are analyzed individually and sequentially in an iteration loop where convergence difficulties can occur. In a strongly coupled case, the whole problem is solved simultaneously providing full system eigen frequencies and stability features.

ANSYS is a general purpose software package based on the finite element analysis (FEA). This allows full 3-dimensional simulation without compromising the geometrical details. The analysis can be static, harmonic, modal and transient; linear or nonlinear. Electrical and mechanical lumped elements facilitate discrete circuit simulation. ANSYS parameter design language (APDL) provides a convenient way to realize a weak coupling.

The central element of many MEMS devices is a comb structure. It is essentially a capacitor with variable geometry. The capacitor plates can deform and move relative to each other due to electrostatic, inertia or mechanical forces, resulting in a measurable capacitance change. This change transduces electrical and mechanical energy and provides information about the location of the plates. By applying appropriate voltage to the plates, the gap between the plates can be controlled.

The paper describes a new electro-mechanical transducer element, EMT, (Figure 1) to realize strong coupling between ANSYS mechanical and electric circuit elements. System analysis of MEMS devices can be conveniently analyzed by EMT in an FEA domain or in circuit simulator (Figure 2). The paper presents examples for the analysis of coupled system static stability (Figure 3), prestressed eigen frequencies, and prestressed harmonic response (Figure 5). Transient signal propagation has also been successfully investigated with EMT, but these results can not be included in the paper due to space limitations.

TRANSDUCER ELEMENT

EMT shown in Figure 1 has the electrical potential u and the mechanical gap displacement x (in the nodal coordinate system) degrees of freedoms (DOF) as the across variables whereas the current i and the force f are the through variables. This allows strong coupling between ANSYS solid mechanical and electrical circuit elements (Figure 2). During the solution phase a "multi-physics" FEA system matrix is assembled and the structural and electrical DOFs are solved simultaneously. This eliminates convergence problems of weakly coupled techniques and robustly accelerates the nonlinear iteration process.

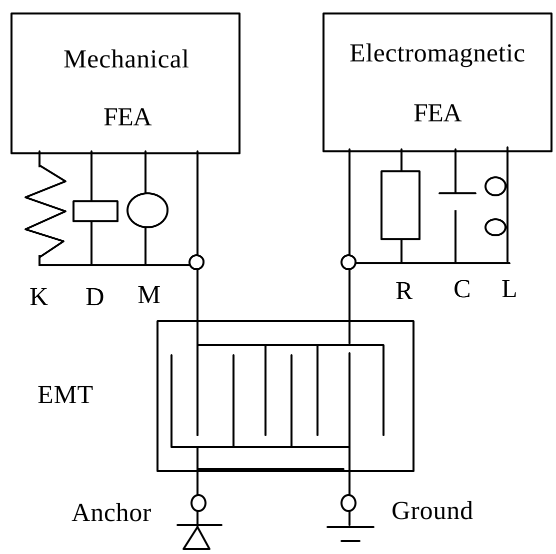

Figure 1: System analysis flow chart

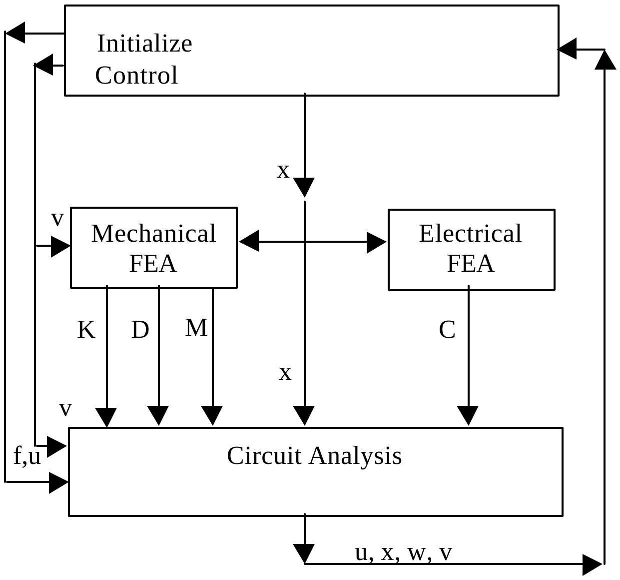

Figure 2: Coupled system with EMT

Please note the importance of the *x* and *u* DOFs. They are both the natural DOFs of solid mechanics and electric circuit analyses. On the mechanical side *x* is the primary variable, nodal displacement is the unknown of all classical FEA formulations. On the electrical side a charge or flux formulation would prevent the analysis of steady state currents. Moreover the *u* DOF allows the application of the computationally efficient nodal potential method which is conforming to standard FEA technology.

An alternative of the two DOF EMT is a single DOF formulation exploiting an electrical-mechanical analogy. Whereas there have been several excellent results obtained with analogy methods, their scope is definitely limited. Here is why. Replacing mechanical elements by electrical circuit elements prevents distributed FEA models since solids must be reduced to discrete elements. Thus, the reverse way is more promising because FEA allows connecting lumped and distributed mechanical elements. (This is a standard feature in ANSYS). Unfortunately an *x* - *u* DOF translation is not feasible because there are gaps in the analogy listed in Table 1. Using velocity *v* instead of *x* (since a *v* - *u* translation has no gaps as shown in Table 2.) a steady circuit solution with constant *u* would translate to constant *v* i.e., continuously increasing *x* - a transient on the mechanical side. An integrator element may help, but an integrator has zero main diagonal entries, preventing the direct application of many equation solver techniques.

Even if there were some feasible way to apply an analogy with a nonstandard implementation there remains the need for a "physics" transformer. An ideal transformer is conservative: the power coming in one port is equal to the power going out in the other port. A capacitor transducer is also conservative but the power of the two ports are different in general: the difference is stored in the form of electrostatic energy.

displ.	*x*	*u*	Volt
veloc.	*v=dx/dt*	*w=du/dt*	Vrat
force	*f*	*i*	Curr
	-	*i=1/L* $\boldsymbol{Y}$	ind.
spring	*f=K x*	*i=1/R u*	res.
damp	*f=D v*	*i=C du/dt*	cap.
mass	*f=M dv/dt*	-	

Table 1: Displacement-Voltage Analogy

veloc.	*v*	*U*	volt
displ.	*x= v dt*	$\boldsymbol{Y}$= *u dt*	flux
force	*f*	*i*	curr
damp	*f=D v*	*i=1/R u*	res.
mass	*f=M dv/dt*	*i=C du/dt*	cap.
spring	*f=K x*	*i=1/L* $\boldsymbol{Y}$	ind.

Table 2: Velocity-Voltage Analogy

The through and across variables are related by:

$$i = \frac{d}{dt}\left[C(x)u\right] = uv\frac{dC}{dx} + Cw = i\left(u,\ x,\ w,\ v\right) \qquad (1)$$

$$f = \frac{d}{dx}\left[\frac{C(x)}{2}u^2\right] = f(u, x, w, v) \qquad (2)$$

where C(x) is the capacitance and w is the voltage rate. The first term in (1) is the motion induced current; the second is the current due to voltage change with fixed capacitor plates. (2) can be easily obtained from virtual work principle.

For small changes the through variables are:

$$i = i_0 + K_{uu}du + K_{ux}dx + D_{uu}dw + D_{ux}dv \qquad (3)$$

$$f = f_0 + K_{xu}du + K_{xx}dx + D_{xu}dw + D_{xx}dv \qquad (4)$$

where:

$$K_{uu} = \frac{di}{du}; K_{ux} = \frac{di}{dx}; K_{xu} = \frac{df}{du}; K_{xx} = \frac{df}{dx}$$

$$D_{uu} = \frac{di}{dw}; D_{ux} = \frac{di}{dv}; D_{xu} = \frac{df}{dw}; D_{xx} = \frac{df}{dv}$$

where i_0 and f_0 are the entries of the coupled system Newton-Raphson restoring force vector corresponding to a large signal nonlinear solutions vector u_0 and x_0; K_{uu}, K_{ux}, K_{xu} and K_{xx} as well as D_{uu}, D_{ux}, D_{xu} and D_{xx} are the entries of the tangent coupled system stiffness and damping matrices, respectively.

EXAMPLES

The static nonlinear solution is obtained by the Newton Raphson iteration until the external loads are balanced by the restoring forces. In general there may be many solutions or no solution at all. ANSYS converges robustly to both stable and unstable solutions (Figure 3) if started in the vicinity of a root and returns with an error message if there is no solution. At the pull-in voltage there is only one solution.

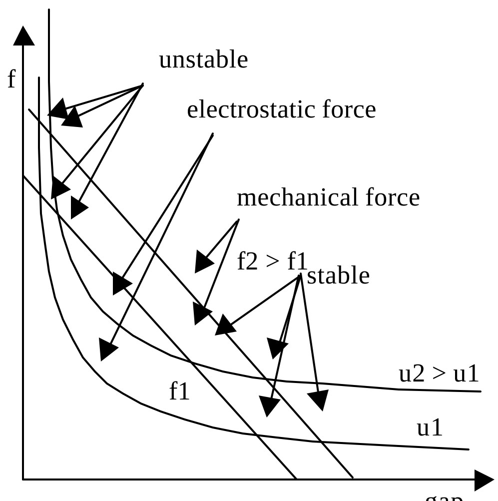

Figure 3: Static stability

A longitudinal beam on Figure 4.b has been analyzed with the data in [1]. The computed stable and unstable roots are in excellent agreement with the analytic solution and the results in [1] (Table 3).

	unstable		stable		
volt	analytic	ANSYS	analytic	ANSYS	[1]
70	.62878	.62881	.10920	.10915	.11
80	.54379	.54386	.16064	.16063	.16
90	.40609	.40612	.26553	.26540	.26

Table 3: Longitudinal beam displacement [μ]

Voltage [V]	Angle [mrad]	Gap [micron]	Torque [nJ]
0.00	0.0000	3.4200	0.0000
10.00	0.2833	3.2769	-4.0543
19.00	1.6266	2.5986	-23.2748
19.50	1.9545	2.4330	-27.9667
19.63	2.1789	2.3197	-31.1772

Table 4: Torsional beam solution

A torsional beam on Figure 4.d has been analyzed with the MMV-type mirror actuator data in [2]. The capacitor plate has side sizes of *b=1300 m* and *a=a2-a1=150 m* The zero voltage gap is *3.42 m* The computed data in Table 4 are in excellent agreement with [2]. The computed pull-in voltage in [2] is 19.358 V, measured value is 19.5 V, ANSYS returns 19.65 V.

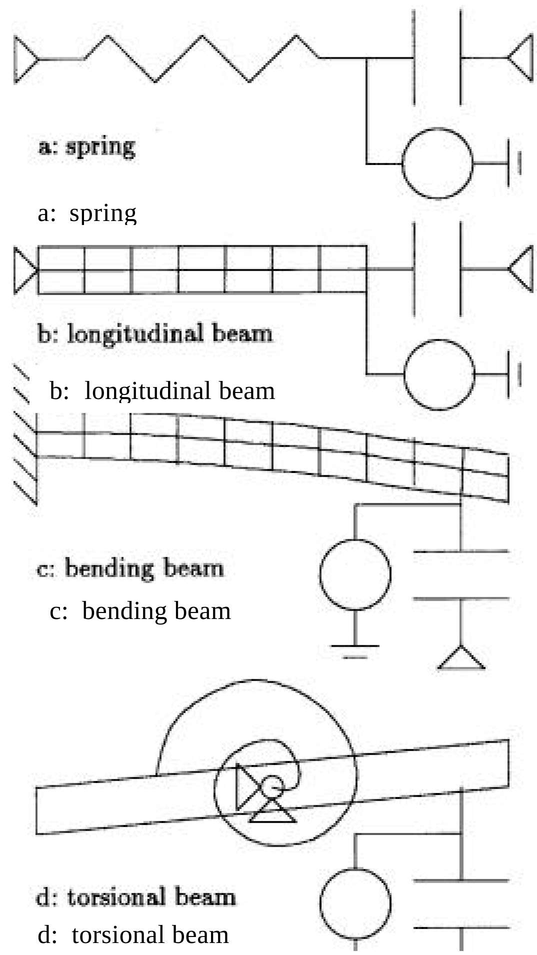

Figure 4: Beam-EMT systems

The eigen frequencies or frequency response of MEMS resonator or filter can vary based on the DC bias voltage applied to the comb drive. To account for these "pre stress" effects a nonlinear static analysis should precede the small signal linearized harmonic/modal analysis characterized by the tangent stiffness and damping matrices given by (3) and (4).

A beam longitudinally prestressed by a comb drive in Figure 5 has been analyzed with the data in Table 5. The analytic eigen frequencies are [3]

$$f_i = \frac{l_i^2}{2pl^2}\left[\frac{EI}{m}\right]^{\frac{1}{2}} \quad (5)$$

$$l_i^2 = i^2 p^2 \left[1 + \frac{fl^2}{EIi^2 p^2}\right] \quad (6)$$

$$f = \frac{e_0 A_c u^2}{2g^2} \quad (7)$$

The analytic and computed data show excellent agreement (Table 6); the frequency response from a transverse excitation has sharp peaks at the analytic eigen frequencies (Figure 6).

E	169x10^3	mN/mm^2	Young modulus
I	$bh^3/12$	mm^4	Inertia
l	150	mm	beam length
b	4	mm	beam width
h	2	mm	beam thickness
m	2.332x10^{-15}	Kg/mm^3	mass density
e_0	8.854x10^{-6}	pF/mm	free space permittivity
A_c	100	mm^2	capacitor plate area
u	150.15	$Volt$	DC bias voltage
g	1	mm	capacitor gap

Table 5: Pinned-pinned beam data

	No prestress		Prestress	
Mode	analytic	ANSYS	analytic	ANSYS
1	343.1	343.1	351.7	351.7
2	1372.5	1372.1	1381.2	1380.8
3	3088.1	3086.2	3096.8	3094.9

Table 6: Pinned-pinned beam eigen frequencies [kHz]

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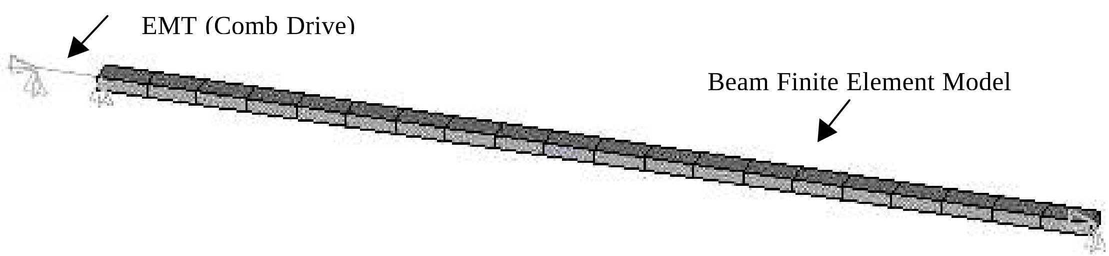

Figure 5: Pinned beam attached to comb drive

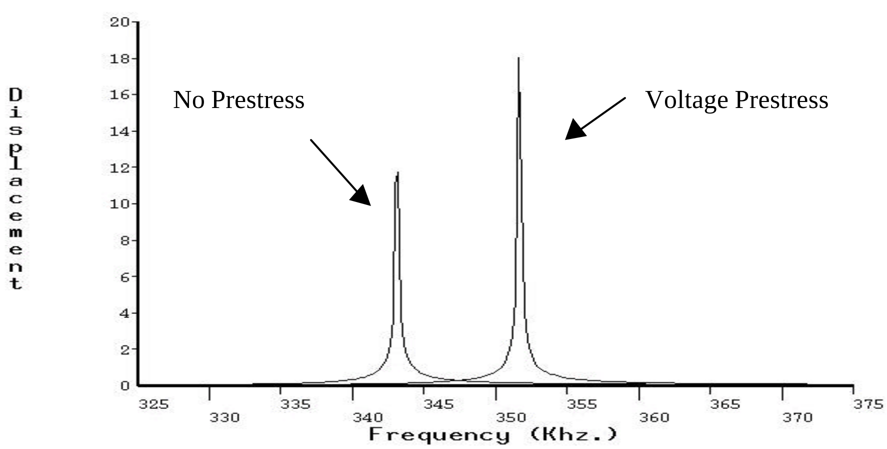

Figure 6: Beam center displacement from a transverse harmonic excitation