Modeling of Electrostatic MEMS Components

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ABSTRACT

This paper describes an analytical approach for the network-type modeling of pull-in comb drives based on a mathematical field description using conformal mapping methods. The model was coded in MAST® for the SABER® simulator and its accuracy was compared to FDM and boundary element simulations using MAFIA and MEMCAD®. This approach leads to a library of network-type multi domain component models for the system simulation of MEMS, providing nearly the accuracy of field-solving codes at network-type simulation speed.[1]

Keywords: network-type modeling, MEMS, SABER, comb model, conformal mapping

INTRODUCTION

Comb structures on MEMS sensors are straight or curved fingers which serve both as actuators (converting electric energy into mechanical energy) and as sensing devices, to detect displacements between the substrate and the moving parts of the sensor. The combs come in interdigitated pairs: one part, called the stator is anchored to the substrate physically but isolated from it electrically, and the other is attached to the moving structure. The displacement between the combs is determined by measuring the electrical capacitance between them. The forces between the combs depend on their relative position and the applied voltages. Thus, the system level model of a comb structure must accurately model both the position-dependent capacitance and the forces between the stator and the movable comb.

CAPACITANCE CALCULATION

The position dependent electrostatic field between the combs is the key to the capacitance and force calculation. In order to find an analytical field approximation, some essential simplifications have to be made:

1. Just two variables describe the position of the movable comb, one for the horizontal pull-in and one for the vertical displacement. Angular motions or the horizontal displacement perpendicular to the comb fingers are neglected.

2. There is no restriction for the overlapping range of the comb fingers.

3. The field distribution between the fingers is the same as for a comb structure with an infinite number of fingers. As shown in Fig. 1, the remaining three dimensional field problem can therefore be reduced to an electrode arrangement of two half fingers and the substrate. The substrate voltage is the boundary condition of the bottom of the depicted box. For symmetry reasons, all other sides of the box have Neumann boundary conditions.

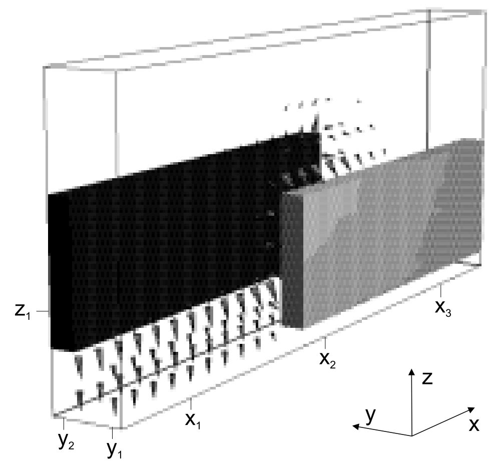

Figure 1: Elementary comb cell with two half fingers

4. The electrostatic field of Fig. 1 is decomposable into several two dimensional field problems as shown in Fig. 2.

Considering Fig. 2, the capacitances of the equivalent electrical circuit Fig. 3 are given by:

$$\begin{aligned} C_{\mathrm{S}} &= n_{\mathrm{s}}\left(C_{\mathrm{S1}} + C_{\mathrm{S2}} + C_{\mathrm{S3}}\right) \\ C_{\mathrm{SM}} &= n_{\mathrm{s}}\left(C_{\mathrm{SM1}} + C_{\mathrm{SM2}} + C_{\mathrm{SM3}} + C_{\mathrm{SM4}}\right) \\ C_{\mathrm{M}} &= n_{\mathrm{s}}\left(C_{\mathrm{M1}} + C_{\mathrm{M2}} + C_{\mathrm{M3}}\right) \end{aligned} \tag{1}$$

[1]MAST® is a registered trademark of Analogy, Inc. SABER® is a registered trademark of American Airlines, Inc., licensed to Analogy, Inc. MEMCAD® is a registered trademark of Microcosm Technologies, Inc. MAFIA is a product of Computer Simulation Technology.

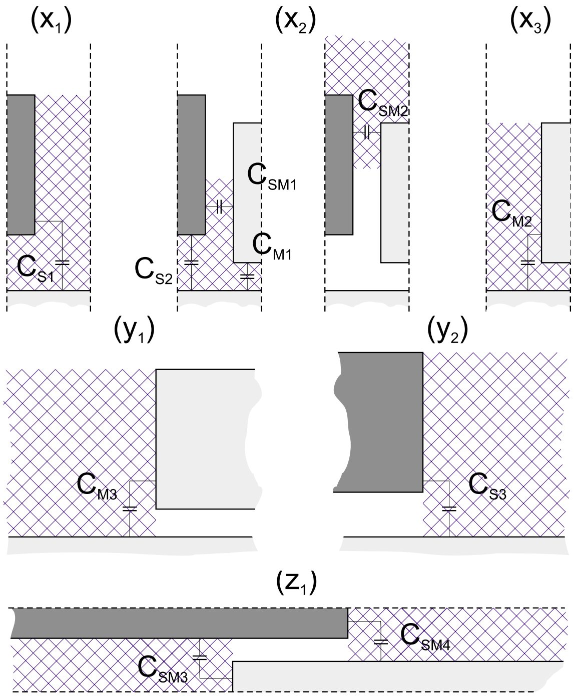

Figure 2: 2D fields in the y-z-, x-z- and x-y-plane

where n_s is the number of elementary cells that compose the investigated comb structure. In the setup of Fig. 1,

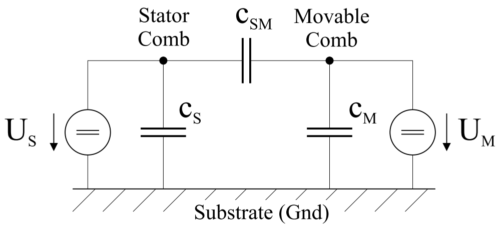

Figure 3: Equivalent circuit

the main contribution to the capacitance C_{SM} is due to the overlapping region of stator and moving comb. The corresponding partial capacitances are C_{SM1} and C_{SM2}. They are products of the overlapping length and the corresponding per-unit-length capacitances. The latter are obtained by analytical determination of the 2D electrostatic field in the cross-section at $x = x_2$.

In contrast to the other hatched areas of Fig. 2 the lower part of the cross-section is a three conductor problem. It can be converted into a two conductor problem. Setting the first of the three electrodes to a fixed potential and the other two to ground leads to a sum of two capacitances. This procedure has to be repeated for the remaining two conductors yielding three linear equations for C_{S2}, C_{SM1} and C_{M2}.

For the analytical 2D field calculations, the method of conformal mapping is used: In a complex $(w = u+iv)$-plane, the electric potential $\Phi(u,v)$ of an ideal plate capacitor is represented by a complex potential $\chi(w) = w$

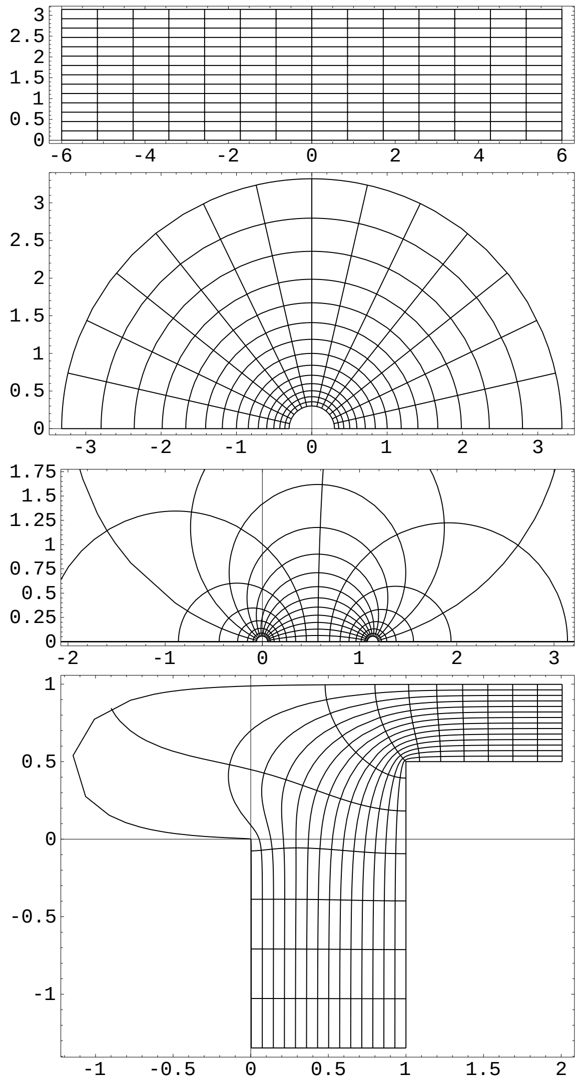

Figure 4: Stages of conformal field conversion

with $Im\{\chi\} = \Phi$ (first picture of Fig. 4). By successive steps, we map the plate capacitor to the originally given field area. Firstly, by

$$w_1 = e^w \tag{2}$$

it is mapped to a split-plane-capacitor (second picture of Fig. 4). Secondly, by

$$w_2 = \frac{aw_1 + b}{cw_1 + d} \tag{3}$$

this one is mapped to the upper complex plane but with the real axis divided into three equipotential intervals (third picture of Fig. 4). Finally, the Schwarz-Christoffel-Integral maps the upper plane to the initial problem geometry, the w_3-plane (For details about the Schwarz-Christoffel-Integral we would like to refer to standard textbooks about complex variables like [1] or [2].) The last picture of Fig. 4 shows the plot of the complex potential function that relates to one of the three two-conductor field problems of the lower cut of $x = x_2$. In opposite to the cutaway view of Fig. 2, the field plot is depicted upside down, with the substrate at the top and the charged finger in the lower right corner. The complete derivation of the mapping function for the problem shown in Fig. 4 is far to involved to be included in this paper. However, the mapping function $w_3 = w_3(w_2(w_1(w)))$ is the basis for the calculation of the per-unit-length capacitances.

In a similar way all partial capacitances of the hatched areas in Fig. 4 can be expressed by a corresponding mapping function that finally leads to the capacitances of the equivalent circuit in Fig. 3.

Verification

A verification for the analytically calculated capacitance C_{SM} within the working range of micro machined comb structures can be seen in Fig. 5. The diagrams show the the relative differences between the analytical approximations and numerical results versus the horizontal and vertical displacement of the movable comb. The first set of numerical results have been obtained

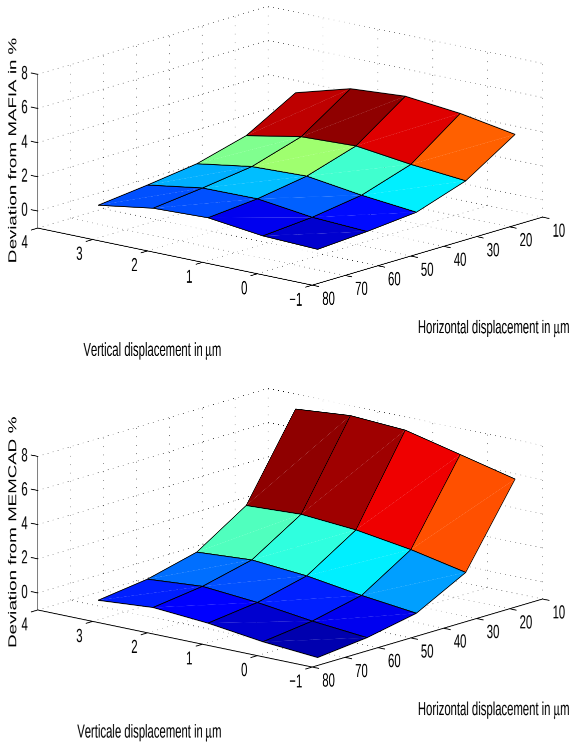

Figure 5: Comparison with MAFIA and MEMCAD

with the finite difference program MAFIA using a high density net with 600 000 nodes. The used model is shown in Fig. 1. The second plot is based on calculations with a boundary-element solver from the MEMCAD package using 16 352 panels. Because of the lack of Neumann boundary conditions the model consisted of two combs with three fingers each. Hence, the model included all stray capacitances of a complete three finger comb structure. In order to meet the results of the considered elementary cell the simulated capacitance values were divided by five.

FORCE CALCULATION

Using Equ. (1), the charges of each comb can be calculated by:

$$\begin{bmatrix} Q_S \\ Q_M \end{bmatrix} = \begin{bmatrix} C_{\mathrm{S}} + C_{\mathrm{SM}} & -C_{\mathrm{SM}} \\ -C_{\mathrm{SM}} & C_{\mathrm{SM}} + C_{\mathrm{SM}} \end{bmatrix} \begin{bmatrix} U_S \\ U_M \end{bmatrix} \quad (4)$$

Applying the physical principle of virtual work leads to the pull-in force F_x and the levitation force F_z.

$$F_n = -\frac{1}{2}\frac{\delta}{\delta n}\left(\mathbf{U}^{\mathrm{T}}\mathbf{Q}\right) \quad (5)$$

Where n stands for either x or z. To remedy the deviations of the capacitance functions and to boost the simulation speed, all three capacitance functions are fitted to a two dimensional polynomial function of x and z.

All necessary conformal mapping functions as well as the used least square fitting routines are part of a newly developed SABER comb model. In correspondence to the other parts of the BOSCH MEMS library [3] the comb model is equipped with a general mechanical interface, consisting of 3 translational and 3 angular network nodes. Altogether they define the relative position of a corresponding physical point in space as “across” and the related forces and torques at this point as “through” variables. The initial coordinates of this interface point, the comb geometry and the comb position must be given as model parameters.

The SABER comb model supplies the forces and torques at the mechanical node by adding up the horizontal and vertical comb forces of each movable comb finger. The forces at the comb fingers are calculated according to the potentials of the six mechanical nodes and the electrical finger potentials, see Fig. 6.

An experimental verification of calculated forces based on the MAFIA model (Fig. 1) can be found in [4].

SENSOR EXAMPLE

A design for the investigation of new manufacturing technologies shall serve as an example to demonstrate the idea of a network-type system model of MEMS structures. The structure in Fig. 7 is made of galvanically deposited nickel and was recently fabricated at the research department IMSAS of the university of Bremen, Germany. The sensor structure consists primarily of a middle mass which is suspended by four beams and two rigid trusses on both sides. The middle mass can be driven into a resonant vibration by the two outer

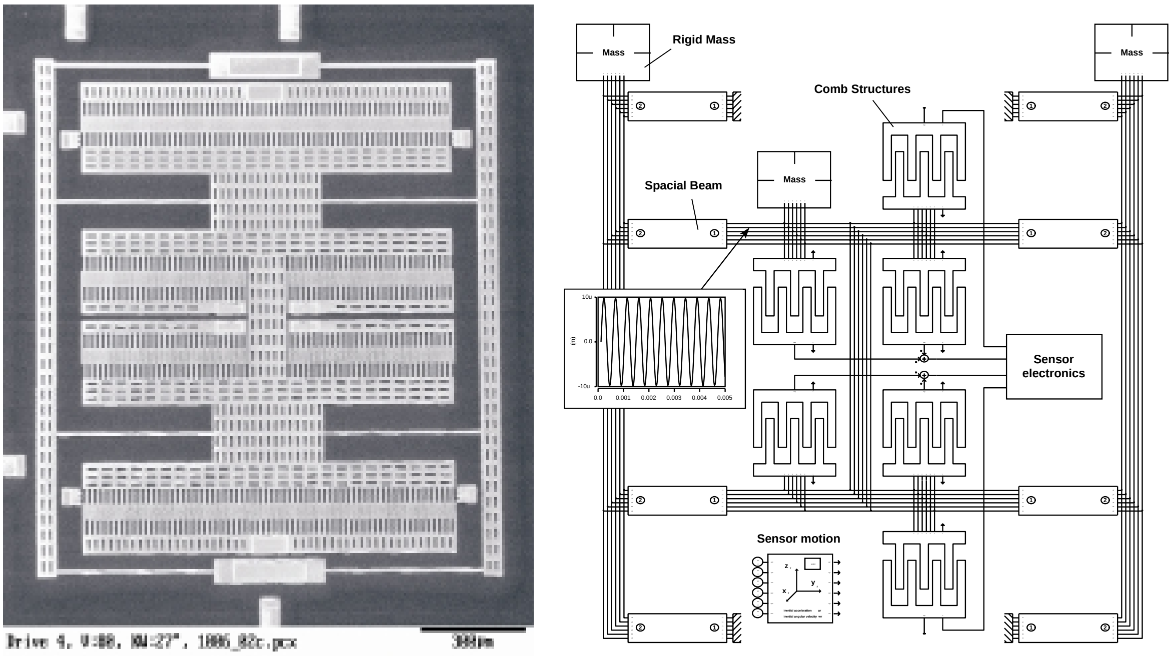

Figure 7: SEM picture and SABER Sketch model of a micro machined resonator

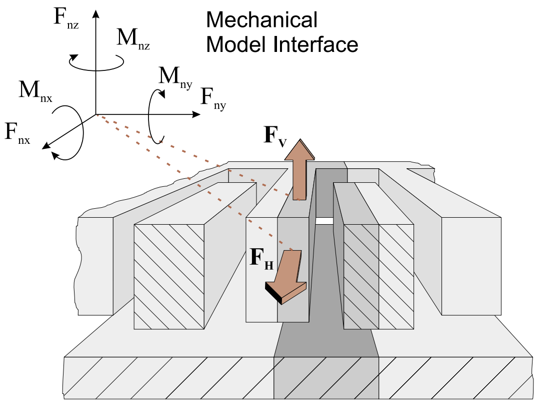

Figure 6: Conversion of the segment forces

comb drives. The internal combs are used for the detection of the motion of the structure. Furthermore, Fig. 7 depicts the SABER Sketch appearance of the corresponding network-model. The mechanical sensor part is modeled using rigid mass and spatial beam components [5]. The conversion of electrical voltage into mechanical forces and torques as well as a position dependent comb capacitance is calculated by the discussed comb models.

CONCLUSION

An analytical model for micro machined comb structures has been presented. The approach, based on an analytical approximation of the 3D-field distribution between the comb fingers, gives an instant answer to geometrical changes like the finger thickness or the gap size between the substrate and fingers. Geometrical changes on the MEMS models can be completed in a few seconds and simulated within minutes.

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