Analysis of Unstable Behavior Occurring in Electro-Mechanical Microdevices

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ABSTRACT

We analyze the instability which is inherent to electrostatically driven microdevices. Further, we propose a homotopy method to overcome this difficulty during simulation of these devices. Starting from a simplified model, the governing differential equations are formulated and their stability behaviour is analyzed. Based on this analysis, a homotopy method is presented which overcomes this instability. Thus the simulation of microdevices with both rigid and flexible structures becomes possible with no regard to stable and unstable areas of operation. The algorithm presented here is based on an iterative coupling of commercially available FEM- and BEM- solvers. Numerical results are presented for a micromirror and a membrane, including the contact problem.

Keywords: micromechanical devices, stability analysis, electrostatic drive, simulation, micromirror, membrane

INTRODUCTION

Most electrostatically driven microdevices exhibit an inherent instability in their operating behavior. Typical examples include comb drives, membrane-driven pumps, microrelays, and micromirrors on the actuator side, or gyroscopes and pressure sensors on the sensor side, where the unstable behavior becomes apparent as the so-called snap-down effect. This phenomenon is an inevitable consequence of the simultaneous competitive action of elastomechanical and electrostatic forces. We discuss the governing equations and perform a stability analysis of a generic model problem. On the basis of these results we are led to a method that allows the analytical treatment and numerical analysis of the regions of instability in the whole operating area of the device.

ANALYSIS OF UNSTABLE OPERATION

As a model problem let us consider a parallel plate capacitor, where one plate is movable and connected to a mechanical spring as shown in Fig.1. We assume a linear spring force with the spring constant k. The area of the capacitor plate is A, the distance between the plates in the equilibrium position at zero voltage is d. The parallel displacement of the movable plate is denoted by x. When a voltage U is applied at the capacitor,

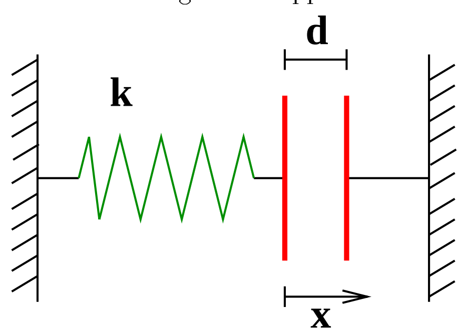

Figure 1: Idealized lumped element model of an electromechanical device

the stationary displacement $x(U)$ is determined by the implicit equation

$$F(U,x) = \frac{1}{2}\frac{U^2 A\epsilon}{(d-x)^2} - kx = 0$$

describing the balance of the mechanical and the electrical force. The diagram in Fig.2 illustrates the balance of forces in a point of equilibrium. The mechanical and the electrical force are drawn as a function of the displacement of the movable electrode, with the applied bias voltage as curve parameter. The hyperbolas represent electric forces, the straight line represents the mechanical force. Balance of forces is attained at the intersectional points of mechanical and electrical force graphs. The left point (square symbol) denotes a stable equilibrium configuration, while the right intersection point (circle symbol) marks an unstable equilibrium condition. When the voltage is increased, the hyperbola representing the electric force moves upward and, thereby, shifts the point of stable equilibrium further to the right, which corresponds to a larger displacement. With further increasing voltage, the stable and unstable point of intersection merge in one single point. The equilibrium of forces becomes unstable and the snap-point is reached. If the applied voltage lies above the snap-down voltage, the movable electrode pushes to the touch-down point on the rigid counterelectrode (triangle symbol),

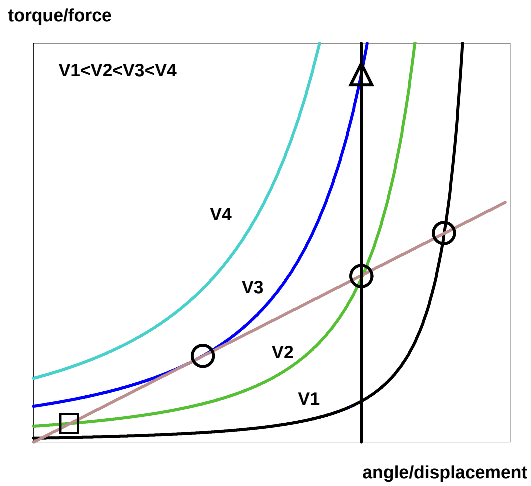

Figure 2: Mechanical and electrical forces vs. membrane displacement for the idealized membrane drive model. Curve parameter is the voltage applied at the electrodes. Circles denote unstable equilibrium points, squares stable ones. The triangle is the position where the insulated capacitor plates touch.

the position of which is represented by a vertical line at $x = d$.

For a characterization of the trajectory $x(U)$ in the $x - U-$plane, we differentiate the identity $F(U, x) = 0$ implicitly with respect to U. In our example, this leads to

$$\frac{dx}{dU} = \frac{-\frac{\partial F(U,x)}{\partial U}}{\frac{\partial F(U,x)}{\partial x}} = \frac{-\frac{UA\epsilon}{(d-x)^2}}{\frac{U^2 A\epsilon}{(d-x)^3} - k}$$

This equation, the so-called Davidenko equation, constitutes a differential equation for $x(U)$. Since the enumerator $\frac{\partial F(U,x)}{\partial U}$ is always non-zero, we encounter two different cases along the solution curve

(a) $\frac{\partial F(U,x)}{\partial x} \neq 0$: The curve can be uniquely continued in a U-neighborhood.

(b) $\frac{\partial F(U,x)}{\partial x} = 0$: The tangent is not defined.

Case **(b)** characterizes the snap-down point where the equilibrium becomes unstable. For finding the continuation of the trajectory we could change the local parametrization from $x(U)$ to $U(x)$. However, this is only possible for structures where the displacement is restricted to one degree of freedom. In the general case (e.g. for flexible structures) we have an infinite number of degrees of freedom. Finding the inverse function is not possible in this situation.

A more general way to tackle the problem is to introduce an additional homotopy parameter, which serves for a local parametrization along a solution trajectory. In our case, the charge Q stored on the capacitor plates is chosen, since this quantity is a unique and monotonous function of any possible device configuration (see Fig.3).

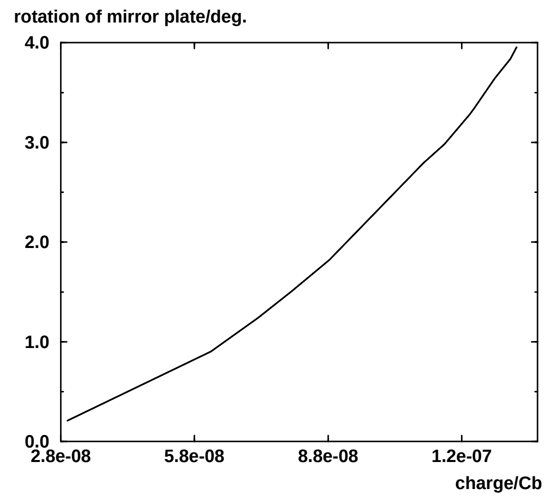

Figure 3: Deflection angle of a micromirror vs. charge stored on the electrodes.

Generally, this approach can be formulated as

$$\begin{pmatrix} F(U, x) \\ p(U, x, Q) \end{pmatrix} = \begin{pmatrix} F(U(Q, x), x) \\ Q - C(x) \cdot U \end{pmatrix} = 0$$

where p denotes an additional scalar equation which establishes the parametrization. Substituting the electrical force $\frac{1}{2} \frac{U^2 A\epsilon}{(d-x)^2}$ by $\frac{Q^2}{2A\epsilon}$ in the model problem we can easily calculate the differential equation for $x(Q)$ which is:

$$\frac{dx}{dQ} = -\left(\frac{\partial F(Q,x)}{\partial Q}\right)^{-1} \frac{\partial F(Q,x)}{\partial x} = \frac{Q}{A\epsilon \cdot k} \qquad (*)$$

NUMERICAL SOLUTION USING HOMOTOPY

As in the example above, the charge Q is also a suitable homotopy parameter for the numerical analysis of the behavior of realistic devices. A direct access to the solution $x(Q)$ would be the integration of the Davidenko equation $(*)$ [2]. This method requires to know the derivatives $\frac{\partial F(Q,x)}{\partial Q}$ and $\frac{\partial F(Q,x)}{\partial x}$, where usually the latter is difficult to obtain from commercially available solvers. Therefore an estimated derivative must be constructed which may be numerically difficult and expensive.

Another access to the solution, which is our approach, is an iterative method which follows a Gauss-Seidel-like relaxation scheme [1]. The mechanical subdomain is calculated using the Finite Element Method (FEM) and the electrical subdomain is calculated by means of the Boundary Element Method (BEM). Both domains are coupled through the common domain interfaces. With

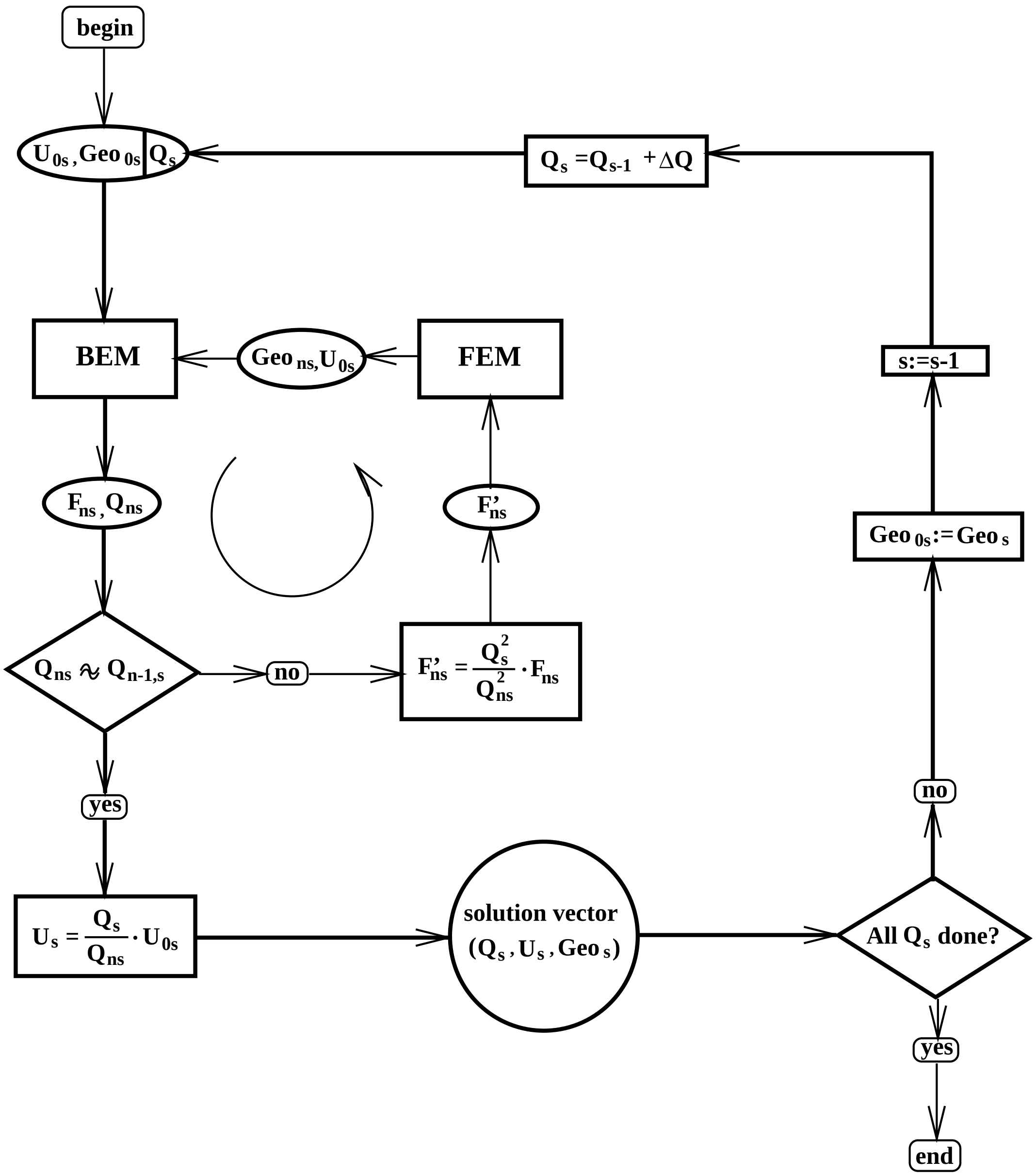

Figure 4: Charge-controlled homotopy algorithm for rigid and flexible structures.

the homotopy parameter Q kept at a given value, a Gauss-Seidel iteration between BEM and FEM is performed until convergence to the desired value Q is obtained. In this procedure, the voltage U is not a control variable but rather a result of the iteration, which delivers a sequence of solution point triples $(U_i, x_i, Q_i)_i$, see Fig.4.

This approach requires no extra treatment of the unstable region beyond the snap-down point but shows stable convergence in the whole range of operation. Fig.5 displays the numerical solution trajectory $U(Q)$ for a micromirror characterized by a single degree of freedom (rotation of mirror). Both the stable and the unstable branch of the numerical solution trajectory are shown, illustrating the general stability analysis in section 2 for a realistic device structure.

As another example, we studied a micromembrane, the dynamics of which is described by an infinite number of degrees of freedom. Fig.6 shows the solution trajectory $U(Q)$of the two stable branches (1,3) and the unstable branch 2. The stable branch 3 corresponds to the situation where a part of the membrane touches the insulated counterelectrode. The more charge is loaded the larger becomes the contact area between membrane and counterelectrode. If voltage were used as control variable, the response of the membrane would be described by a "snap-down and release" hysteresis as indicated by the dashed lines in Fig.6. Along the unstable branch 2,

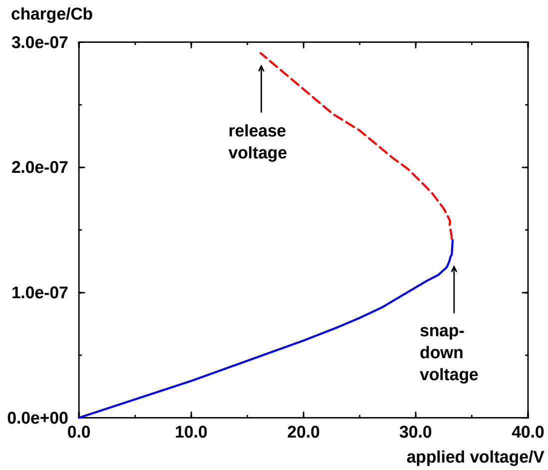

Figure 5: Charge stored on the electrodes of a deflectable micromirror vs. applied bias voltage.

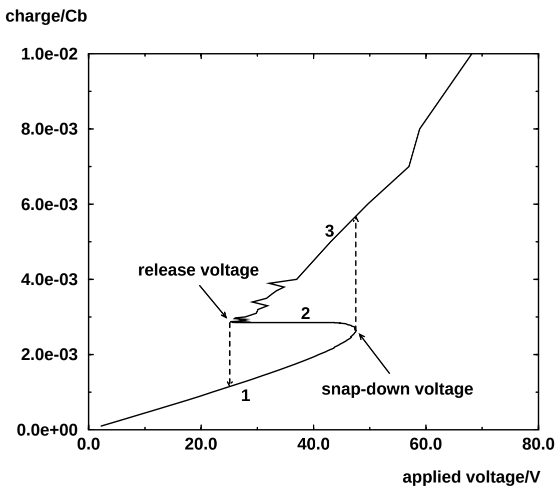

Figure 6: Charge stored on the movable plate of a micromembrane vs. applied bias voltage.

we find a rapid change of membrane shape and voltage with Q. But with Q chosen as homotopy parameter, the bending of the $Q-V$ characteristics near the snap-down point as well as its negative slope portion can be calculated without problems. The obvious numerical roughness of branch 3 is caused by the contact algorithm. Fig.7 shows the deformation of the membrane shape under the action of the controlling charge. As it can be noticed, the shape deformation is smooth with increasing Q until the electrode gets in contact with the counterelectrode. No snapping occurs under charge control. The apperent gap just before the membrane meets the counterelectrode is a consequence of the equidistant discretization of charge used in the numerical procedure but does not indicate snap-down. At a certain charge Q_t, a single point contact is attained. With further increase of Q, the membrane shape becomes more and more rectangular.

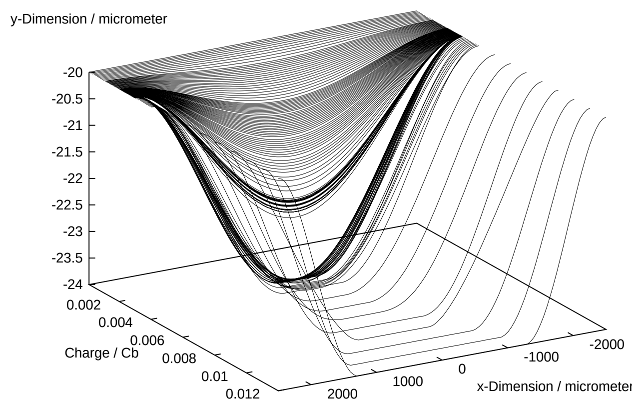

Figure 7: Deformation of the membrane vs. applied charge

CONCLUSION

Based on the stability analysis of the describing equations of electrostatically driven microdevices, a homotopy method was developed to simulate the behaviour of this class of microdvices. This method uses the stored charge as a homotopy parameter which is a unique and monotonous function of the geometric configuration. This homotopy method allows to simulate structures with a finite number of degrees of freedom (i. e. rigid structures like micromirrors) as well as structures with an infinite number of degrees of freedom (i. e. flexible structures like pump membranes).

The numerical approach is to couple a FEM solver and a BEM solver by a Gauss-Seidel relaxation scheme, incorporating the homotopy method.

Numerical results show the feasibility of this approach. The behaviour of the micromirror was calculated from zero position to touch-down, covering both its stable and unstable branch. Snap-down voltage, release voltage, and mirror position were calculated.

The behaviour of the membrane is similar to the micromirror between zero position and touch-down. Snap-down voltage, release voltage, and membrane shape were calculated here as well. After touch-down, membrane operation continues due to the flexibitity of the membrane. Solving the contact problem after touch-down within the FEM solver, the behaviour and shape of the membrane can be simulated in all areas of its operation.

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