Optimum Algorithm for Electronic System Design

A. Zemliak

Physics and Mathematics Department, Puebla Autonomous University
Av. San Claudio s/n, C.U., Puebla 72570, Mexico, azemliak@fcfm.buap.mx

ABSTRACT

The evaluation of operations number for the system design has been done for general design strategy. More general methodology for the system and circuit design was elaborated by means of optimum control theory formulation. This approach generalizes the design process and generates infinite number of the different design strategies. The main equations for this general design methodology were elaborated. These equations include the special control functions that are introduced into consideration artificially to generalize the total design process. Optimum dependencies of these control functions have been obtained by the special optimization procedure and give us the minimum computer design time. Numerical results that were obtained demonstrate the efficiency of the proposed approach.

Keywords: Optimum system design, optimum control theory, maximum principle.

INTRODUCTION

The electronic system design by traditional methodology includes the formulation of the principal equation system, definition the number of independent variables K and the number of dependent variables M and using some type of optimization procedure. The principal equation system can be formulated as algebraic system or integral-differential system. The systems model can be determined as the equation system relation between independent and dependent variables. From the optimization problem point of view this system can be determined as the system of constrains for the objective function optimization.

On the other hand it is possible to use the idea of general optimization [1,2] for the electronic system design. On this way the independent variables vector includes arbitrary number of the systems components from K to $K+M$. In that case the objective function includes additional penalty terms that simulate the relation equations. This strategy can reduce the total computer design time.

In this paper one approach for the system design is proposed. This method is based on the optimum control theory formulation and serves as the generalization of different design strategies. It can reduce considerably the necessary computer design time.

OPERATIONS NUMBER EVALUATION FOR THE GENERAL DESIGN STRATEGY

For the computer time comparison of different kinds of design strategy and for optimal algorithm elaboration it is necessary to evaluate the operations number.

By general design strategy, in case when the number of independent parameters is variable and equal to $K+Z$ the following two systems are used:

$$\frac{dx_i}{dt} = -b \cdot \frac{d\ F(X)}{d\ x_i} \tag{1}$$

$j = 1,2,\ldots,K+Z$

and

$$g_j\ (X) = 0 \tag{2}$$

$j = Z+1, Z+2, \ldots, M$

where $F(X) = C(X) + \frac{1}{e} \sum_{j=1}^{Z} g_j^2(X) \cdot$

In this case the total operations number N for the solution of the systems (1), (2) is equal to:

$$N = L\{K+Z+(1+K+Z)\{C+(P+1)Z+S\cdot[(M-Z)^3+(M-Z)^2(1+P)+(M-Z)P]\}\} \tag{3}$$

when the Newton's method is used.

In case of $Z=0$ the formula (3) gives us the operations number for traditional design strategy and when $Z=M$ formula it is a modified traditional design strategy.

Sometimes the necessary operation number C for the objective function $C(X)$ calculation has no dependency from the independent parameters number $K+Z$, but for the majority of electronic systems is in proportion to the sum $K+Z$ ($C = c\ (K+Z)$). Formula (3) in this case is transformed into following expression:

$$N(Z) = L \cdot \{K+Z+(1+K+Z)\{c(K+Z)+(P+1)Z$$
$$+S \cdot [(M-Z)^3+(M-Z)^2(1+P)+(M-Z)P]\}\} \tag{4}$$

Analysis of the operations number N as the function of Z by formula (4) gives us the conditions of the minimum computer time. In case when the system (2) is the linear one this general design strategy almost has no preference in computer time as shown in [1]. Formula (4) gives the optimum point Z_{opt} that is within the region $[\,0, M]$ for the nonlinear system (2).

In more general case, when the system's model can be separate on two parts as linear and nonlinear we have the following systems :

a) the nonlinear part is

$$g_j(X) = 0$$

$$j = 1,2,\ldots,r\,(M - Y) \qquad (5)$$

b) the linear part is

$$A\,X = B$$

where $r \in [0,1]$; A and B are matrices of the order $(1 - r)\cdot(M - Z)$. For this case the formula for the operations number has the following form:

$$\begin{aligned} N(Y,Z) = {} & L\ \{K + Y + Z + (1 + K + Y + Z) \\ & \cdot\{C + (M+1)Z + [M(1-r) - Z]^3 + M(1-r) - Z + (P+1)Y \\ & + S\cdot[(M\cdot r - Y)^3 + (M\cdot r - Y)^2(P+1) + (M\cdot r - Y)P]\}\} \end{aligned} \qquad (6)$$

Analysis by this formula shows that for the majority of the practice problems it is correct that the optimum point of the function *N(Y,Z)* is within the dominion. This case is illustrated in Figure 1.

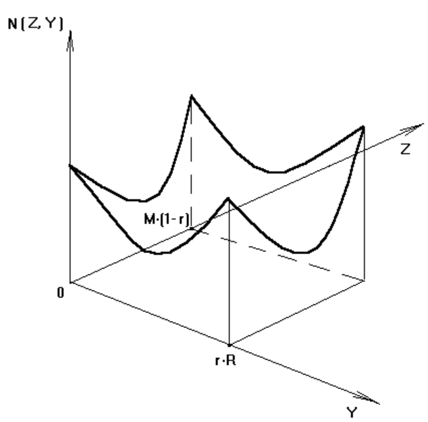

Figure 1: Behavior of the function *N(Y,Z)*.

The optimum point (Y_{opt}, Z_{opt}) minimizes the necessary computer time for the large system design and has dependency from the electronic system size and topology. This optimal point can be fined by different methods, for example by ordinary gradient method.

The optimization of the space dimension number of independent parameters leads to reduction of the total operation number and therefore to reduction of the total computer time for electronic system design. The analysis of different types of electronic systems shows that the optimal space dimensions of independent parameters can reduce the total computer time in 10 - 50 times. This optimal space dimension has dependency from electronic systems' size and topology. In this work the problem of optimum order of the space dimension is solved by Newton's method and by optimal control theory method. The total computer time is served as the objective function for the optimal algorithm finds.

DESIGN STRATEGY BY CONTROL THEORY FORMULATION

It is possible to define the problem of the optimum algorithm construction for more general case. We can determine the problem of a large system design as the problem of optimal control.

The principal equations system can be determined as:

$$\frac{dx_i}{dt} = f_i(X,U) \qquad (7)$$

$$i = 0,1,\ldots,N$$

where $N=K+M$; X is the variables vector $X = (x_1, x_2, \ldots, x_K, x_{K+1}, x_{K+2}, \ldots, x_N)$; U is the vector of control variables $U = (u_1, u_2, \ldots, u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.

The sense of the variable u_j is presence (when $u_j = 0$) or absence (when $u_j = 1$) of the equation number j in the system (2). The function $f_0(X,U)$ is determined as the necessary calculation time for one step of the system (7) integration. In this case the variable x_0 is determined as the total computer time for the electronic system design.

The functions of the right part of the system (7) are determined as:

$$f_i(X,U) = -b\left\{\frac{d\,C}{d\,x_i} + \frac{1}{e}\frac{d}{d\,x_i}\left[\sum_{j=1}^{M} u_j g_j^2(X)\right]\right\} \qquad (8)$$

for $i = 1,2,\ldots,K$ and

$$f_i(X,U) = -b\cdot u_{i-K}\left\{\frac{dC}{dx_i} + \frac{1}{e}\frac{d}{dx_i}\left[\sum_{j=1}^{M} u_j g_j^2(X)\right]\right\} + \frac{(1-u_{i-K})}{d\,t}\left\{-x'_i + h_i(X)\right\} \qquad (8')$$

for $i = K+1, K+2, \ldots, N$;

where x'_i is equal to $x_i(t - dt)$; $h_i(X)$ is the implicit function ($x_i = h_i(X)$) that is determined by the system: $(1 - u_j)g_j(X) = 0$; $j = 1,2,\ldots,M$.

In this case we determine the problem of some system design as the classical problem of the optimal control. In that context the aim of optimal control is to result each function $f_i(X,U)$ to zero for the final time t_{fin} , $f_i\left(X\left(t_{fin}\right),U\left(t_{fin}\right)\right)=0$ and minimize the total computer time x_0 . The minimum-time problem for the system (7) with non-continued or non-smoothed functions (8) can be solved most adequately by means of Pontryagin's maximum principle [3].

For the classical Pontryagin's form optimal control problem formulation it is necessary to define the conjugate system for the additional functions y_i :

$$\frac{dy_i}{dt}=-\sum_{l=0}^{N}\frac{\partial f_l(X,U)}{\partial x_i}\cdot y_l \qquad (9)$$

$$i=0,1,\dots,N$$

Hamiltonian is determined as :

$$H(X,U,\Psi)=\sum_{i=0}^{N} y_i \; f_i(X,U) \qquad (10)$$

This function has supreme value during the optimal trajectory with the Pontryagin's maximum principle:

$$M(X,\Psi)=\sup_{u\in\Omega} H(X,U,\Psi) \qquad (11)$$

The main problem of the maximum principle application in that formulation is unknown vector Ψ_0 of initial values of the functions y_i . This problem has adequate solution only for linear functions $f_i(X,U)$, for example in [4]. For the nonlinear case it is possible to use one iterative algorithm for the solution of the problem (7) - (11).

This method in maximum principle formulation that is used for the solution of the optimal control problem (7)-(11) is based on the boundary problem solution for $2\times(N+1)$ order equations system (7), (9). The iteration process for the numerical integration of this system includes consecutive iterations of Cauchy problem solution.

The strategy of this method is that:

1. The initial value of vector X_0 has been given, because it is known ; $X_0=\left(x_{10},x_{20},\dots,x_{N0}\right)$.
2. The initial value of vector Ψ_0 has been given arbitrary; $\Psi_0=\left(y_{10},y_{20},\dots,y_{N0}\right)$.
3. The vector of control variables U is fined by the formulas (10), (11).
4. Two systems (7), (9) are solved in one time step Δt and new values of vectors X and Ψ are determined.
5. The conditions $\left|f_i(X,U)\right|<e$ are verified for all index i . If this conditions are right, in this case we pass to step number 6, if they are not right we return to step 3.
6. In that case we have the solution of the problem (7)-(11). We have the functions *X(t), U(t),* $\Psi(t)$ and the total computer design time T that is equal to x_0. This solution is not optimal because it has been obtained with arbitrary value of the vector Ψ_0 that is not correct. However, this solution is the first approximation to the optimal solution.

To minimize the total computer design time T it is necessary to improve the initial approximation Ψ_0 . This problem can be solved by different methods. First of all it is possible to use some gradient method with the calculation of the T function's gradient $\nabla T=\left(\frac{\partial T}{\partial y_{10}},\frac{\partial T}{\partial y_{20}},\dots,\frac{\partial T}{\partial y_{N0}}\right)$ and movement along anti-gradient. Other way is the solution of the equations system $\frac{\partial T}{\partial y_{i0}}=0$; $i=1,2,\dots,N$ by Newton's method. In that case it is necessary to calculate the matrix of the second derivatives but the number of iterations can be reduced significantly.

EXAMPLES

For the demonstration of the optimum control theory approach two simple circuits have been investigated.

Example 1

In Figure 2 there is a circuit that has three independent variables (*K=3*) as admittance y_1,y_2,y_3 and two dependent variables (*M=2*) as nodal voltages V_1,V_2 at the nodes 1, 2. The total number of different design strategies by means of general design idea is equal to 2^M. We suppose that the non-linear admittance has dependency by the law $y_n=a_n+b_n\cdot V_2$.

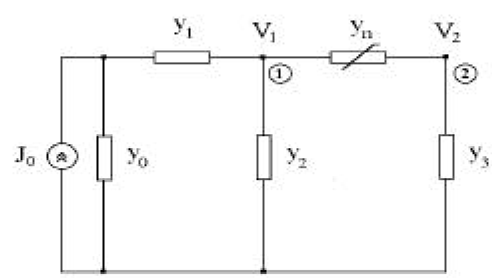

Figure 2: Circuit topology for 3 independent and 2 dependent parameters.

The results of the analysis of this circuit for two different values of the non-linearity parameter b_n (10^{-4} and 0.5 respectively) show that the modified traditional strategy $\left(u_1=u_2=1\right)$ is the optimum one for the first practically linear case. The total design time is equal to 6.37 sec. The time gain is equal to 17% with respect to the traditional design strategy. However, this strategy is not optimum one in general. It is necessary to find the optimal strategy by means of some optimization procedure or by maximum principle. The optimum trajectory has three switching points and the total design computer time for this case is equal to 5.6 sec. The optimum vector of the control functions has four regions of a constancy (10); (01); (00); (11). In that case this optimum strategy has the gain that is equal to 12 % with respect to modified traditional strategy.

The results of this circuit analysis for the non-linear situation when $b_n = 0.5$ show that neither traditional design strategy nor modified traditional strategy are not optimal ones. The strategy that is determined on the basis of general design idea $(u_1 = 0;\ u_2 = 1)$ has the minimum computer time (7.36 sec.) among all others. However, this strategy is not optimum one as for the first case. The optimum strategy has three switching points and has minimum computer time that is equal to 6.04 sec. The optimum vector of the control functions has four regions of a constancy (01); (11); (00); (11). We have the time gain in that case 1.5 times with respect to the modified traditional strategy.

Clearly it is impossible to define optimal dependencies of the control functions without additional investigation. These optimum dependencies are the result of the special optimization procedure application.

Example 2

In Figure 3 there is a nonlinear circuit that has 4 independent variables (*K=4*) as admittance y_1, y_2, y_3, y_4 and 3 dependent variables (*M=3*) as nodal voltages V_1, V_2, V_3 at the nodes 1, 2, 3.

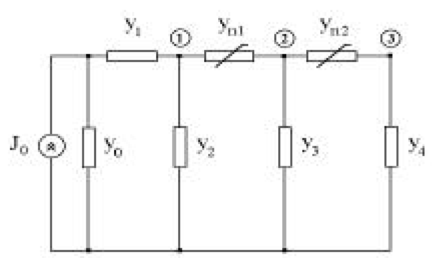

Figure 3: Circuit topology for 4 independent and 3 dependent parameters.

There are eight different design strategies for this example from the general design strategy point of view. The total computer design time for the traditional design strategy and for modified traditional design strategies are equal to 16.53 sec. and 46.91 sec. respectively. Among the rest of the strategies there is one $(u_1 = 1;\ u_2 = 0;\ u_3 = 0)$ that has the minimum computer time (9.28 sec.). However, the optimum strategy can be fined by means of some optimization procedure only. In this case the optimum vector of the control functions has three regions of a constancy (001); (100); (111). The computer time for this strategy is equal to 4.45 sec. This strategy has two switching points and has the time gain almost four times with respect to the traditional design strategy.

These data show that it is impossible to determine the optimal behavior of the control functions without some special optimization procedure because these dependencies have no any definite law.

CONCLUSIONS

Optimal design algorithm is depended on the number and the order of the equations that are excepted from the main system. The problem of the optimum algorithm construction can be solved more adequately by the control theory approach. In that case as the result we have the optimal trajectory X and optimal dependency of the control functions u_j. These optimal control functions can be used for the optimize computer design time of some systems that have similar topology. In that case it is possible to reduce the total design time of very large scale integrated circuits to many times.

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