Reduced-order modeling of Lorentz force actuation with modal basis functions

M. Varghese*, V. L. Rabinovich** and S. D. Senturia*

*Massachusetts Institute of Technology, Cambridge, MA, USA, varghese@mit.edu
**Microcosm Technologies Inc., Cambridge, MA, USA, rvl@memcad.com

ABSTRACT

We present a method for generating dynamic reduced-order models for Lorentz force actuation in microelectromechanical systems (MEMS). These models, derived from a set of meshed simulations, are analytical representations of the magnetic co-energy of a system. The representations are expressed as functions of harmonic mode amplitudes. By calculating analytical gradients of the co-energy with respect to the mode amplitudes, we find the Lorentz forces. Magnetic and mechanical energy domains are then coupled together by inserting Lorentz forces and mechanical forces into modal equations of motion.

Keywords: macro-model, energy method, MEMS, Lorentz force, magnetics

INTRODUCTION

There now exists a small but growing number of microactuators and microsensors that operate in the magnetic energy domain. Among these are devices that utilize the Lorentz force for either sensing or actuation; examples of which are resonant magnetometers [1], scanning mirrors[2], gyroscopes[3], and flexural plate wave devices [4]. We perceive a need arising among designers of this group of devices for tools that make the task of modeling and design both easier and faster. The essential physics of these devices lies in the coupling between the magnetic and mechanical energy domains. There already exist commercial finite element tools that have the capability to model this interaction [5], but the designer often finds that the coupling must be handled manually, or through specifically written scripts. In addition, the task of meshing the devices is often complicated by the need to mesh the air around the device. Lastly, the task of performing a mechanically dynamic coupled simulation, required for system level modeling, is often too slow with a finite element tool.

We propose to use an energy-based method that has proven successful in electromechanical systems [6] in solving the same problems that we now face in magnetomechanical systems. Our method generates, from a fully meshed device model, a reduced-order model that captures the magnetomechanical interaction of interest. The model is compact, fast, and relatively easy to insert into system level simulators to model the dynamics of the system.

THEORY

An overview of our macro-modeling strategy is shown in Figure 1 and follows that developed in [7].

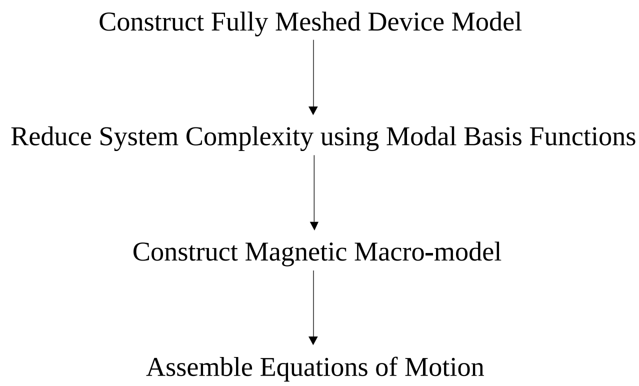

Figure 1: Overview of macro-modeling strategy.

We begin with a fully meshed model of our device, and from this generate the first few harmonic modes. These are the eigenvectors of the equation:

$$[M]\ddot{\vec{y}} + [K]\vec{y} = 0 \qquad (1)$$

M is the mass matrix, K is the stiffness matrix, and y is a vector representing the positional state of the meshed system. We then make the assumption that a well chosen set of harmonic modes can represent the practical deformations of our device accurately according to:

$$\vec{y} = \vec{y}_{eqm} + \sum_{i=1}^{m} q_i \vec{\varphi}_i \qquad (2)$$

where y_{eqm} represents the equilibrium positional state of the system, φ_i represents the harmonic mode shape, q_i represents the modal amplitude, and m is the number of modes we choose to use.

The deformation of the system is thus expressed in modal co-ordinates, q_i, and the equation of motion can be transformed into this co-ordinate system.

$$[M_G]\ddot{\vec{q}} + [K_G]\vec{q} = \vec{f}_m(\vec{q}, \vec{I}) \tag{3}$$

M_G is the global mass matrix, K_G is the global stiffness matrix, f_m is the generalized magnetic force, and I is the vector of currents involved in the problem. Before moving on, we should note that the approach above develops errors if displacements are large enough to cause stress stiffening [7]. The use of the linear stiffness matrix in our formulation does not account for this effect.

M_G and K_G are found from the original modal analysis and are known to be diagonal matrices. Thus, the only part left undetermined is f_m . This force is not necessarily linear in q. It can even couple the different modes together. Our macro-model allows us to obtain f_m from an analytic representation of the magnetic co-energy of the system, W^*_m. Since M_G and K_G are diagonal matrices we can write the matrix equation (3) into a coupled set of second order differential equations:

$$m_i\ddot{q}_i + k_i q_i = f_{m(i)}(\vec{q}, \vec{I}) \tag{4}$$

$f_{m(i)}$ is then given by the derivative of the magnetic co-energy of the system with respect to the modal amplitude, q_I:

$$f_{m(i)}(\vec{q}, \vec{I}) = \left.\frac{\partial W_m^*}{\partial q_i}\right|_I \tag{5}$$

Construction of the macro-model

The heart of our macro-model is the analytic co-energy function from which we calculate forces. This function is obtained by fitting data from a set of full 3-D meshed simulations. The data set must be representative of the practical deformations experienced by our device, and an approach to picking this data set is presented in [7].

In the following, we will consider a device modeled electrically as an n-port inductive circuit, and mechanically by m harmonic modes.

We follow the outline shown in Figure 2. Our first task is to generate data. We pick a point in mode space, $\{q^i_1, q^i_2, \ldots, q^i_m\}$, and then displace the positional state of the mesh, y, according to equation (2). We then perform a simulation using the tool MEMHenry [8] to calculate the n-port inductance matrix associated with this mesh. This procedure is repeated to generate the data set.

The next step is to fit each entry in the inductance matrix, using the generated data, to a rational polynomial in modal co-ordinates:

$$L_{kl} = \frac{\sum_{i_1=0}^{R_1}\sum_{i_2=0}^{R_2}\cdots\sum_{i_m=0}^{R_m} a_{i_1 i_2 \cdots i_m} q_1^{i_1} q_2^{i_2} \cdots q_m^{i_m}}{\sum_{i_1=0}^{S_1}\sum_{i_2=0}^{S_2}\cdots\sum_{i_m=0}^{S_m} b_{i_1 i_2 \cdots i_m} q_1^{i_1} q_2^{i_2} \cdots q_m^{i_m}} \tag{6}$$

L_{kl} is the inductance seen between ports k and l, a and b are fitting co-efficients. The resulting analytic inductance functions are now as accurate as the 3-D meshed simulation from which they were derived.

The final step is to construct the magnetic co-energy function [9], shown here for a two port device:

$$W_m^* = \frac{1}{2}L_{11}I_1^2 + L_{12}I_1I_2 + \frac{1}{2}L_{22}I_2^2 \tag{7}$$

To obtain Lorentz forces we can take the analytic derivative of this function with respect to modal amplitudes, and then insert these forces into the equations of motion.

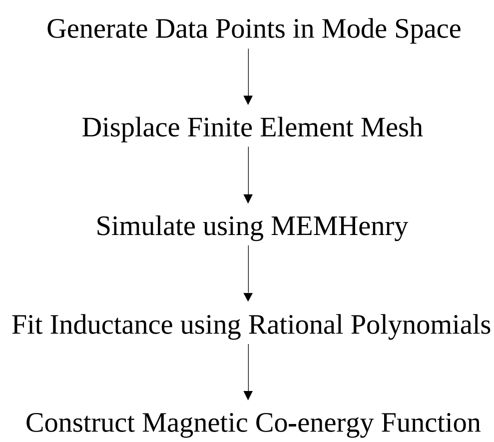

Figure 2: Overview of macro-model construction.

RESULTS

We demonstrate our method using a device representative of typical Lorentz force actuated MEMS. The device, depicted in Figure 3, is an elastic beam forming one side of a square, conducting loop. The device is placed in an homogenous z-directed magnetic field. The beam experiences a Lorentz force along its length and deforms.

Since MEMHenry only calculates inductance associated with conductor loops, we cannot readily insert the boundary condition of a z-directed magnetic field into our macro-modeling methodology. We circumvent this problem by creating this field with a cylindrical shell around our structure. The magnetic field inside a long cylindrical shell is given by:

$$H_z = K_\phi \quad (8)$$

The system is now, electrically, a two port element with port one belonging to the square loop, and port two belonging to the shell. The current entering the shell controls the magnetic field.

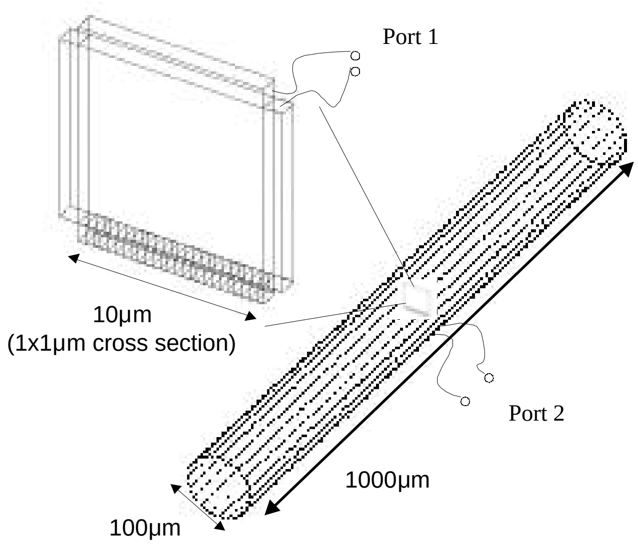

Figure 3: Example of a Lorentz force device. The elastic beam, shown as the bottom segment of the conducting loop, has a Young's Modulus of 130GPa. The cylindrical shell has a slot cut out of it to form port 2 and generates an axial magnetic field (z-direction). Current I_1 flows into port 1 and I_2 flows into port 2.

With this in place, our next step is to construct the magnetic macro-model. We simplify this example for clarity by using only the fundamental mode (q_1) and the first harmonic mode (q_2) of the elastic beam in our macro-model. For small deflections, we expect only the fundamental mode to be excited.

Figure 4 shows the fitted inductance L12 as a function of the fundamental mode amplitude. Figure 5 shows the static deflection of the beam for varying external magnetic fields. Our results compare well with those of a finite difference model of the beam in a magnetic field.

Figure 6 shows the deflection of the beam as a function of primary current with no external magnetic field. This is due to a "self-force" and acts to enlarge the current loop. This effect is negligible for typical Lorentz force devices because the current densities necessary for actuation are impractical. However, this "self-force" is significant in devices with magnetically permeable material, and our demonstration indicates that our methodology might be applicable to this class of devices.

Once the initial effort is made to construct the macro-model, further analysis is very fast. Static analysis, such as the deformation shown in Figure 5, takes only a few seconds to perform in MATLAB while retaining high accuracy. Dynamics simulations are also very fast as force calculations require only a function call to the analytic macromodel.

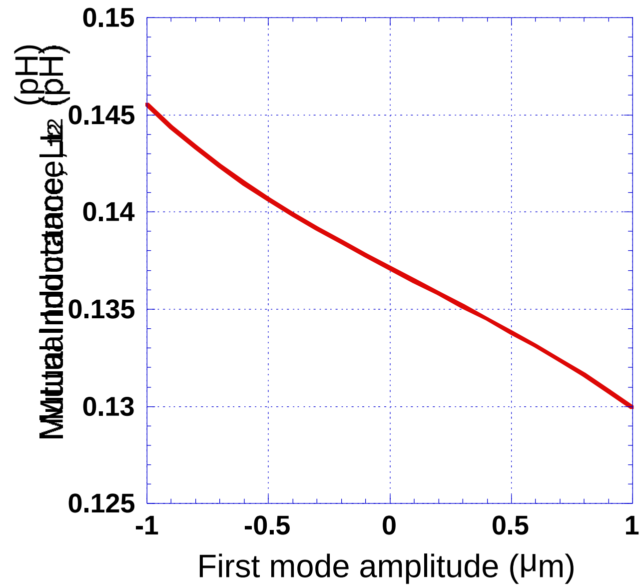

Figure 4: Mutual inductance, L_{12}, as a function of the fundamental mode amplitude.

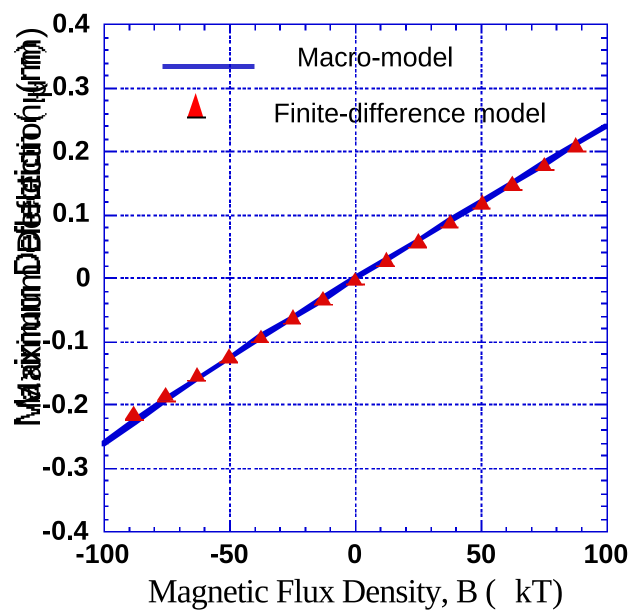

Figure 5: Maximum deflection of the elastic beam as a function of the axial magnetic flux density. The large magnetic field required for actuation is due to the high stiffness of this short beam. Practical devices are typically actuated in fields on the order of 1T or below.

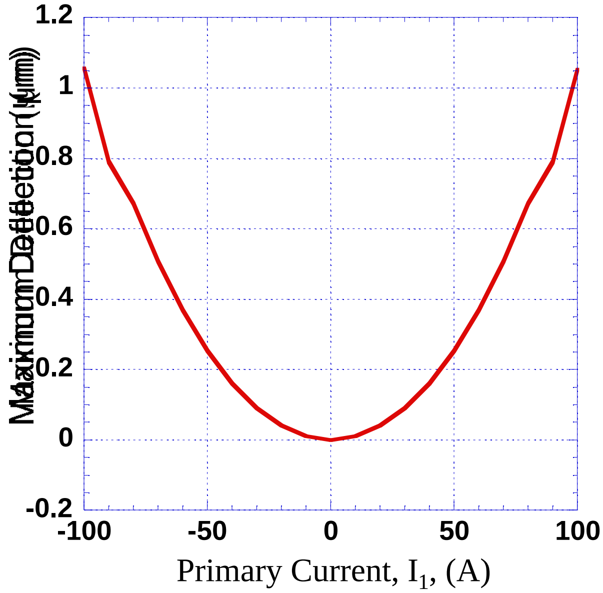

Figure 6: Maximum deflection of the elastic beam as a function of the loop current I_1, with no external field. The beam bends to increase the area of the loop independent of the direction of the current.

CONCLUSION

We have demonstrated a methodology for generating energy-based macro-models using harmonic mode shape amplitudes as generalized co-ordinates. The use of mode shapes enables us to reduce the size of our problem to a set of a few coupled ordinary differential equations, and the use of energy methods enables us to use fast analytic forms for non-linear force calculations.

Further work is necessary to quantify the speed, accuracy, and robustness of our methodology. We must also explore methods for determining the minimum size of the data set required to construct a useful macro-model.

REFERENCES

[1] D. Wickenden, T. Kistenmacher, R. Osiander, S. Ecelberger, R. Givens, J. Murphy, “Development of Miniature Magnetometers,” Johns Hopkins APL Technical Digest, Vol. 18, No. 2, 271-278, 1997

[2] B. Shen, W. Allegretto, Y. Ma, B. Yu, M. Hu, and A. Robinson, “Cantilever Micromachined Structures in CMOS Technology with Magnetic Actuation,” Sensors and Materials, Vol. 9, No. 6, 347-362, 1997.

[3] A. Harris, J Burdess, D. Wood, R. Langford, G. Williams, M. Ward, M. McNie, “Issues associated with the design, fabrication and testing of crystalline silicon ring gyroscope with electromagnetic actuation and sensing,” J. Micromech. Microeng., No. 8, 284-292, 1998.

[4] S. Martin, M. Butler, J. Spates, M. Mitchell, W. Schubert, “Flexural plate wave resonator excited with Lorentz forces,” Journal of Applied Physics, Vol. 83, No. 9, 1998.

[5] ANSYS 5.4, Ansys, Inc., Southpointe, 275 Technology Drive, Canonsburg, PA 15317, 1998.

[6] L. Gabbay, S. Senturia, “Automatic generation of dynamic macro-models using quasi-static simulations in combination with modal analysis,” Technical Digest, Solid State Sensors and Actuators Workshop, 197-200, 1998.

[7] L. Gabbay, “Computer Aided Macromodeling for MEMS,” Ph.D. Thesis, Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, MA, 1998.

[8] MEMCAD 4 Manual, Microcosm Technologies, Inc., 5511 Capital Center Dr., Raleigh, NC, 1999.

[9] H. Haus, J. Melcher, “Electromagnetic Fields and Energy,” Pretence Hall, 1989.