Simulation of Silicon Piezoresistive Sensors

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ABSTRACT

The temperature influence of the heavy doped silicon piezoresistive sensors is simulated. In this simulation, the density of state functions for impurity band and band edge tail are taken into account for carrier and ionized impurity concentrations, and the Fermi level is determined by appling a Newton iteration scheme for a charge neutrality condition. The piezoresistance factors are calculated as parameters for the impurity concentration and the power law of the total relaxation time on energy. Simulation results for the ionized impurity scattering show that the heavy doped silicon of the donor concentration $10^{17}\ cm^{-3}$ and the acceptor concentration $3 \times 10^{20}\ cm^{-3}$ reduces both the temperature coefficient and the sensitivity of gauge factor by about 10%. We can gain the signal to noise ratio if the temperature coefficient is regarded as noise. Combined with recent result on good linearity, we can show a guideline for the excellent piezoresistive sensors design simulation.

Keywords: piezoresistance, temperature, impurity, DOS, sensor.

INTRODUCTION

We had made a graphical representation of silicon piezoresistive (PR) sensors, in which the PR coefficients were expressed as a function of crystal orientation, impurity concentration and temperature [1], [2]. In the design of PR sensors the heavily doping procedure is commonly adopted in order to surpress the temperature noise. However, practical simulation on this approach has not been exactly investigated yet. Our previous approximate Fermi level expression, in which the Fermi integral and the square-root law for the density of state were used, shows that the deviation of the calculated values from the experimental ones occured for higher impurity concentration. Nowadays, more precise simulation on the heavily doping procedure is required from sensor designers and users.

The purpose of this paper is to investigate the temperature influence of the heavy doped silicon PR sensors by means of computer simulation.

THEORY

The Fermi level is detemined by the charge neutrality condition

$$p(F) + N_D^+(F) = n(F) + N_A^-(F) \tag{1}$$

where $n(F)$, $p(F)$, $N_A^-(F)$, and $N_D^+(F)$ are the total electrons and holes, ionized acceptor and ionized donor concentrations respectively, which are functions of the Fermi energy F and given by the following equations:

$$n(F) = \int_{-\infty}^{\infty} f_0 \rho_n dE \tag{2}$$

$$p(F) = \int_{-\infty}^{\infty} (1 - f_0) \rho_p dE \tag{3}$$

$$N_D^+ = N_D f_D = \frac{N_D}{1 + 2\exp\left(\frac{F - E_D}{kT}\right)} \tag{4}$$

$$N_A^- = N_A f_A = \frac{N_A}{1 + 2\exp\left(\frac{E_A - F}{kT}\right)} \tag{5}$$

where f_0 is the Fermi-Dirac distribution function, E is the carrier energy, k is the Boltzmann constant, T is the temperature, F is the Fermi energy, ρ_n and ρ_p are the density of state functions of n- and p-type materials, N_D and N_A are the donor and acceptor concentrations, and E_D and E_A are the energy levels of donor and acceptor, respectively.

In this computation, equation (1) is solved by means of a Newton-iteration scheme until a consistent solution is reached. The flow-chart is shown in Figure 1. The integrals appeared in the calculations of carrier concentration are same as the Fermi integral except for that the density of state functions are used in stead of the square-root density of states and the limits of the integrations extend to the impurity band energy.

Density of State Function

At high impurity concentrations, the impurity atoms interact with each other, so that the wave functions of their associated electrons are going to overlap, which causes a spliting of the impurity energy levels and results in the formation of impurity band. The shape of this impurity band has been calculated by Morgan [3] as

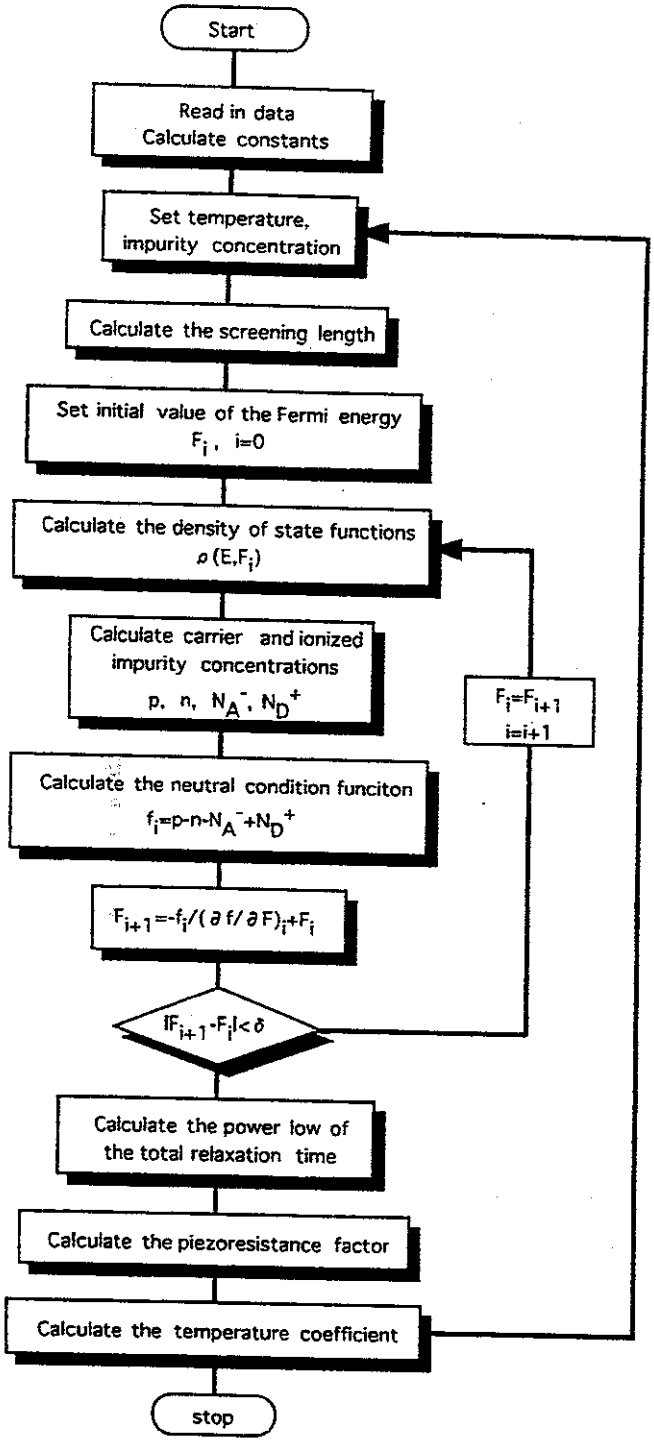

Figure 1: Flow-chart of the computation scheme.

follows,

$$\rho_i(E) = 2N_D(2\pi\sigma_e^2)^{-1/2}\exp\left(-\frac{(E-E_D)^2}{2\sigma_e^2}\right) \quad (6)$$

where σ_e is the effective standard deviation of the impurity band:

$$\sigma_e = (1.03)\sigma\exp(-(11.3806\pi\lambda^3(N_D+N_A))^{-1/2}) \quad (7)$$

where σ is the standard deviation of the impurity band:

$$\sigma = \left(\frac{2\pi q^4}{\epsilon^2}\right)^{1/2}\lambda^{1/2}(N_D+N_A)^{1/2} \quad (8)$$

where q is the electron charge and ϵ is the parmittivity of silicon. And λ is the screening length and characterizes the local potential, which for non-degenerate material where Boltzmann statistics can be applied, reduces to the well-known Debye length $\sqrt{kT\epsilon/q^2(N_D+N_A)}$, while for degenerate material where Fermi-Dirac statistics can be applied, depends on $(N_D-N_A)^{-1/6}$.

The local potential of high impurity concentration makes formation of a band tail, instead of a well-defined band edge. According to the theory of Kane [5] and Bonch-Bruyevich [6] the modified density of state function ρ_c of the conduction band becomes,

$$\rho_c(E) = m_e^{*3/2}(2^{3/2}\sigma)^{1/2}\pi^{-2}\hbar^{-3}y\left(\frac{E}{\sigma\sqrt{2}}\right) \quad (9)$$

where

$$y(x) = \pi^{-1/2}\int_{-\infty}^{x}(x-u)^{1/2}\exp(-u^2)du$$

where $\hbar$ is the Planck's constant divided by 2π and m_e^* is the effective electron mass. The corresponding equations for holes are obtained by replacing m_e^* by m_h^*(Effective hole mass) , N_D by N_A, and E_D by E_A, and modifing direction and reference of the energy definition in the above equations. The density of state functions versus energy are shown in Figure 2. Total density of states function is then given by the envelope of ρ_i and ρ_c.

RESULTS

Fermi Energy

The calculations for Fermi level, the carrier concentrations and the ionized impurity concentrations in a equilibrium state of heavy doped silicon are made when the values of N_D, N_A, and temperature are given as parameters. Some of these computation results are shown in Figures 3 and 4, in comparison with a result of the Fermi integral calculation which we discussed in the previous paper [1] and shows that the material is degenerate in the concentrations higher than about $3\times10^{19}cm^{-3}$. As shown in this figure, Fermi levels calculated by means

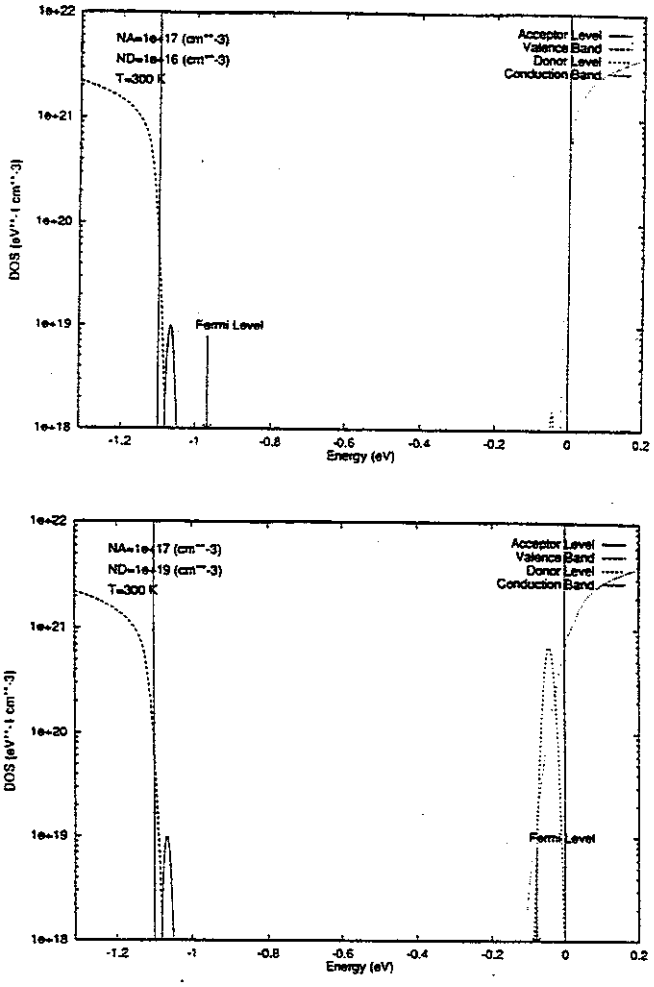

Figure 2: Density of state functions versus energy for impurity compensated silicon ($N_A = 10^{17}\ cm^{-3}, N_D = 10^{16}\ cm^{-3}$ and $N_A = 10^{17}\ cm^{-3}, N_D = 10^{19}\ cm^{-3}$). The energy is chosen zero in the conduction band edge.

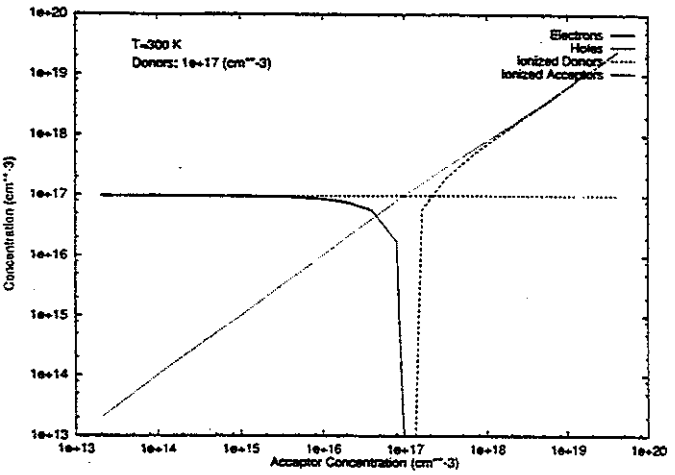

Figure 3: Concentrations of electron, hole, ionized acceptor, and ionized donor versus the acceptor concentration. The donor concentration is $10^{17} cm^{-3}$.

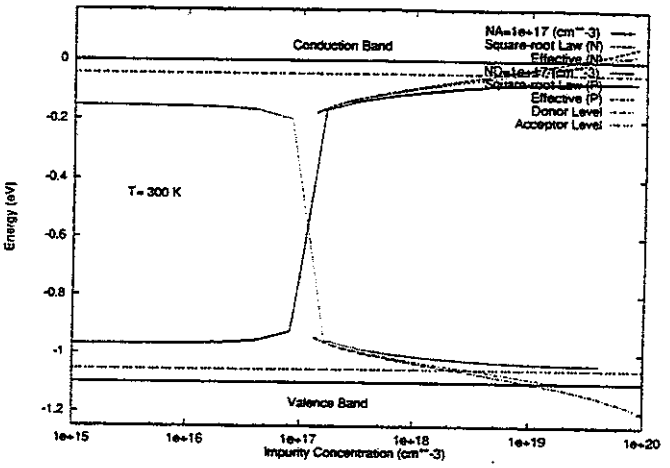

Figure 4: Fermi levels versus impurity concentrations. The acceptor concentration or the donor concentration is $10^{17} cm^{-3}$. Our previous results are denoted as "Square-root Law". The plots denoted by "Effective" are the results by use of the effective band edge.

of the density of state functions are abnormally saturated in higher impurity concentrations. This effect should be attributed to a view that the band edge energy of heavy doped material becomes dim and is no longer same as the well-defined band edge energy. In view of this we re-define the effective Fermi energy such that the calculated major carrier concentration equals the general Fermi integral. The result of this calculation is also shown in Figure 4, in which the band edge energy for the effective Fermi energy is not absolute but is just relative. The effective Fermi energy will be used in the computation here.

Piezoresistance Factor

The first order PR coefficient π is defined by

$$-\frac{\Delta\sigma_c}{\sigma_c} = \pi P \tag{10}$$

where σ_c is the conductivity and P is the stress.

According to the general carrier transport theory, the conductivity depends on the carrier concentration, and consequently depends on the impurity concentration and the temperature. The PR coefficient with impurity concentrations (N_D, N_A) and at temperature T can be written in the form

$$\pi(N_D, N_A, T) = \mathcal{P}(N_D, N_A, T)\pi(300\ \mathrm{K}) \tag{11}$$

where $\mathcal{P}(N_D, N_A, T)$ is the PR factor which is given by the Fermi integral $\mathcal{F}$ as follows,

$$\mathcal{P}(N_D, N_A, T) = \frac{300}{T}\frac{\mathcal{F}_{s-(1/2)}(F/kT)}{\mathcal{F}_{s+(1/2)}(F/kT)}. \tag{12}$$

where s is the exponent appeared in the relaxation time $\tau = \tau_0 E^s$, which is shown in Table 1 for typical scattering mechanisms. Calculation results of the PR factor are shown in Figure 5 for various acceptor concentrations. As shown in this figure, the PR factor does

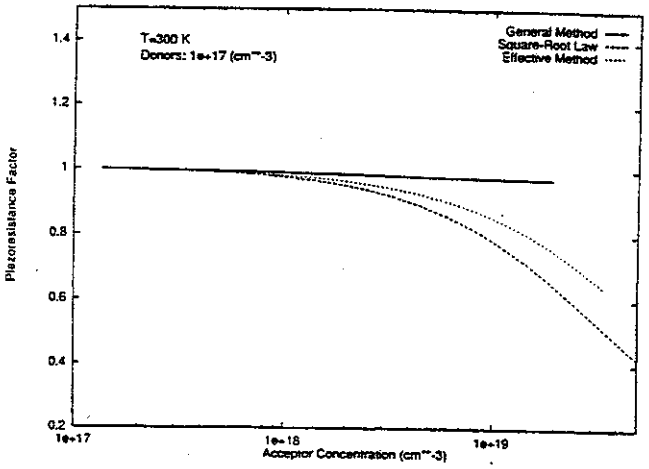

Figure 5: Piezoresistance factor versus acceptor concentration as parameter for scattering mechanism. The donor concentration is $10^{17}\ cm^{-3}$.

not depend on the scattering mechanism and becomes 1 in lower impurity concentrations because Boltzmann distribution function becomes good approximation and $\mathcal{F}_j(F/kT) \simeq \exp(F/kT)$, while in higher impurity concentrations where the material is degenerate, reduces prominently and weekly depends on the scattering mechanism. The simulation result for various scattering mechanisms shows that the PR factor becomes the largest for the impurity scattering.

Temperature Coefficient

The temperature coefficient (TC) is defined as the coefficient when the PR factor is expanded by temperature [7].

$$\mathcal{P}(N_D, N_A, T) \simeq \mathcal{P}(N_D, N_A, 300\,\mathrm{K})(1 + \beta \Delta T) \qquad (13)$$

where $\Delta T = T - 300(\,\mathrm{K})$ and β is the TC. The simulation result of the TC is shown in Figure 6 as parameter for scattering mechanism.

It is known from the general theory of the PR coefficient that the TC is inversively proprtional to $-T$ at lower impurity concentrations. For example, the TC becomes -0.0033 at 300 K. However at higher impurity concentrations, this situation becomes different. As shown in Figure 6, the calculation results show that the TC decreases with the impurity concentration, that is just the reason why we use the heavy doped material in order to surpress the noise attributed to the temperature flickering, also the TC is going to depend remarkably on

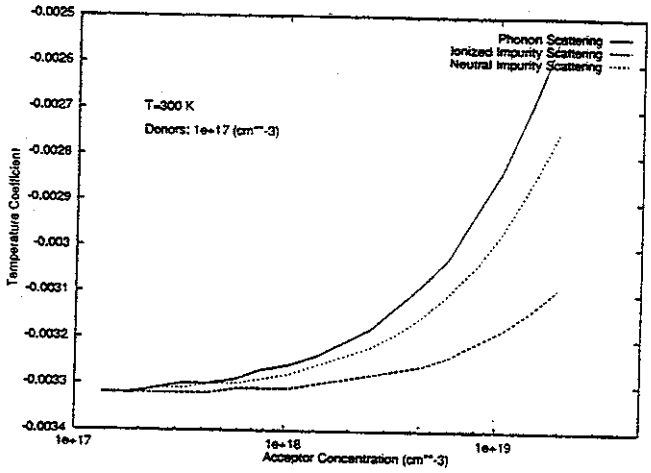

Figure 6: Temperature coefficient versus acceptor concentration as parameter for scattering mechanism. The donor concentration is $10^{17}\ cm^{-3}$.

the scattering mechanism with the impurity concentration. It is worth noiticing that the temperature coefficient for the phonon scattering becomes the smallest among other scattering mechanisms.

DISCUSSION

In our previous study [1], the PR factor for heavily doped p-type silicon was discussed with the experimental values of Mason *et al.* [8] in which the strain gauge factor was used. The strain gauge factor G is commonly used to characterize the strain sensor, which is defined as the fractional change in resistance $\Delta R/R_0$ per unit strain and is given by

$$G = (\Delta R/R_0)/e = 1 + 2\nu + Y\pi \qquad (14)$$

where e is the strain, ν is the Poisson's ratio and Y is the Young's modulus. In this relationship the PR coefficient can be obtained from the experimental gauge values.

Now we shall again discuss with the presently computed PR factors. Reduction ratios of the PR factor, which are defined by the ratios of the PR factor for acceptor concentration of $3 \times 10^{19}\ cm^{-3}$ to that for $5 \times 10^{18}\ cm^{-3}$, are tabulated in Table 1 for various scattering mechanisms, in comparison with the experimental value and our previous result. The present computation results show that the PR factor for the phonon scattering is better agreement with the experimental value than the previous one.

For sensor design, it is desired to decrease the temperature coefficient and increase the PR factor. According to our simulation results the two requirements contradict each other, but if we regard the noise as the temperature coefficient the signal to noise ratio becomes almost same in all cases. Then if we need a smaller temperature coeffieint, a heavily doped material is available

Table 1: Reduction ratios of the PR factor according to heavily doping and scattering exponents of the relaxation time for typical scattering mechanisms.

Scattering Mechanisms		s	Reduction ratios of the PR factor		
			Experiment	Previous	Present
Impurities	Ionized	$\frac{3}{2}$		–	0.91
	Neutral	0		–	0.80
Acoustic phonons	Deformation potential	$-\frac{1}{2}$	(0.63)	0.44	0.73
Optical phonons	Deformation potential	$\sim -\frac{1}{2}$		0.44	0.73

without failing the signal to noise ratio.

In the general computation scheme of the relaxation time it is necessary to consider several scattering processes for a given temperature at the same time. The magnitude of each scattering prosess has the temperature dependency and the power low on energy. The relaxation times for the impurities scatterings can be calculated from N_A^+ and N_D^- obtained by the present computation procedure. The total relaxation time can be obtained by adding the reciprocal relaxation time for each scattering process,

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} \tag{15}$$

where τ_i is the relaxation time of each scattering process. These calculations, however, are needed to determine more accurate PR factor for wide temperature range remain as a future ploblem.

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