Accurate Lumped-Parameter modeling for Dynamic Simulation of Electrostatic MEMS actuators.

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ABSTRACT

This paper introduces a novel approach of accurately modeling electrostatically actuated MEMS structures, using a lumped parameter electrical analogy based on voltage controlled devices in Spice. This analysis takes into account the changes in capacitive force and squeeze film damping because of movement of the structures, by modeling them by voltage controlled devices in Spice. The value of the spring constant and the capacitance, as a function of airgap between parallel plates is derived from commercially available CAD tools by static analysis of the MEMS structures. The speed of actuation and movement of the electrostatic MEMS devices, on applying a pulse voltage is simulated using HSpice. This paper provides a way of co-simulating digital, analog and sensor/actuator parts put together, on a single **System on a chip**.

Keywords: Hspice, Memcad, Static parameter extraction, Dynamic electrical equivalent.

INTRODUCTION

The generation of electrical equivalent of mechanical systems, is a well known concept. A mechanical mass-spring-damper can be modeled in SPICE by making an electrical analogy to an inductor-capacitor-resistor (RLC) circuit. Either the series-connected or the parallel connected RLC circuit can model the second-order mechanical system; which is the basis for all electrostatic MEMS actuators. Most of the literature [1,2] on this concept of an electrical equivalent have been rather simple with constant lumped parameters, which does not accurately model the dynamic behavior of MEMS structures.

This paper gives a unique way of dynamic simulation of the MEMS structures by taking their electrical analogy and simulating it in a circuit simulator. This electrical analogy of the mechanical system takes into account the changes in the value of the parameters due to the movement of the structures, dynamically. This is achieved by modeling the lumped parameters, not as constant resistors, inductors, voltages and capacitors, but by voltage controlled devices. Moreover, the value of the spring constant for various geometry of these MEMS structures is derived from the static simulation of the devices, by commercially available MEMS CAD tools. The capacitance between the two plates in an electrostatically actuated MEMS device, is a function of the distance between the plates and is derived by using Memcad simulator. It is then modeled in Spice using a voltage controlled voltage source. A feedback voltage loop to the input, ensures the dynamic behavior of the MEMS structures. Using this technique, the MEMS devices, arrayed together, and actuated by electronic control circuitry can be co-simulated using a circuit simulator.

MOTIVATION

A beam steering system being designed in North Carolina State University for a Laser Radar system has an array of micro-actuators, controlled by underlying CMOS electronic circuitry. A peripherally mounted electronic drive circuit with closed loop optical or electrical feedback is proposed to support the VLSI-MEMS based reflective beam steering system. The drive circuitry consists of 8-bit D/A converters (14-bit DAC's may be necessary to meet design performance targets) which receive digital input from the system controller. The resulting analog drive voltages are then routed to the corresponding MEMS piston group. Using the technique described in the paper, the MEMS array along with the underlying electronic drive circuitry will be co-simulated using a circuit simulator.

STATIC ANALYSIS

Recently, a number of CAD tools for MEMS have come up with finite element method analysis for the MEMS structures. The basic step in MEMS design starts with a 2D layout creation, with a VLSI or mechanical layout tool. The next step involves the conversion of this 2D structure, to a 3D model where the heights of the devices become apparent. Meshing and finite element modeling is the next step, where the beams and structures are broken down into small parts and are individually treated. A co-simulator simulates the electrostatic and the mechanical properties together iteratively, by balancing the electrical and mechanical forces acting on the device. For calculating the spring constant of a complex MEMS structure shown in Fig 3, the Memcad tool developed by Microcosm Inc., (together with I-DEAS and Fastcap) was used for its static simulation. This simulation result gives the value of the spring constant by applying an incremental force on the device and calculating the deformation of the top plate, which is linear with the force. Similarly, by applying an incremental voltage, the value of capacitance between plates is calculated for different air-gap distances. These values of spring constant and capacitance are used in the next section for dynamic simulation of the structure.

DYNAMIC SIMULATION

The mechanical motion with one degree of freedom, for an electrostatic MEMS actuator is given by the equation:

$$\mathbf{M\ d^2y/d^2t + B(y)\ dy/dt + Ky = F(y,t)}$$

which is the basic second order spring equation and represented by the diagram in Fig 1.

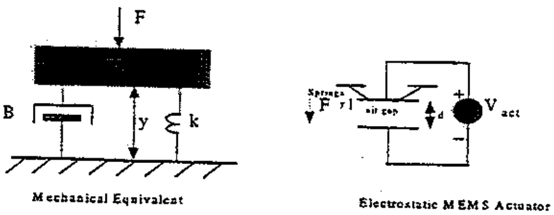

Fig 1: Mechanical Equivalent of an electrostatic MEMS actuator

where m is the mass, k is the spring constant and B denotes the squeeze film damping coefficient and is given by

$$\mathbf{B = K_{Bz}\,(L_x/L_y)\ \ (\mu L_y L_x^3)/(d-y)^3}$$

K_{Bz} (L_x/L_y) is a factor to account for finite plate length as suggested in (1). For a hexagonal plate, a standard K_{bz} value is 0.42. This mechanical mass-spring-damper can be modeled in SPICE by making an electrical analogy to an inductor-capacitor-resistor (RLC) circuit (Fig 2). Choosing a Voltage-Force analogy, the electrical circuit is a simple series RLC circuit, if force and the damping coefficient are constants. A scaling factor may be needed, to keep the electrical impedance values of the RLC circuit from being too small, which may lead to convergence problems in Spice. The original airgap distance is d, movement of the top plate is given by y, C is the capacitance between the plates, F is the actuation force, m is the mass of the body and B is the damping coefficient. The conversion is given below:

$L(m) = m$; $R[B] = B(y)$; $C(k) = 1/k$
$V[F]=F(y)=Cv_{act}^2/2(d-y)$
$V[y]=F(y)/k$

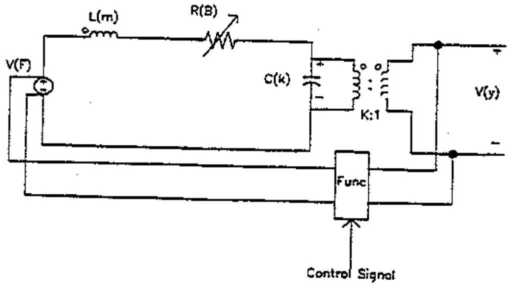

Fig 2: Electrical analogy to a mass-spring-damper system

The voltage source, analogous to the force between the two capacitor plates, changes with the distance between the plates. This has been modeled by a voltage controlled voltage source in Spice. The voltage across the capacitor is analogous to the mechanical force generated by the spring. A step down transformer gives the voltage proportional to the distance between the plates for a linear spring. The input voltage to the circuit, analogous to the capacitive pullin force between the plates, is a function of this stepped down voltage. This is realized by using a polynomial voltage function in Spice. Similarly, for the damping coefficient, the corresponding electrical equivalent resistance is a voltage controlled resistor as it changes with the airgap. With the added value of the spring constant from Memcad, which depends on the geometry of the device, the dynamic simulation is modeled more accurately.

MODEL PARAMETER GENERATION

The MEMS structure, simulated by Memcad and used for dynamic simulation using HSpice, is shown in the Fig 3. This hexagonal structure is used for steering an optical beam by electrostatic actuation. Voltage is applied to the top plate which comes down due to the electrostatic force between the two parallel plates. The three elongated structures attached to the top hexagonal plate, act as springs. The graph of force versus the displacement of the top plate is shown in Fig 4. Assuming a linear spring, the value of the spring constant is the slope of this curve.

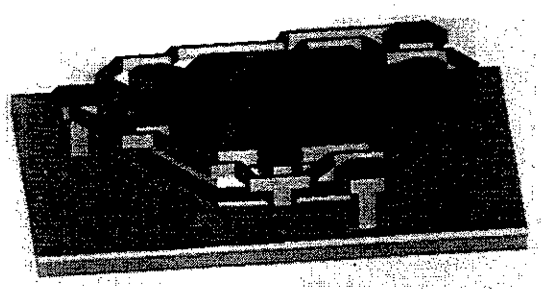

Fig3: Electrostatic Actuator for beam steering

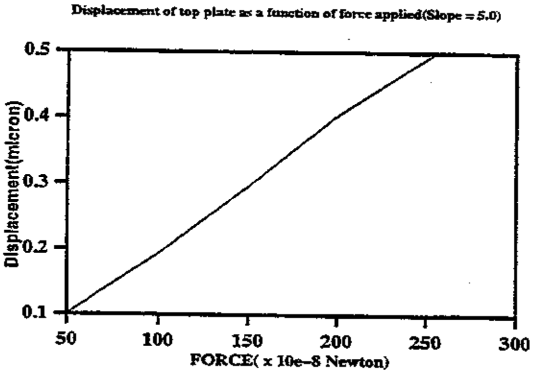

Fig 4: Displacement vs. Force

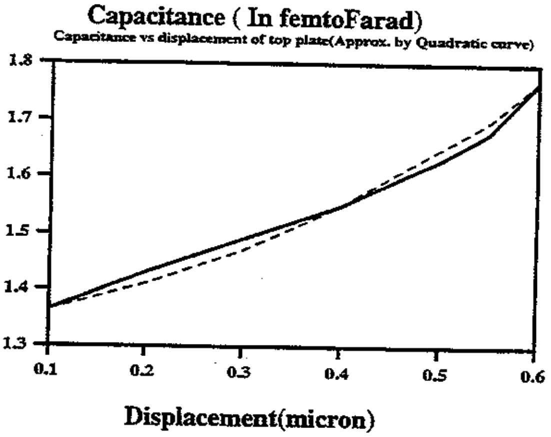

Fig 5: Capacitance vs. Displacement

A scaling factor of 10^6 is used for arriving at proper convergence within Spice. The plot of the capacitance between the two hexagonal plates versus the displacement of the top plate is shown in Fig 5. This plot is arrived at, by using the Memcap electrostatic simulator from Microcosm. The curve is approximated as a quadratic curve (shown in Fig 5 as a dashed line) with $C(y) = 0.0096y^2 + 0.14 \times 10^{-8} y + 1.3404 \times 10^{-14}$. This equation is used to calculate the voltage to the input of the circuit using a polynomial function $F = CV_{act}^2/2(d-y)$.

The area of the top hexagonal plate is 2500 sq. micron and thickness is 4.5 microns. Density of polysilicon used for the top plate is 2.7 gm/cc.

$M = 30.38 \times 10^{-12}$ kg
$\mu = 1.79 \times 10^{-5}$ Pa-s (Viscosity of air at atmospheric pressure)
$B = 5.8 \times 10^{-6} (1 + 3y/d)$ Pa-s (Expanding as a series and taking the first term)
$K = 5.0$ Newton/m (From Slope of Curve in Fig 4)

SIMULATION RESULTS

The spice netlist is shown in Fig 6. Simulation result in Fig 7 shows the actuation of the MEMS device with a voltage of 22V. The top plate comes down by 0.5 micron, which corresponds to 500mV voltage shown in the graph(Scale factor is 10^6). The maximum frequency by which this device can be actuated is 25kHz. Any frequency greater than 25kHz does not allow the top plate to stabilize. Decreasing the damping coefficient results in greater oscillations of the mass body as predicted. Natural frequency of oscillation for this device is 60kHz which is measured with zero damping coefficient as shown in Fig 8.

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* Transient characteristic for a MEMS device FIG 4
* Somsubhro Pal Chaudhury
.option post
.option method =GEAR
* Spring Constant k
.param k = 5.0
Lmass 1 2 30.38uH
GRDampingcoeff 2 3 VCR 4 0 5.8 8.7 IC = 0.0
Cspring 3 0 0.2uF
* ETRANS is the transformer with turns ratio k:1
ETRANS 4 0 TRANSFORMER 3 0 k
* Voltage corresponding to the Capacitance between the
plates[A^2 x C(y)]
EVSCAP 8 0 POLY(1) 4 0 0.0134 0 0.0096 IC = 0.0
* Polynomial function V(9,0) = 156.25xV(8,0) + 78.125
V(8,0)xV(4,0) + * 39.06V(8,0) V(4,0)^2)
EVSPOLY 9 0 POLY(2) 8 0 4 0 0 0.25 0 0 0.125 0 0 0
0.0625 IC = 40.0,20.0
*Voltage input control
VinControl  10 0 PULSE (0 484.0 2us 1us 1us 30u 60u)
Vin  11 0 PULSE (0 22.0 2us 0.1us 0.1us 30u 60u)
* Actuation Voltage of 22V
* Input voltage to LCR circuit V(1,0) = V(8,0) x V(9,0)
EVINPUT 1 0 POLY(2) 9 0 10 0 0 0 0 0 1
* Simulation time and timestep
.TRAN 1us 80u
.END
```

Fig 6: Spice Netlist

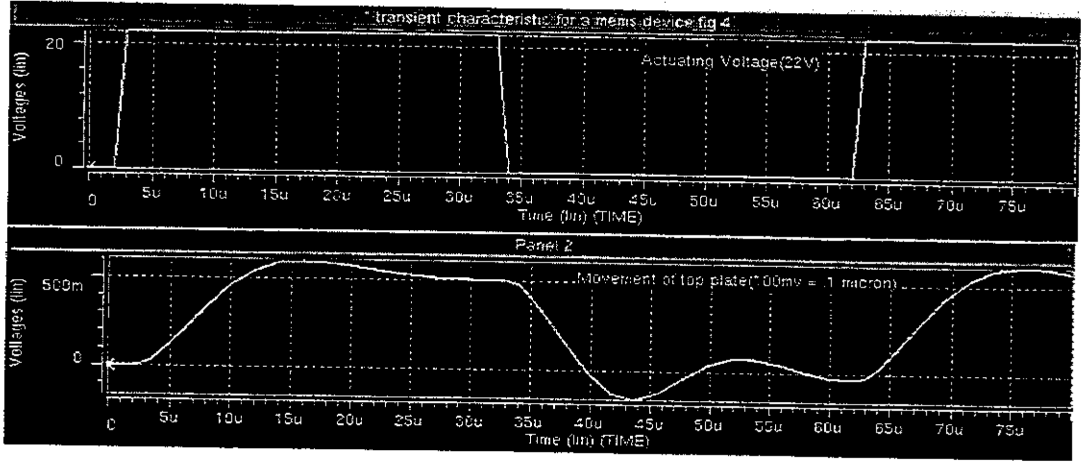

Fig 7: Movement of the top plate with a 22V pulse

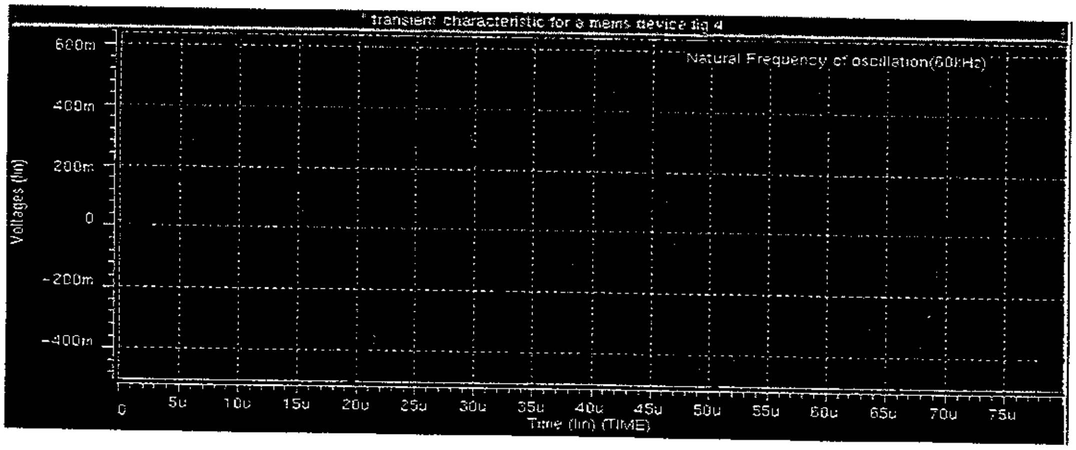

Fig 8: Natural frequency of oscillation

virtually impossible to obtain a frequency response, since many transient simulations would have to be run with different input magnitudes.

CONCLUSIONS

The model of a digital, micromachined accelerometer derived in this work proved to be a valuable tool to predict and evaluate the system performance before implementing the sensor in hardware. SPICE is suitable to implement models of non-electrical components which are described at a behavioral level, consequently, the entire microsensor system can be simulated comprising the sensing element and the interface electronics. The simulation can help to develop alternative interface electronics and control strategies [6]. The approach relies on readily available simulation tools and is relatively easily applicable to other micromachined sensors (e.g. gyroscopes [7]) provided that a lumped parameter mathematical model for the micromachined part is available.

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