Numerical Fracture Analysis of MEMS devices

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ABSTRACT

Non-conventional finite element analysis (FEA) based on linear elastic fracture mechanics (LEFM) is applied to fractured MEMS specimens with notches. The objective of this paper is to test the applicability of LEFM at mesoscale and to evaluate the application of newly developed FEA methods to MEMS fracture analysis. The displacement discontinuity method and the element free Galerking meshless method are used. The tested cases consisted of specimens in the form of notched cantilever beams, and double cantilever beams suspended by an anchor by two thin beams. A good correlation with some experiments is obtained.

***Keywords*:** Linear elastic fracture mechanics, MEMS reliability, Numerical methods, Meshless Methods, Boundary Element Methods.

1. INTRODUCTION

As MEMS popularity and applications are increasing, questions are being asked about their reliability. Unlike integrate circuits and similar devices from which MEMS technology borrowed a great deal, MEMS perform mainly repeated mechanical actions making them prone to fatigue and fracture failures.

A number of experimental programs have been conducted to extract the fracture properties of silicon for MEMS applications. A comprehensive review of experiments for the determination of fracture toughness for silicon MEMS was conducted by the authors [1, 2]. It was found that the fracture toughness of single crystal silicon at micro and macrosdcale was around 0.84- 0.9 Mpa.m$^{1/2}$, whereas polycrystalline silicon displays different behaviors at macro and microscale. Unfortunately, most publications reporting fracture of silicon do not report the details of the dimensions and boundary conditions of the test specimens, making it difficult to simulate their experiments numerically and to assess the validity of conventional linear elastic fracture mechanics for MEMS fracture. Four experiments among those reviewed provided enough data for modeling and were used as benchmark for numerical simulation using two non-conventional BEM and FEM method namely the displacement discontinuity method (DDM) and the element free Galerkin method (EFGM). The DDM is explained in details in references [3, 7] and the EFGM is briefly described here.

2. THE ELMENT FREE GALERKIN METHOD

The element free Galerkin method (EFGM) is a meshless method and is suitable for problems with changing geometry such as crack propagation since it does not require any meshing. The crack is simply considered as a boundary extension. The numerical procedure is quite similar to that of finite element except that, in EFGM, least square interpolants are used to approximate the dependant variables. These interpolants use an influence domain to define the connectivity between nodes.

For an arbitrary point $x \in \Omega$, we define a small domain Ω_x surounding this point with $\Omega_x \subset \Omega$. Considering a function $u(x)$ where $x=(x, y)$ defined on the domain Ω. Thus for any given point $x \in \Omega_x$, the function $u(x)$ is approximated by :

$$u(x)=\sum_{j}^{m} p_j(x)\, a_j(x)=p^T(x)a(x) \tag{1}$$

where $p_j(x)$ are monomials in the space coordinates $x^T= [x, y]$ and $a_j(x)$ are coefficients that are function of x. a(x) are obtained by using the L-norm which consists in minimizing the expression:

$$J=\sum_{I}^{n} w(x-x_i)\left[p^T(x_I)a(x)-u_I\right]^2 \tag{2}$$

where n is the number of points in the neighborhood of x where the weight function $w(x-x_I)\neq 0$, and u_I is the nodal value of u for $x = x_I$. Equation (2) leads to the following linear relations between a (x) et u_I:

$$A(x)\, a(x) = B(x)\, u \tag{3}$$

where A(x) and B(x) are defined as:

$$A(x)=\sum_{I}^{n} w_I(x)\; p^T(x_I)\; p(x_I) \tag{4}$$

$$B(x)=\left[w_1(x)p(x_1),\, w_2(x)\; p(x_2),\ldots,\; w_n(x)\; p(x_n)\right] \tag{5}$$

by defining the shape functions as :

$$f_I(\mathrm{x}) = \sum_{j}^{\mathrm{m}} p_j(\mathrm{x})\ (\mathrm{A}^{-1}\mathrm{B}(\mathrm{x}))_{jI} \qquad (6)$$

$u^h(\mathrm{x})$ can be writen as :

$$u^h(\mathrm{x}) = \sum_{I}^{n} f_I(\mathrm{x})\ u_I \qquad (7)$$

In this work, the exponential weight function is selected and is defined as:

$$w_I\left(d_I^{2k}\right) = \begin{cases} \dfrac{\mathrm{e}^{-(d_I / c)^{2k}} - \mathrm{e}^{-(d_{mI} / c)^{2k}}}{(1-\mathrm{e}^{-(d_{mI} / c)^{2k}})}, & d_I \le d_{mI} \\ 0, & d_I > d_{mI} \end{cases} \qquad (8)$$

Finally, the problem is solved by a stiffeness equation Ku = f where K and f are composed of submatrices K_{IJ} (2×2) and f_I (2×1) given by:

$$\mathrm{K}_{IJ} = \int_\Omega \mathrm{B}_I^T \mathrm{D} \mathrm{B}_J\, d\Omega - \int_{\Gamma_u} f_I \mathrm{S} \mathrm{N} \mathrm{D} \mathrm{B}_J\, d\Gamma - \int_{\Gamma_u} \mathrm{B}_I^T \mathrm{D}^T \mathrm{N}^T \mathrm{S} f_J\, d\Gamma \qquad (9)$$

and $$\mathrm{f}_I = \int_{\Gamma_t} f_I\, \bar{\mathrm{t}}\, \text{äd}\Gamma - \int_\Omega f_I \mathrm{b}\,\text{äd}\Omega - \int_{\Gamma_u} \mathrm{B}_I^T \mathrm{D}^T \mathrm{N}^T \mathrm{S} \bar{\mathrm{u}}\, d\Gamma$$

(10)

where D is the elastic matrix and

$$\mathrm{B}_I = \begin{bmatrix} f_{\mathbf{I},x} & 0 \\ 0 & f_{\mathbf{I},y} \\ f_{\mathbf{I},y} & f_{\mathbf{I},x} \end{bmatrix} \qquad (11)$$

$$\mathrm{N} = \begin{bmatrix} n_x & 0 & n_x \\ 0 & n_x & n_x \end{bmatrix} \qquad (12)$$

$$\mathrm{S} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \qquad (13)$$

$$s_{x,y} = \begin{cases} 1 \text{ if } u_{x,y} \text{ is prescribed on } \Gamma_u \\ 0 \text{ if } u_{y,x} \text{ is prescribed on } \Gamma_u \end{cases} \qquad (14)$$

Fracture mechanics concepts are introduced by defining the domain of the J integral near the crack tip.

3. APPLICATION TO MEMS FRACTURE

The element free Galerkin method is used to simulate fracture experiments conducted by Ballarini et al (1997) and Suwito and Dunn (1997).

Case study 1

The test setup of Ballarini et al (1997) is shown in Fig.1(a) and the numerical model in Fig.1(b). The length *x* which corresponds to the uncracked ligament has three values 6, 10 and 20mm . A probe was used to open the notch (the notch is considered as sharp crack in the mathematical model) to cause fracture where the prescribed displacement u=4mm corresponds to the probe wedge. Polycrystalline silicon was considered isotropic with a Young's modulus and a Poisson's ratio respectively equal to 160 GPa and 0.22. In this example, we considered the same assumptions as Ballarini et al.

600 uniformly distributed nodes were used. Figures 4.(a) and (b) show the variation of the stress intensity factor in terms of the distance of the probe tip from the crack tip. Both methods gave perfectly concordant results with those obtained by Ballarini et al.

Case study 2

Suwito and Dunn (1997) studied the effect of notch depth on a 3-point beam and the small width on T structure made both of anisotropic single crystal silicon. The specimens are shown in Fig. 3, 4, and 5.

The dimensions of the first specimen are L = 20 mm, b=1.5 mm, h=1.08 mm and a was equal to either 0.093mm, 0.147 mm, 0.164 mm or 0.210 mm.

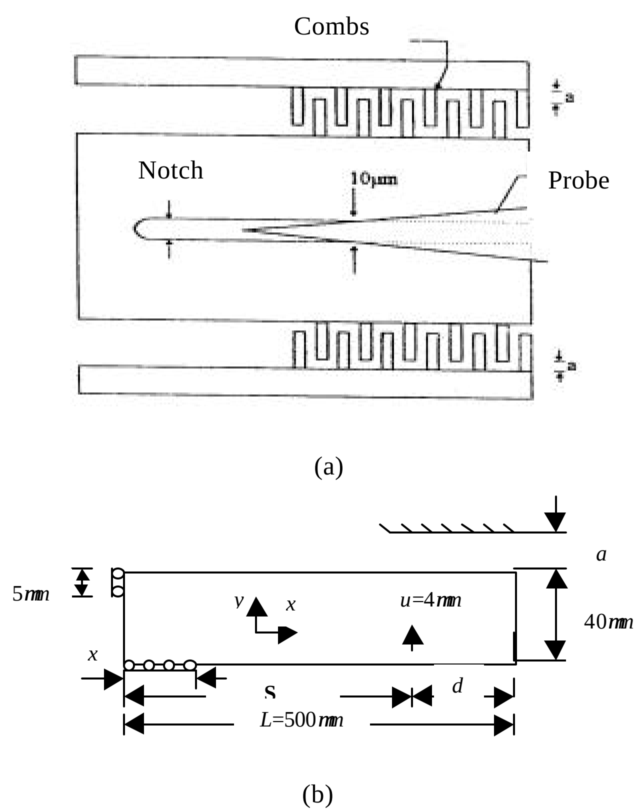
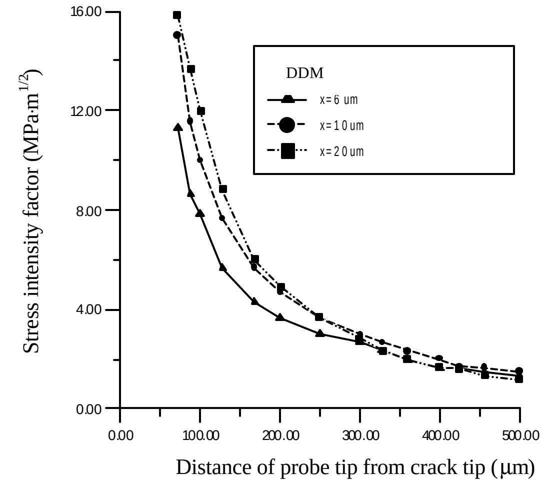
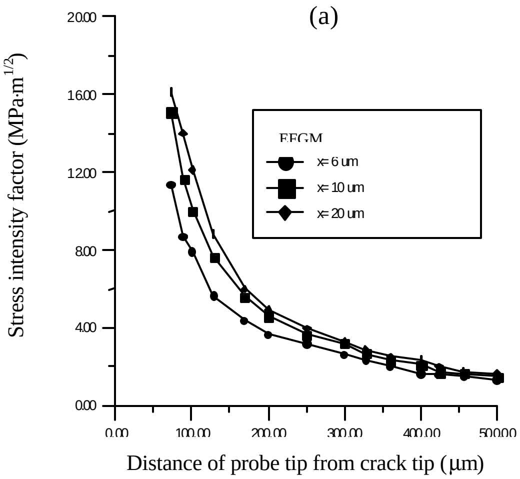


Figure 1: (a): Schematic of the Test setup, (b): Half of the mathematical model



(a)



(b)

Figure 2: Stress intensity factor vs. probe position for comb distance equal to infinity, (a): DDM, (b): EFGM.

For this example, 800 uniformly distributed nodes were used. Results obtained by using EFGM are compared to those obtained by Suwito and Dunn in Table1.

For the second specimen, the dimensions are L=20mm, b=1.5mm and h=1.08mm, the notch depth a=0.211mm and 0.213mm for two values of d=0.184mm and 0.029mm. Here also 800 uniformly distributed nodes were used. Results obtained by using EFGM are compared to those obtained by Suwito and Dunn in Table2.

The forth experiment considers the effect of sharp corners in T- shaped specimens of widths w_2=500 μm and w_1=8 and 28μm. Here again 800 uniformly distributed nodes were used and results are shown in Table 3.

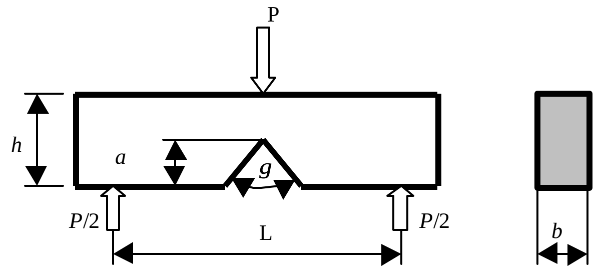


Figure3: 3 point flexure beam used by Suwito (notch angle = 70.53°).

Table 1: Stress intensity factor of 70.53° notched beams with four different notch depths.

	K_I (MPa·m$^{0.5}$)	
	a=0.093mm σ_r =86.19 MPa	a=0.147mm σ_r =61.90 MPa
Suwito	0.81	0.73
EFGM	0.68	0.69
	K_I (MPa·m$^{0.5}$)	
	a=0.164mm σ_r =61.15 MPa	a=0.210mm σ_r =52.60 MPa
Suwito	0.76	0.74
EFGM	0.65	0.67

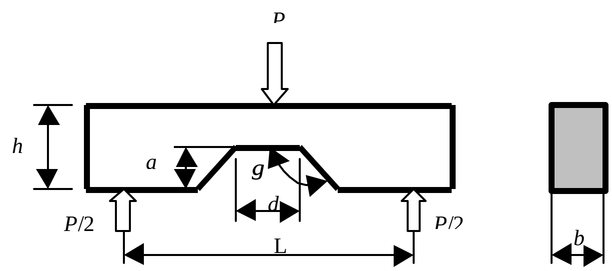


Figure 4: 3 point flexure beam used by Suwito (notch angle γ= 125.26°) wiith a flat notch part d.

Table 2: Stress intensity factor of 125.26° notched beams with different notch depths and flat notch part

	K_I (MPa·m$^{0.5}$)	
	a=0.211mm d=0.184mm σ_r =186.11 MPa	a=0.213mm d=0.029mm σ_r =157.18 MPa
Suwito	6.35	6.60
EFGM	3.945	4.129

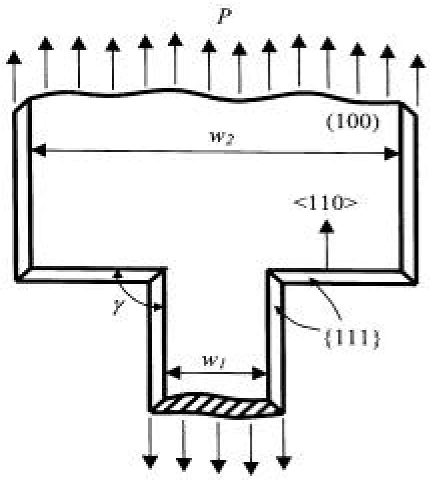


Figure 5: T structure

Table 2: Stress intensity factor of the T tructure for three different values of w_1.

	K_I (MPa·m$^{0.5}$)	
	w_1=8mm s_r=951.4MPa	w_1=28mm s_r=674.2Mpa
Suwito	2.09	2.09
EFGM	1.62	1.53
	K_I (MPa·m$^{0.5}$)	
	w_1=48mm s_r=554.7Mpa	
Suwito	2.10	
EFGM	1.652	

5. CONCLUSION

A non-conventional finite element method was presented as a tool to simulate fracture experiments of MEMS specimens with the assumption of the applicability of linear elastic fracture mechanics. Simulations were the most successful for polycrystalline silicon. For single crystal silicon, the sharper the corners or notches, the less accurate the simulation. Further research is needed to derive an appropriate finite element formulation for anisotropic fracture of single crystal silicon.

REFERENCES

[1] Tayebi, N. and Tayebi, A. K., “Assessment of MEMS Structural Reliability by Conventional Linear Elastic Fracture Mechanic”, to be presented in the second conference of micro nano-technology in aerospace application. April 1999.

[2] Tayebi, N. and Tayebi, A. K., “Review of silicon MEMS fracture toughness”, to be published.

[3] Crouch, S. L and Starfield, A. M., “Boundary element method in solid mechanics”. Allen and Unwin (1983).

[4] Belytschko, T., Gu, L., and Lu, Y.Y., “Fracture and Crack Growth by Element-Free Galerkin Methods’’, Journal of Mater. Sci. Eng, 1995, pp. 519-534.

[5] Ballarini, R., Mullen, R.L., Yin, Y., Kahn, H., Stemmer, S., and Heuer, A.H., “The Fracture Toughness of Polysilicon microdevices”, J. Mater. Res, Vol.12, N° 4, Apr 1997, pp. 915-922.

[6] Suwito, W., “Fracture of Notched Silicon Microstructures”, PhD Dissertation, University of Colorado at Boulder, 1997.

[7] Tayebi, N., “Etude du phenomene de rupture des structures MEMS”, Ingenieur d’Etat Thesis, Ecole Nationale Polytechnique.