Sensitivity Calculation Model using the Finite-Difference Method

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ABSTRACT

The paper describes an approach to general-purpose design sensitivity analysis for electromagnetic devices.
Micro system technology often requires the assessment of manufacturing techniques or effects of tolerances. Emphasis is therefore put on the adaptability to different requirements, depending on desired accuracy, computational effort and significance. By introducing a distributed sensitivity function, the effect of small contour distortions can be described. The design sensitivity is based on a magnetic double-layer model. It is shown that sensitivity can be expressed in terms of virtual anti-parallel double-layer currents, flowing in a movable contour. The sensitivity is explicitly derived for two-dimensional coordinate systems using the finite-difference method within a commercially available field calculation program. The proposed method is demonstrated by means of an example of two magnetic planes facing each other.

Keywords: Sensitivity analysis, tolerance analysis, finite differences, magnetostatics, global field quantities

INTRODUCTION

Especially in the field of micro system technology the prediction of the effect of manufacturing tolerances increasingly gains importance in order to facilitate the assessment of manufacturing techniques. This consequently amounts to the so-called tolerance analysis.
Sensitivity information is also required in many optimization methods. The determination of the "direction" of optimization steps [1,2] is decisive for those methods to work properly. Thus an efficient sensitivity analysis at moderate accuracy is desirable.
In general, however, field calculation programs, i.e. existing finite-difference and elements codes, are not designed to perform sensitivity analysis to system parameters.
Normally, interest is taken in the optimum design of a specific electromagnetic device. On the other hand, for economic reasons a method that can be adapted to different cases is desirable as well.

The method for discrete sensitivity analysis [3] is very general in that they are applicable to many problems.
However, sensitivity calculation on the discretized equations often requires access to the source code. It is therefore desirable to have a sensitivity evaluation method which is more generally applicable and can be implemented without extensive access to and knowledge of the insides of standard finite-difference codes.
In the continuum approach of sensitivity analysis this aim is achieved by differentiating the variational governing equations before they are discretized [4].

The intention of this paper is to implement shape design sensitivity analysis which takes advantage of existing finite-difference codes in connection with post processing data from one single field calculation of the system being analyzed. It is shown that sensitivity can be expressed in terms of virtual anti-parallel double-layer currents, flowing in a material contour. This may offer a way to meet the issues mentioned above.

SENSITIVITY AND DOUBLE-LAYER MODEL

Sensitivity Formula

Sensitivity analysis defines the relative sensitivity function for time-independent parameters as

$$S_{i,j} = \frac{\partial x_i(\mathbf{p})}{\partial p_j} \quad (1)$$

whereas x_i represents the i-th state variable; p_j is the element j of the parameter vector. Hence the sensitivity is given by the so-called sensitivity matrix **S**, containing the sensitivity coefficients $S_{i,j}$, equation(1).
The direct approach of numerically differentiating by means of numerical field calculation software will lead to diverse difficulties [1,3].
Therefore, some ideas to overcome those problems aim at performing differentiations necessary for sensitivity analysis prior to any numerical treatment. Further calculations are then carried out with a commercially

available field calculation program. Such approaches have already been practiced successfully [5].

In the following sections, it is started from the assumption, that sensitivity can be expressed in terms of virtual surface current densities by means of which an equivalent system can be solved to obtain sensitivity information; this will be proofed.

Magnetic Double Layer

Starting from the relationship of the magnetic intensity boundary condition at the interface between two media with present surface current density and the general magnetic double-layer formulation, given by equations (2,3),

$$\mathbf{H}_1 - \mathbf{H}_2 = \mathbf{K}_S \times \mathbf{n} \quad (2)$$

$$\mathbf{A}_1 - \mathbf{A}_2 = \mu \cdot \mathrm{dl} \cdot \mathbf{K}_D \times \mathbf{n} \quad (3)$$

information on the behavior of the tangential components H_t and A_t is available. In words, two surface current densities with the magnitude $|\mathbf{K}_D|$ at the distance dl (thickness of the magnetic double layer) produce a discontinuity of the magnetic vector potential, whereas the total sum of current densities $\mathbf{K}_S$ produce a discontinuity of the magnetic intensity. The cross product vector of the normal vector $\mathbf{n}$ and the respective current density $\mathbf{K}_S$, $\mathbf{K}_D$ lies in the surface.

Referring to Figure 2, two different surface current densities K_1, K_2, flowing on either side of the interface between two media at the distance dl from each other, can be defined (6). The permeability μ has to be split into two portions, since the double layer stretches across the material contour. So equations (4,5) are modifications of (2,3).

$$H_{1t} - H_{2t} = K_S \quad (4)$$

$$A_{1t} - A_{2t} = \frac{\mu_1 + \mu_2}{2} \cdot K_D \cdot \mathrm{dl}. \quad (5)$$

$$K_1 = \frac{K_S}{2} + K_D \qquad K_2 = \frac{K_S}{2} - K_D \quad (6)$$

Equivalent System

A magnetostatic arrangement can be described by Poisson's equation.

$$\Delta\mathbf{A} = -\mu \cdot \mathbf{J} \quad (7)$$

This equation can formally be differentiated with respect to parameter p_j. By doing so, Poisson's equation is conveyed into Laplace's equation for sensitivity $\mathbf{S}(\mathbf{r},\mathbf{p})$, providing that the current density $\mathbf{J}$ is kept constant, i.e. not depending on the parameter p_j. It should be remarked, that the original current density has now been eliminated. Differential equations of the form

$$\Delta S_j = 0 \quad (8)$$

representing components of the vector Laplace equation are to be solved. In the following, the α component of a cylindrical problem is to be considered. In order to obtain a unique solution, the question for inner boundary conditions raises. The boundary values in question are obviously related to the parameter p_j. Hencefore, these parameters are referred to as movable material contours (r-z cylindrical coordinates) as outlined in Figure 1.

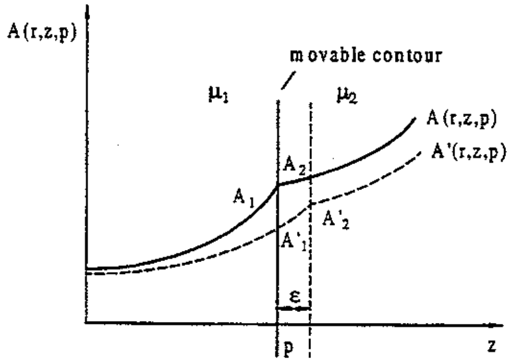

Figure 1. Magnetic vector potential at a movable contour

After virtually moving the contour p by the distance ε, the vector potential at location p+ε of now A'_1/A'_2 must remain continuous. For the α component of A(r,z,p), this can be described by the two-dimensional Taylor series

$$A_1 + \frac{\partial A_1}{\partial p} \cdot \varepsilon + \frac{\partial A_1}{\partial z} \cdot \varepsilon = A_2 + \frac{\partial A_2}{\partial p} \cdot \varepsilon + \frac{\partial A_2}{\partial z} \cdot \varepsilon \quad (9)$$

yielding

$$\frac{\partial A_{1t}}{\partial p} - \frac{\partial A_{2t}}{\partial p} = \frac{\partial A_{2t}}{\partial z} - \frac{\partial A_{1t}}{\partial z}. \quad (10)$$

This structure of relationship applies to all field quantities being continuous at the interface between two materials. However, this is not directly applicable as boundary values. By inspecting equations (4,5) and differentiating with respect to p, one can find

$$\frac{\partial H_{1t}}{\partial p}-\frac{\partial H_{2t}}{\partial p}=Kp_S \qquad (11)$$

$$\frac{\partial A_{1t}}{\partial p}-\frac{\partial A_{2t}}{\partial p}=\frac{\mu_1+\mu_2}{2}\cdot Kp_D\cdot dl. \qquad (12)$$

The surface current densities Kp_S, Kp_D, carrying the dimension A/m^2, are used to define Kp_1, Kp_2 that will serve as input data to the existing finite-difference coding. These current densities must not be mistaken for current densities originating from **J**!

Direct Results

The foregoing provides some direct information on the relationship between the differences of sensitivity and the corresponding continuous field components on the contour. A first estimate of sensitivity may be found by considering the following equations (13-16); especially equation (13) results from (10).

$$\frac{\partial A_{1t}}{\partial p}-\frac{\partial A_{2t}}{\partial p}=B_{1t}-B_{2t} \qquad (13)$$

$$\frac{\partial A_{1t}}{\partial p}-\frac{\partial A_{2t}}{\partial p}=(\mu_1-\mu_2)\cdot H_t \qquad (14)$$

Equation (14) is of special interest for later considerations, since H_t is part of the field computation output data; thus it can be used without any alteration. Referring to equation (10) again, analogue expressions can be derived for the continuous components of the magnetic intensity H_t and flux density B_n. By using $\mathrm{curl}\mathbf{H}=0$ as well as $\mathrm{div}\mathbf{B}=0$, equation (15) and (16) have been formed in a way that only derivatives of continuous field components remain on the right sides.

$$\frac{\partial H_{1t}}{\partial p}-\frac{\partial H_{2t}}{\partial p}=\left(\frac{1}{\mu_2}-\frac{1}{\mu_1}\right)\cdot\frac{\partial B_n}{\partial r} \qquad (15)$$

$$\frac{\partial B_{1n}}{\partial p}-\frac{\partial B_{2n}}{\partial p}=(\mu_1-\mu_2)\cdot\frac{\partial H_t}{\partial r} \qquad (16)$$

Concluding it is to be seen, that only field quantities on the contour need to be taken into account, which can directly be derived from data obtained by the initial field computation performed.
This is essential when proceeding to the numerical implementation of a suitable algorithm.

NUMERICAL APPROACH

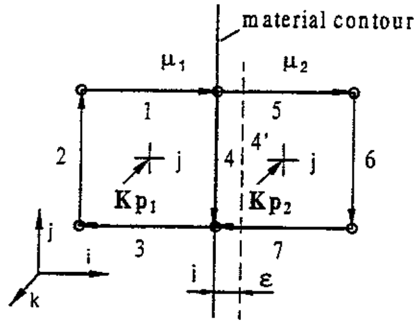

Figure 2. Meshes at the contour

Figure 2 shows a sector of the discretized space of the equivalent system which is sub-divided by the boundary in question, carrying two surface current densities Kp_1, Kp_2 through one mesh on either side. The meshes have the step size of Δz in i direction and Δr in j direction whereas i, j act as local coordinates. The step size is only needed to be equidistant at the contour.
This configuration represents the said magnetic double layer.

Equivalent Current Densities

The boundary conditions obtained earlier (equation (10,14-16)), are to be transformed into the equivalent surface current densities Kp_1, Kp_2.
More thorough investigations revealed that the total current Kp_S is not depending on the mesh size within the given scope of accuracy. The equivalent current densities can now be stated only with data available from the initial field calculation.

$$Kp_1=\frac{\mu_1-\mu_2}{2\cdot\mu_1\mu_2}\cdot\frac{\partial B_n}{\partial r}+\frac{2\cdot(B_{1t}-B_{2t})}{\Delta z\cdot(\mu_1+\mu_2)} \qquad (17)$$

$$Kp_2=\frac{\mu_1-\mu_2}{2\cdot\mu_1\mu_2}\cdot\frac{\partial B_n}{\partial r}-\frac{2\cdot(B_{1t}-B_{2t})}{\Delta z\cdot(\mu_1+\mu_2)} \qquad (18)$$

The structures of equations (17,18) can be compared with the ones given by (6). The first term in (17,18) referes to the sum Kp_S of the virtual current densities (see equation(11,15)), and the second term to the difference Kp_D of those current densities (see equation (10,11,13).

The current density data input to the field calculation program for sensitivity calculation is realized through its standard current input interface.

An interesting aspect of calculating the sensitivity of vector potential by means of this equivalent system is that only linear computations are required. Given that the initial field calculation has properly been performed, the distribution of the (in general) non-linear permeability is known. Only this distribution in the working point is necessary for calculating the equivalent system.

The following features are achieved:
- no change of the existing finite-difference coding
- no re-meshing between calculation steps
- only data required from one single initial field calculation of the original system

Distributed Sensitivity

Apart from calculating a single sensitivity coefficient, one may wish to investigate the influence of not only the entire contour, but also small distortions of it. The following function be called distributed sensitivity:

$$S_{i,j}(r,z,p)=\frac{\partial A(r,z,p)}{\partial p_j}. \qquad (19)$$

Summing up $S_{i,j}(r,z,p)$ will yield the above-mentioned sensitivity coefficient. For the evaluation of, for instance, small irregularities at a contour to be carried out, only the virtual current densities at the specified location are necessary to be known. One run of the field computation in linear mode will provide the desired sensitivity information along the contour as well as in the entire discretized space.

Global Field Quantities

For the design of magnetostatic devices to be evaluated in satisfactory way, it is often sufficient to know the respective global field quantities (force, field energy, inductance). This restriction to global field quantities offers an opportunity to dramatically reducing the set of equations to be solved. While for local field quantities the complete meshed area has to be taken into account, global quantities only require the node positions along the contours to be calculated.

The force on an interface between two magnetic materials can be formulated as the integral of the force density $\mathbf{f}_A$ over the surface **A**.

$$\mathbf{F}_A=\int \mathbf{f}_A dA \qquad (20)$$

The derivative of the force density f_A with respect to a design parameter p becomes in cylindrical coordinates:

$$\frac{\partial f_A}{\partial p}=\frac{\mu_1-\mu_2}{\mu_1\cdot\mu_2}\left(B(r,z,p)\frac{\partial B(r,z,p)}{\partial p}+\mu_1\cdot\mu_2\cdot H(r,z,p)\frac{\partial H(r,z,p)}{\partial p}\right) \qquad (21)$$

Besides the distribution of permeability μ_1, which is known from the initial (non-linear) field computation, only the continuous field components of **B** and **H** are relevant.

This suggests solving the differential equation (8) along the respective contour, saving the computational effort for all other node positions.

This can be done by treating the remaining stripe-shaped region like a so-called sub-region within the entire meshed area (an option provided by the field calculation program) with own boundary values. The boundary conditions along the contour have to be formulated as Neumann boundary values, but care is required because the problem of differences of nearly equal numbers makes itself strongly felt.

There are sufficient boundary values to obtain a unique solution, since Dirichlet boundary values apply to the remaining boundaries. In order to define the Neumann boundary values, additional conditions need to be considered at j position on the boundary. In similar fashion (see Figure 2) to equation (9), with first-order approximation of the derivatives, and equation (22)

$$B_1+\frac{\partial B_1}{\partial p}\cdot\varepsilon+\frac{\partial B_1}{\partial z}\cdot\varepsilon-\frac{\partial B_1}{\partial r}\cdot\frac{\Delta r}{2}=$$
$$B_7+\frac{\partial B_7}{\partial p}\cdot\varepsilon+\frac{\partial B_7}{\partial z}\cdot\varepsilon+\frac{\partial B_7}{\partial r}\cdot\frac{\Delta r}{2}, \qquad (22)$$

equation (23) can be found.

$$\frac{\partial B_{1n}}{\partial p}-\frac{\partial B_{7n}}{\partial p}=\frac{\partial B_{7n}}{\partial z}-\frac{\partial B_{1n}}{\partial z} \qquad (23)$$

Replacing the flux density by the vector potential on the left side and using the slightly modified equation (14) yields the behavior of the α component of vector potential at positions 4 and 6.

$$\frac{1}{r_1}\frac{\partial A_1}{\partial p}-\frac{1}{r_7}\frac{\partial A_7}{\partial p}+\frac{\partial^2 A_1}{\partial p\partial r}-\frac{\partial^2 A_7}{\partial p\partial r}=\frac{\partial B_{7n}}{\partial z}-\frac{\partial B_{1n}}{\partial z} \qquad (24)$$

$$\frac{\partial A_2}{\partial p}-\frac{\partial A_6}{\partial p}=(\mu_1-\mu_2)\cdot H_t \qquad (25)$$

The described procedure has been repeated for the flux density at positions 3 and 5 in analogue manner to have an additional equation for averaging purposes.

$$\frac{1}{r_3}\frac{\partial A_3}{\partial p}-\frac{1}{r_5}\frac{\partial A_5}{\partial p}+\frac{\partial^2 A_3}{\partial p \partial r}-\frac{\partial^2 A_5}{\partial p \partial r}=\frac{\partial B_{5n}}{\partial z}-\frac{\partial B_{3n}}{\partial z} \qquad (26)$$

It is necessary to introduce finite differences at this state, since formulas need to be discretized for implementation. By applying the divergence theorem of the magnetic field in discretized form to the right side of equations (25,26) and collecting terms, the Neumann boundary values are finally obtained in the following form:

$$\frac{\partial A_4}{\partial z}=\frac{(\mu_1+\mu_2)\cdot(4j^2-1)}{32j^3\cdot\Delta z}\cdot\left((2j+1)\cdot H_{j+1}+(2j-1)\cdot H_{j-1}\right)+\frac{(8j^2-1)\cdot\mu_1-\mu_2}{8j^2\cdot\Delta z}\cdot H_j \qquad (27)$$

$$\frac{\partial A_6}{\partial z}=\frac{(\mu_1+\mu_2)\cdot(4j^2-1)}{32j^3\cdot\Delta z}\cdot\left((2j+1)\cdot H_{j+1}+(2j-1)\cdot H_{j-1}\right)+\frac{(8j^2-1)\cdot\mu_2-\mu_1}{8j^2\cdot\Delta z}\cdot H_j \qquad (28)$$

The magnetic intensity in these two equations is meant to be the tangential component of the magnetic intensity on the contour as it comes from the initial field calculation. Since the magnetic intensity on the contour has been used in the foregoing equations (27,28) instead of the ones from both sides, sensitivity computations will yield approximate values that correspond with the analytic solution. This will be reflected in the example below.

EXAMPLE

In order to verify the theoretical findings, several analytically solvable examples have been worked. The corresponding numerical solutions have been obtained from a commercially available field calculation program [6].

For the purpose of better significance, examples with non-symmetric material distribution had been chosen [7].

Let us consider the arrangement in Figure 3. It shows two plates in r-z cylindrical coordinates, facing each other with the distance z_2-z_1 and with a current loop at the coordinate origin.

The boundaries in r direction and the left one in z direction go off to negative or positive infinity; thus all field quantities will vanish there.

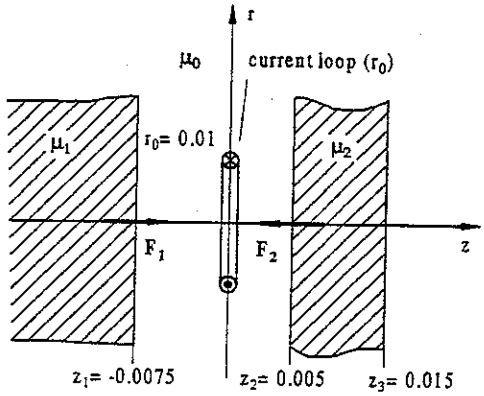

Figure 3. Two-plates arrangement

Contour z_1 is considered movable in z direction, and we are interested in finding the sensitivity of both the vector potential S(r,z) and the force S_{F1} with respect to z_1.

This system has been treated semi-analytically, applying the inverse Fourier transforms numerically to finally obtain the sensitivity of vector potential.

For convenience, the plots of the sensitivity of vector potential have been multiplied with a normalizing factor.

The plot shown in Figure 4 illustrates the kind of results obtained; the derivative of the vector potential with respect to the geometric parameter "contour z_1". The magnetic double layer, located at z_1, is well indicated by the edges. This curve is a sector from the complete 2D solution that has been computed with the virtual double-layer currents according to equations(17,18).

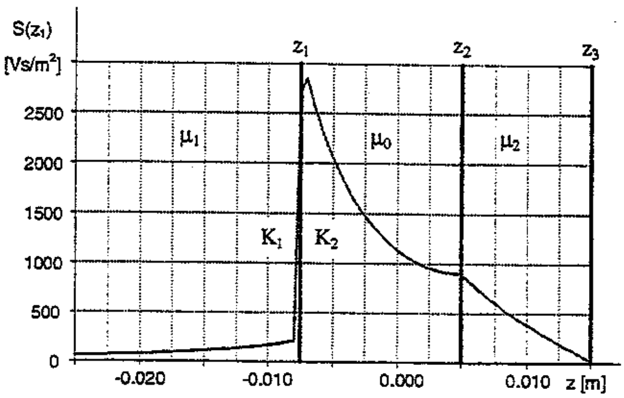

Figure 4. Sensitivity S(r,z) of magnetic vector potential (at r = 0.010 m)

Apart from the regions adjoining the contour (finite thickness of the double layer) the numerically calculated curve comes considerably close to the analytically calculated result (see Figure 5).

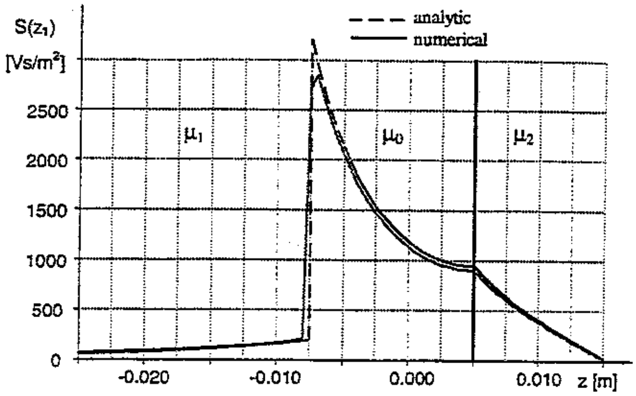

Figure 5. Analytically and numerically calculated sensitivity

If only the sensitivity of the force F_1 is required, one can save computational effort by solving the reduced set of equations concerning the meshes adjoining contour z_1. The plot shown in Figure 6 has been computed that way, applying the Neumann boundary values according to equations (27,28). As already remarked, this result may directly be compared with the respective left and right-hand-sided values obtained analytically.

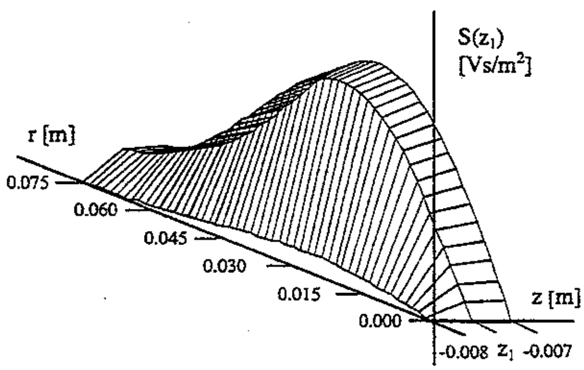

Figure 6. Sensitivity S(r,z) of magnetic vector potential at contour z_1

Accuracy, of course, varies on how many influences had been taken into consideration, especially the order of approximation (only first-order approximation used here). The maximum values from Figure 6 illustration have been gathered and compared with the corresponding analytic values (see Table 1).

Table 1. Comparison of the sensitivity of the normalized vector potential (left and right-hand side) and force F_1

Sensitivity	Analytic	Numerical	Deviation
Left [Vs/m²]	224	290	29%
Right [Vs/m²]	3149	3061	-2.8%
Force [N/m]	132.5	143.9	8.6%

CONCLUSION

An approach to sensitivity analysis concerning several field quantities has been presented. This approach offers a possible way toward a cost-efficient sensitivity and tolerance analysis, to support the design of electromagnetic devices. The method only requires post-processing data, based on one single field calculation of the system in questions, taking advantage of an existing finite-difference coding.
The proposed magnetic double layer model fits linear as well as non-linear cases.
Recently, the method is applied to supporting the design of a real electromagnetically operated bearing system.

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