SUNRED: a field solver and compact model generator tool based on successive node reduction

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ABSTRACT

The paper presents the new features of the SUNRED field solver program: the visualization of the structures in the 3D version, and the new dynamic simulation methods. Comparisons with other field solvers demonstrate the superior features of SUNRED. A novel type of algorithm, the time constant spectrum analysis is presented in the paper. The algorithm is directly applicable for compact RC model generation of the simulated structure.

Keywords: Thermal simulation, Field solvers, SNR method, Time constant analysis, Compact model generation

INTRODUCTION

Field solvers are very important parts of general-purpose design tools. They solve the Laplace and Poisson equations, the frequently encountered mathematical formulation of many physical phenomena, e.g. electrostatic fields or heat conduction. SUNRED is the first realization of the *Successive Node Reduction* method [1].

Last year we have presented already the 2D version of the SUNRED program [2]. This year the development of the 3D version of the program has been finished and extended also for calculation in the time and frequency domains, both for the 2D and 3D versions. The principle of the successive node reduction (SNR) method is described in details for the 2D case in our last year publication [2]. In the SNR method the structure is divided into elementary rectangular cells, rectangles in the 2D,cubes in 3D case, which are modeled by a lumped RC network. After the definition of the 3D cell model, see Fig.1, the 3D extension was straightforward - the algorithm is the same as in the 2D case. The additional difficulties arose from the visualization of the 3D structures, so this is what we present in the next section. Comparisons with similar algorithms are presented after this. The rest of the paper introduces the novel feature of the 3D SUNRED program, the time constant density analysis that can directly deliver compact models of the analyzed structure.

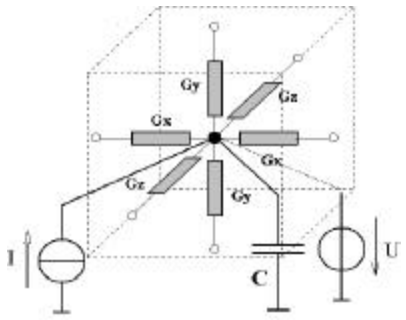

Figure 1. Circuit model of a 3D cell in SUNRED

VISUALIZATION OF THE RESULTS

The 2D SUNRED program was designed to accept picture-like problem input: even micro-photographs can be used to input the structure [2]. SUNRED 3D tried to keep as much as possible from the simplicity and visual power of the problem definition of SUNRED 2D. This aim is clearly visible from the Figures 2. and 3, produced by the graphical user interface of SUNRED 3D.

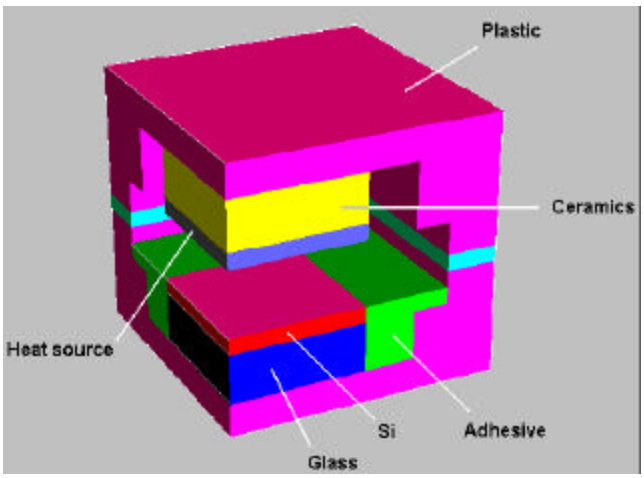

Figure 2. The structure of the micropump

Fig. 2 and 3 illustrate the problem definition and the presentation of the steady state results in the case of the temperature distribution simulation of a thermally operated micro-pump MEMS. Using the symmetries of the

structure, only one quarter of the micro-pump has been simulated.

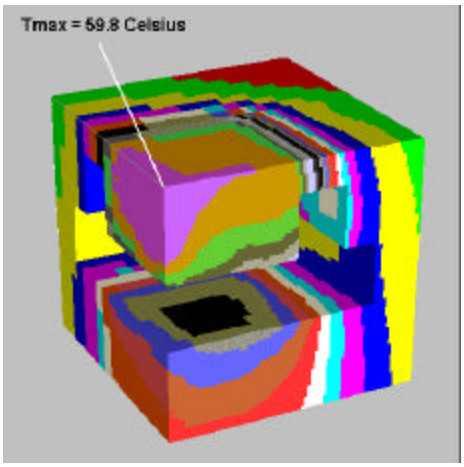

Figure 3. Static temperature distribution in the micro-pump

SPEED OF THE SIMULATION

The successive node reduction algorithm results in efficient calculation of high accuracy results. The run-time properties of the 2D SUNRED algorithm were analyzed in [2]. In the last period we evaluated the time requirement of the 3D version as well. In Fig 4 the requested solution time of SUNRED 3D is compared with four very frequently used sparse solvers in the function of the node numbers of the analyzed structure. The comparison shows that over about 1800 nodes the SUNRED algorithm is much faster, demonstrating that SUNRED can be very effectively used in the case of high accuracy calculations. It is worth to note that for a good resolution simulation at least 16^3-32^3 elementary cells are needed. This means that in practical cases the number of the nodes is 4000-32000, definitely more than the 1800 nodes of the crossing point.

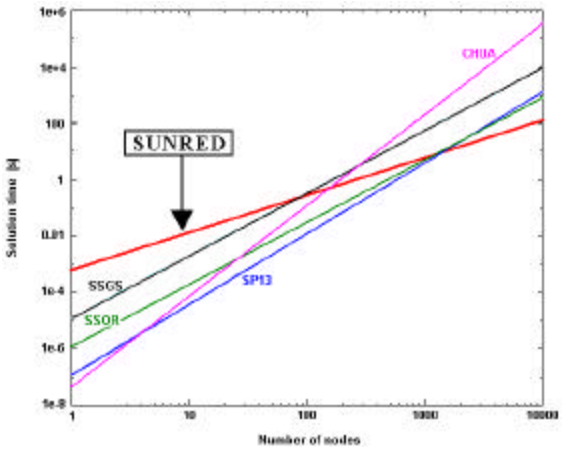

Figure 4. Comparison of the 3D SUNRED solution times with traditional methods

CHUA the solver described by Chua in [8].
SP13 the Sparse 1.3 library from the Univ. of Berkeley, USA [9].
SSGS the iterative sparse symmetric Gauss-Seidel method [7].
SSOR the iterative sparse symmetric successive overrelaxation method [7].

DYNAMIC SIMULATION

The SUNRED program is suitable to simulate dynamic thermal problems as well. Both the frequency domain complex loci of the thermal impedances and the time-domain transients of the system can be calculated by SUNRED. In the latter case the movie-like visualization of the heat-propagation. In our last year paper [2] we put emphasis on the presentation of the transient simulation. The recent developments resulted in two further analysis types both for the 2D and 3D cases. These are the frequency domain simulation and the time constant spectrum calculation.

Based on the dynamic simulation results of both SUNRED versions compact dynamic models can be generated [6] for the examined behavior. The models can be delivered either in the form of a SPICE net-list, or in the form of a behavioral description.

Frequency domain simulation

The new versions of the SUNRED program provide frequency domain simulation as well. In this analysis the admittance matrices of the cells are of complex values. The principle of the successive node reduction remains unaltered, only the calculus had to be modified to handle complex numbers. The solution time is approximately doubled compared to the steady-state calculation. Moreover such a solution is valid only for one frequency value. Aiming to obtain Bode plots or complex loci a sequence of analyses is needed for a considerable number of frequencies (25–100 or even more points). This fact explains that the time requirement of the frequency-domain analysis is usually with two orders of magnitude greater as that of the steady-state simulation.

Results of a frequency-domain simulation made by the 2D-SUNRED are presented in the next two figures. Fig.5 shows the investigated structure. This is a chip, soldered on a layered structure, and we intend to study the effects of a crash and a void on the thermal behavior. The dissipating element lies on the chip's surface (point **A**).

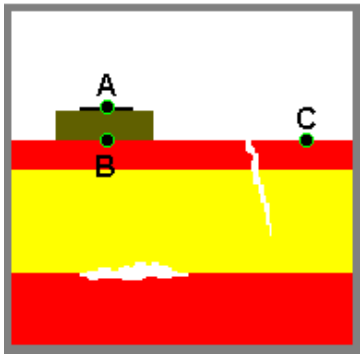

Figure 5. The analyzed structure

We have ivestigated the driving-point thermal impedance at the point **A**, and the thermal transfer impedances of **A**→**B** and **A**→**C**. The impedances were calculated in a frequency range of 5 decades, altogethr in 75 frequency points. The complex loci of the impedances are plotted in Fig. 6. The markers are for 1, 10 and 100 Hz, respectively.

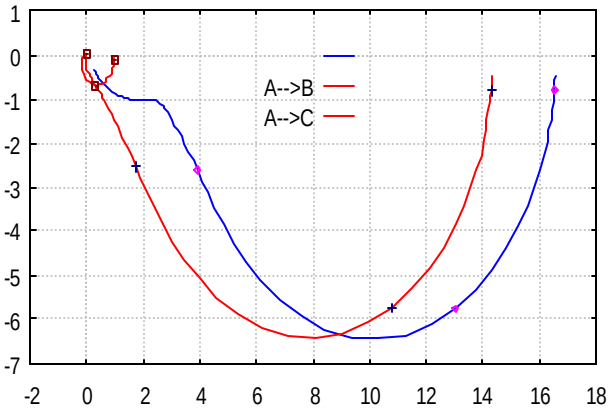

Figure 6. Complex loci of the driving point (A) and transfer (A-B,A-C) impedances

Direct calculation of the time-constant spectrum

A novel and unparalleled feature of the SUNRED program is the suitability to calculate directly the *time-constant spectrum* of the investigated structure.

The time-constant spectrum function R(z), introduced in [3] and [4] is suitable to describe both the driving-point and transfer behavior of distributed RC networks. This function is a generalization of the discrete set of time constants and the related magnitudes are defining a lumped RC equivalent circuit of the system. The function is defined on the

$$z = \ln t, \qquad \zeta = \ln \tau \tag{1}$$

logarithmic time axis by the expression of

$$a(t) = \int_{-\infty}^{\infty} R(\zeta)\,(1-\exp(-t/\exp(\zeta))\,d\zeta \tag{2}$$

where $a(t)$ is the unit-step response of the network. In other words: $R(\zeta)D\zeta$ is the magnitude of the components in the response that belong to the time constant range of $[\zeta, \zeta+D\zeta]$. Equation (2) can be written in an equivalent form as well:

$$\frac{d}{dz}a(z) = R(z)\otimes\exp(z-\exp(z)) \tag{3}$$

where ⊗ is the convolution operator.

Knowing the time-constant spectrum can be useful for two purposes. The first is the time-domain simulation. The unit-step response can be calculated using either (2) or (3), by a convolution operation along the z coordinate. Following this, the transient response may be calculated for arbitrary excitation using convolution again but now in the t time domain. The second benefit is in the generation of compact dynamic thermal models. The time constant spectrum offers a direct way to construct such models (see [5]).

It can be proven (see e.g. [4]) that the $R(z)$ spectrum can be calculated from the $\mathbf{Z}(\mathbf{s})$ complex impedance by using the expression of

$$R(z) = \mp\frac{1}{\pi}\,\mathrm{Im}\,\mathbf{Z}(\mathbf{s} = -\exp(-z)) \tag{4}$$

The use of Eq. (4) requires a great amount of caution. As the equation shows, the imaginary part of the **Z** impedance has to be calculated along the negative real axis of the complex plane. Along this axis, however, singularities lie usually: the poles of the network equation. In such cases we have to avoid to follow exactly the negative real axis, but a line lying close to this axis might be used instead of it that can be given as follows:

$$\mathbf{s} = -(\cos\delta + j\sin\delta)\,\exp(z) \tag{5}$$

This line is shown in Fig. 7 .

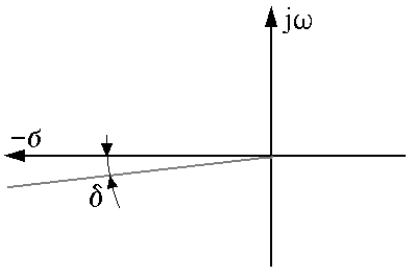

Figure 7. The **s**(z) line on the complex plane

If a field solver program provides the frequency domain calculation then a small extension of the algorithm is enough to generate the time-constant spectrum as well. Only the $j\omega$ imaginary frequency has to be replaced by the **s** complex frequency determined by Eq. (5). The imaginary part of the calculated complex response, multiplied by $1/\pi$, provides the time-constant spectrum according to Eq.(4).

Obviously δ has to be a very little angle ($2–5^{o}$). Even if this angle is small an error is introduced into the calculation, but the value of the error introduced by this calculation can be kept very low. The detailed discussion is beyond the limits of this publication, it will be presented in a longer paper.

A calculated spectrum is plotted in Fig. 8 for a semiconductor chip + package structure. The *x*-axis is scaled in the ω angular frequencies, these are the reciprocals of the time constants: $\tau=1/\omega$. The *y*-axis gives the magnitude of the different time constants occurring in the dynamic response of the system. This diagram is directly suitable to be the base of compact dynamic thermal model generation - by approximating it with a few discrete time constants. This model generation results in a lumped element RC ladder model or in a behavioral description.

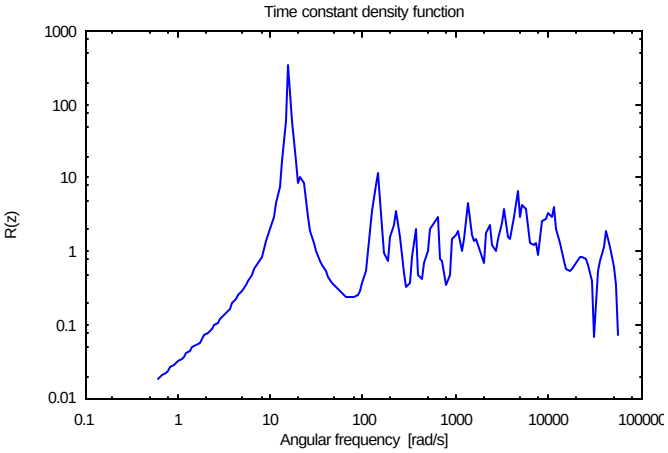

Figure 8. The time-constant spectrum of a semiconductor structure obtained by SUNRED

CONCLUSIONS

The 3D version of the SUNRED field solver is presented in the paper. The program offers a facility for fast thermal or electrostatic simulation of arbitrary shaped 3D structures.

Comparisons with other methods demonstrate that for high accuracy calculations the SUNRED algorithm has superior features compared to the examined widely used methods.

SUNRED is the first program that offers time constant density analysis. This can be used for automated compact RC or behavioral model generation for the examined structure.

The time domain and frequency domain simulations assure various methods to trace the dynamic thermal behavior of arbitrary shaped 3D structures, that can very important in the investigation of thermally operated MEMS.

The algorithms and the solver part of the program are considered to be ready, but the graphical user interfaces are still in the development phase. Currently we focus on developing a user-friendly graphical input that is matching to the fast and easy to use spirit of the solver.

ACKNOWLEDGEMENTS

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