Acceleration of Inductance Extraction by Means of the Monte Carlo Method

G. Leonhardt and W. Fichtner

Integrated Systems Laboratory
Swiss Federal Institute of Technology
Gloriastr. 35, 8092 Zürich, Switzerland, leonhardt@iis.ee.ethz.ch

Abstract

With increasing signal frequencies inductive effects of metallic interconnections in all kinds of packages become more and more important for the electrical behavior of a circuit. During the design process it is therefore of interest to have a stable, fast and accurate simulation tool which is capable of extracting the inductance of large and complicated geometries. This work presents a new program for inductance extraction, INDEX, without consideration of the skin effect. The method is based on a current density computation by means of the Finite Element method [1] with an ensuing energy estimate by means of Monte Carlo sampling. Our previous approach [2] has been considerably improved by application of variance reduction techniques. Ideas are explained and an example is shown below.

Keywords: inductance extraction, Monte Carlo, variance reduction techniques.

Introduction

INDEX extracts partial inductances [3] and mutual inductances from geometries given in structured grid format. Nevertheless the method is applicable to any given three dimensional current distribution. The influence of the skin effect is neglected. Results are valid as long as the skin depth $\delta = 1/\sqrt{\mu\sigma\omega}$ is large compared to diameters of the geometries. Here, μ denotes the magnetic permeability, σ denotes the electrical conductivity the geometries and ω is the angular frequency of the applied current. First method and theory are explained, explains both method and theory are explained, then results are given and finally conclusions are drawn.

Theory

The solution process consists of two steps. The first step is the calculation of the current distribution inside a conductor by means of the Finite Element method [1]. This step could in principle be replaced by any other method, even an analytical method, for current density computation. During the second step the energy of the resulting magnetic field is estimated by integration of Neumann's formula [4].

$$W = \frac{\mu}{8\pi} \int_V \int_V \frac{\vec{J}(\vec{r}) \cdot \vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r' \, d^3r \tag{1}$$

This is a six dimensional integral. V denotes the volume of the conductor, r and r' denote locations in the volumes. Instead of evaluation of the magnetic vector potential

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_V \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r'$$

and calculating

$$W = \frac{1}{2} \int_V \vec{A}(\vec{r}) \cdot \vec{J}(\vec{r}) \, d^3r$$

with a Gaussian integration scheme, Eq. (1) is estimated by means of Monte Carlo sampling [5].

$$\begin{aligned} W &\approx \frac{1}{N} \frac{\mu}{8\pi} \sum_i \frac{\vec{J}(\vec{r}_i) \cdot \vec{J}(\vec{r}_i\,')}{|\vec{r}_i - \vec{r}_i\,'|} \frac{1}{p^2(\vec{r}_i, \vec{r}_i\,')} \\ &= \frac{1}{N} \sum_i W_i \end{aligned} \tag{2}$$

During every sample i two locations $\vec{r}_i$ and $\vec{r}_i\,'$ are chosen randomly according to the probability density $p^2(\vec{r}, \vec{r}\,') > 0$. The probability density depends on both locations $\vec{r}$ and $\vec{r}\,'$ and has to be normalized.

$$\int_V \int_V p^2(\vec{r}, \vec{r}\,') \, d^3r' \, d^3r \equiv 1 \tag{3}$$

The inductance L is computed from W and the total current through a conductor with

$$W = \frac{1}{2} L \, I^2$$

The error of the result ΔW is calculated from the variance of the sampled values W_i Eq. (2)

$$\sigma^2 = \frac{1}{N} \sum_i W_i^2 - \left(\frac{1}{N} \sum_i W_i \right)^2 \tag{4}$$

and from the number of iterations during the Monte Carlo integration.

$$\Delta W = 3 \frac{\sigma}{\sqrt{N}} \tag{5}$$

When the number of samples N is large, according to the central limit theorem. the average W is distributed normal and the above "three σ criterion" gives a measure for the error with higher than 99% confidence.

Mutual Inductances

For N conductors INDEX computes a set of N current distributions J_i with $i = 1, ..., N$, each with the electric potential at one contact where current leaves a conductor set to 1V, 0V at all other contacts. The current distribution resulting from any voltage applied at the contacts can be written as a superposition of the J_i. The energy computation in Eq. (1) is now modified.

$$W_{ij} = \frac{\mu}{8\pi} \int_V \int_V \frac{\vec{J}_i(\vec{r}) \cdot \vec{J}_j(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r' \, d^3r \tag{6}$$

Here W_{ij} is the potential energy of current distribution J_i in the magnetic field of current distribution J_j or vice versa and therefore $W_{ij} = W_{ji}$. The inductive behavior of N conductors is described with a matrix $\mathbf{L}$ with L_{ii} the self inductance of conductor i, whereas the off-diagonal entries L_{ij} stand for the inductive coupling between conductors i and j. I_i is the vector of total currents through all conductors for current density J_i. An expression for the energy in terms of inductances can now be written as

$$W_{ij} = \frac{1}{2} I_i^T \mathbf{L} I_j$$

or in matrix notation

$$\mathbf{W} = \frac{1}{2} \mathbf{I^T L I}.$$

Since $\mathbf{W}$ is symmetric $1/2N(N+1)$ energy terms W_{ij} have to be calculated. Every energy term Eq. (6) is calculated with predefined accuracy as specified in Eq. (5).

Implementation

We use the finite element program SOLIDIS [1] for current density computations. Here, adaptive grid refinement techniques are used for error minimization. Geometries have to be specified in structured grid format.

Current densities have to comply with the continuity equation

$$\nabla \vec{J} = 0.$$

Since $\vec{J} = -\sigma \nabla U$ with σ the electric conductivity of a conductor and U the electric scalar potential, $\vec{J}$ can be obtained from a solution of Laplace's equation

$$\nabla \sigma \nabla U = 0 \tag{7}$$

with specified potential at the contacts. SOLIDIS uses bilinear expansion functions for solution of Eq. (7). The current density in every element is calculated from the derivative in the element's center and is therefore constant in every element.

Variance Reduction Techniques

In order to improve convergence during Monte Carlo sampling, different variance reduction techniques are investigated. The idea is to put as much easily accessible information about the integrand in Eq. (1) into the probability density in order to obtain energy samples in Eq. (2) which are almost constant.
Deployed variance reduction techniques [5] are

- importance sampling,
- stratified sampling,
- combinations of both.

The probability density p^2 is held constant for locations in the same structured grid elements. The computational effort for generation of random locations is constant, independent of the number of structured grid elements.
In order to generate the two random locations independently, the probability density is separated

$$p^2(\vec{r}, \vec{r}\,') = p(\vec{r}) \cdot p(\vec{r}\,'). \tag{8}$$

When no variance reduction techniques are applied, the probability density is constant. Equation (3) leads to

$$p(\vec{r}) \cdot p(\vec{r}\,') = \frac{1}{V^2}, \tag{9}$$

with the conductor volume V.
Stratified sampling is a separation of the integral Eq. (1).

$$W = \sum_{s=x,y,z} \frac{\mu}{8\pi} \int_V \int_V \frac{\vec{J}_s(\vec{r}) \cdot \vec{J}_s(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r' \, d^3r \tag{10}$$

First, the stratum s is selected with probability p_s and then as before two locations are chosen, now according to p_{sl}^2. Clearly due to Eq. (3)

$$\begin{aligned} 1 &\equiv \sum_s p_s \int_V \int_V p_{sl}^2(\vec{r}, \vec{r}\,') \, d^3r' \, d^3r \\ &= \sum P_s. \end{aligned} \tag{11}$$

The expectation value Eq. (2) becomes

$$W \approx \sum_s \frac{P_s}{N_s} \sum_{si} W_{si}, \tag{12}$$

where N_s are the numbers of samples from stratum s and the W_{si} are samples from stratum s only. According to Eqs. (4) and (5) the error estimate is

$$\Delta W = 3 \sqrt{\sum_s \frac{P_s^2}{N_s^2} \sigma_s^2}. \tag{13}$$

Here σ_s^2 denote the variance of sampled values from stratum s.

The idea of importance sampling is to sample large values of the integrand Eq. (1) more often than smaller values. We applied this to Eq. (10) and chose the probability densities

$$p_{sl}^2(\vec{r},\vec{r}\,') \propto |J_s(\vec{r})||J_s(\vec{r}\,')|. \tag{14}$$

They are different for every coordinate direction and they fulfill Eq. (8).

Another idea for importance sampling comes from the notion that the mutual inductance between cubes with constant current density decreases almost proportionally to their inverse center distance [6] when their centers are at least about one edge length apart. We include this knowledge in the probability density by using a coarse "super grid" with constant grid spacing and C cells. Every structured grid element is now associated with the nearest element of the "super grid". The volumes of all structured grid elements associated with one "super grid" element are added and the center of gravity $\vec{r}_c$ is calculated. The probability density is given by

$$p^2(\vec{r},\vec{r}\,') = \frac{\dfrac{1}{|\vec{r}_c(\vec{r}) - \vec{r}_c(\vec{r}\,')|}}{\displaystyle\int_V \int_V \frac{1}{|\vec{r}_c - \vec{r}_c\,'|}\, d^3r'\, d^3r}, \tag{15}$$

where the function $\vec{r}_c(\vec{r})$ is the mapping from a location to the nearest center of gravity. This probability density does not separate. Our approach is now similar to the stratification approach, i.e. first select an interaction between "super grid" elements and then generate two locations with constant probability density analogous to Eq. (9), but now with V the volume of structured grid cells associated with the selected "super grid" cell.

A second idea for stratification comes from the notion that two cubes with reverse current result in two distant peaks in the distribution of samples in Eq. (2) with average somewhere in between. As an example consider both locations in the same cube. The current density now points in the same direction which results in a positive sample. When both locations are, by coincidence, in different cubes, the current densities point in opposite direction which results in a negative sample. The variance of sampled values can be reduced by treating these two peaks separately. So Eq. (10) is separated more into J_s^+ and J_s^- terms, i.e positive and negative current density components. This increases the number of strata in Eq. (10) by a factor of 4.

All tested probability densities are summarized in Tab. 1.

Results

Figures 1 and 2 show the structured grid model of our example and the electric potential after adaptive grid refinement respectively. Table 2 shows the time effort for computation of the 16×16 inductance matrix with

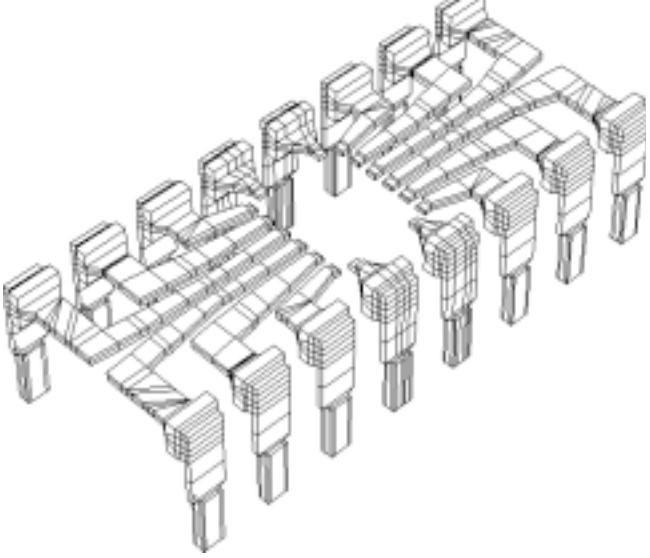

Figure 1: Structured grid model of a DIP package lead frame with 16 pin connectors.

total 136 independent entries. Each entry was computed with 5% accuracy. The number of "super grid" cells was only $2 \times 4 \times 2$. Increasing the number of cells sped up the memory requirements but did not lead to substantial variance reduction.

Simulations have been performed with a Sun UltraSparc/167MHz workstation. Memory requirements have been about 100MB. The preceding 16 current density computations have taken 39mins including 3 adaptive grid refinement steps. Here, memory requirements have been approximately 80MB. Reasonable results could also have been obtained with less adaptive grid refinement steps, and therefore faster current density computations. Nevertheless, an estimate for the error introduced by these coarse current densities can not be given.

Table 2: Time effort for computation of the full 16×16 inductance matrix (example Fig. 2, variance reduction methods Tab. 1).

Variance Reduction	Time
no	5200s
1	1560s
2	1400s
3	4500s
4	1300s
5	340s

Table 1: Tested probability densities for sampling of Eq. (1). p_s probability for selection of stratum s and p_{sl}^2 probability density for selection of locations Eq. (11). $\vec{r}_c(\vec{r})$ is the function which maps locations to the nearest center of gravity of all elements associated with one element of the "super grid" Eq. (15).

variance reduction	no. of strata	p_s	$p_{sl}^2(\vec{r},\vec{r}\,')$
no	1	1	$\frac{1}{V}\cdot\frac{1}{V}$
1	3	$\frac{1}{3}$	$\frac{\lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert}{\int_V\int_V \lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert\, d^3r'\, d^3r}$
2	3	$\frac{\int_V\int_V \lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert\, d^3r'\, d^3r}{\sum_s\int_V\int_V \lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert\, d^3r'\, d^3r}$	$\frac{\lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert}{\int_V\int_V \lvert J_s(\vec{r})\rvert\cdot\lvert J_s(\vec{r}\,')\rvert\, d^3r'\, d^3r}$
3	1	1	$\frac{1}{\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert}\Big/\int_V\int_V\frac{1}{\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert}\, d^3r'\, d^3r$
4	3	$\frac{\int_V\int_V \lvert J_s(\vec{r})\rvert\lvert J_s(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}{\sum_s\int_V\int_V \lvert J_s(\vec{r})\rvert\lvert J_s(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}$	$\frac{\lvert J_s(\vec{r})\rvert\lvert J_s(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert}{\int_V\int_V \lvert J_s(\vec{r})\rvert\lvert J_s(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}$
5	12	$\frac{\int_V\int_V \lvert J_s^{\pm}(\vec{r})\rvert\lvert J_s^{\pm}(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}{\sum_{s\pm}\int_V\int_V \lvert J_s^{\pm}(\vec{r})\rvert\lvert J_s^{\pm}(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}$	$\frac{\lvert J_s^{\pm}(\vec{r})\rvert\lvert J_s^{\pm}(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert}{\int_V\int_V \lvert J_s^{\pm}(\vec{r})\rvert\lvert J_s^{\pm}(\vec{r}\,')\rvert/\lvert\vec{r}_c(\vec{r})-\vec{r}_c(\vec{r}\,')\rvert\, d^3r'\, d^3r}$

Conclusions

We presented an accelerated Monte Carlo sampling method for static inductance extraction. For acceleration, different variance reduction methods and combinations of them have been implemented. We showed a realistic example, where a total speedup of sampling by a factor of 15 has been achieved.

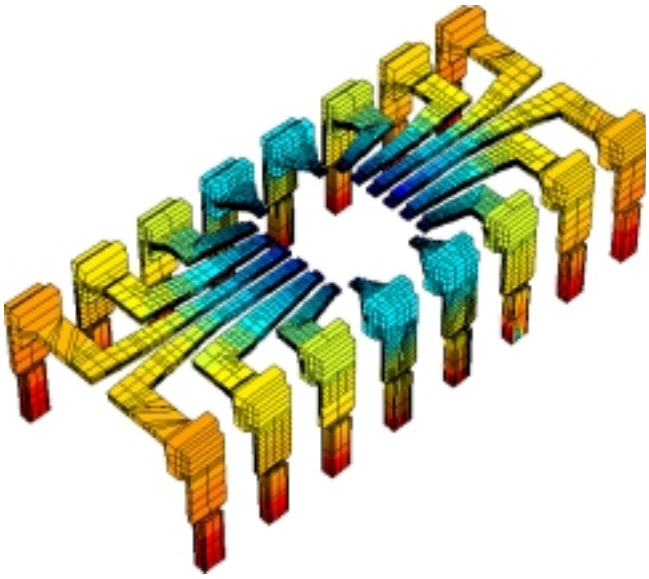

Figure 2: Adaptive refined grid with electric potential. Current densities are modeled with 45912 degrees of freedom.

REFERENCES

[1] ISE Integrated Systems Engineering AG, *SOLIDIS-ISE User Manual and Reference*, ISE AG Zurich, 1997.

[2] G. Leonhardt, Ch. Hager, P. Regli, W. Fichtner, *Inductance Extraction by Means of the Monte Carlo Method*, Proceedings. of the MSM'98 Conf., Santa Clara CA, 1998

[3] A. Ruehli, *Inductance Calculations in a Complex Integrated Circuit Environment* , IBM J. Res. Develop., 16(5), 470-481, 1972.

[4] F. Grover, *Inductance Calculations*, van Nostrand Co., 1946.

[5] R. Rubinstein, *Simulation and the Monte Carlo Method*, J. Wiley & Sons, 1981.

[6] C. Hoer, C. Love, *Exact inductance Equations for Rectangular Conductors with Applications to More Complicated Geometries* , Journal of research of the National Bureau of Standards, 69C(2), 127-137, 1965.